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A self-consistent one-dimensional multi-fluid model of the plasma-wall transition in the presence of two species of negatively biased particles

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Abstract. A one-dimensional three-fluid model of the plasma-wall transition in front of a large, negative, planar electrode is developed. The plasma contains one species of singly charged positive ions, Maxwellian electrons and a second group of singly charged negative particles. These particles can be either electrons with temperature different than that of the basic electron population or singly charged negative ions. The case with a second electron population with a high temperature is analyzed and transitions between the low and the high solutions are studied.

1. Introduction

Plasmas that contain more than one population of negatively charged particles are of considerable importance in many areas of plasma physics. Numerous studies on sheath formation in front of a negative electrode immersed in this type of plasmas can be found in the literature due to the great practical interest – see, e.g., the long list of references in [1] and [2]. A common approach in these studies is that the positive ions are treated by fluid equations, while for the negative particles the Boltzmann relation is assumed [2]. In this work, however, the electrons and the second negative particle species are also treated by their own set of continuity and momentum exchange equations.

2. Model

It is assumed that the continuity equations and equations of motion are valid for all three species of charged particles:

$$\frac{\partial n_i}{\partial t}+\nabla\cdot\left(n_i\mathbf{u}_i\right)=S_i,\quad \frac{\partial n_e}{\partial t}+\nabla\cdot\left(n_e\mathbf{u}_e\right)=S_e,\quad \frac{\partial n_2}{\partial t}+\nabla\cdot\left(n_2\mathbf{u}_2\right)=S_2, \tag{1}$$

$$m_i n_i\left(\frac{\partial \mathbf{u}_i}{\partial t}+\left(\mathbf{u}_i\cdot\nabla\right)\mathbf{u}_i\right)=n_i e_0\mathbf{E}-\nabla p_i+\mathbf{A}_i-m_i\mathbf{u}_i S_i, \tag{2}$$

$$m_e n_e\left(\frac{\partial \mathbf{u}_e}{\partial t}+\left(\mathbf{u}_e\cdot\nabla\right)\mathbf{u}_e\right)=-n_e e_0\mathbf{E}-\nabla p_e+\mathbf{A}_e-m_e\mathbf{u}_e S_e, \tag{3}$$

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$$m_2 n_2 \left(\frac{\partial \mathbf{u}_2}{\partial t} + \left(\mathbf{u}_2 \cdot \nabla \right) \mathbf{u}_2 \right) = -n_2 e_0 \mathbf{E} - \nabla p_2 + \mathbf{A}_2 - m_2 \mathbf{u}_2 S_2 . \tag{4}$$

The meaning of the symbols is the following: m_i is the ion mass, m_e is the electron mass, m_2 is the mass of the second negatively charged particle species, n_i, n_e and n_2 are the respective particle densities, $\mathbf{u}_i$, $\mathbf{u}_e$ and $\mathbf{u}_2$ are the flow velocities, e_0 is the elementary charge, S_i, S_e and S_2 are the source terms, discussed below, $\mathbf{A}_i$, $\mathbf{A}_e$ and $\mathbf{A}_2$ are collision terms. In this work the elastic collisions are neglected. The electric field $\mathbf{E}$ is related to the potential Φ by $\mathbf{E} = -\nabla\Phi$ and the potential profile $\Phi(\mathbf{r})$ is determined by the Poisson equation:

$$\nabla^2 \Phi = -\frac{e_0}{\varepsilon_0} \left(n_i \left(\mathbf{r} \right) - n_e \left(\mathbf{r} \right) - n_2 \left(\mathbf{r} \right) \right). \tag{5}$$

Here ε_0 is the permittivity of the free space. Isothermal flow of ions, electrons and second species of negative particles is assumed and the gradient pressure terms are expressed using the ideal gas law:

$$\nabla p_i = kT_i \nabla n_i, \ \nabla p_e = kT_e \nabla n_e, \ \nabla p_2 = kT_2 \nabla n_2 . \tag{6}$$

Here k is the Boltzmann constant.

The source terms S_i, S_e and S_2 give the difference between the number of created and annihilated charged particles of the respective species per unit volume and per unit time. They can be functions of space (and time) and in models, like the one presented in this work, it is usually assumed that they are given functions of space. Ionization and annihilation can have various physical mechanisms. In this work, it is assumed that the source terms are constants independent of the space coordinates. Of course, the consistency of the model requires that $S_i = S_e + S_2$ since in the steady state the same number of positive and negative particles must be created and annihilated in any selected unit volume. The source terms are therefore written in the following form:

$$S_i = \frac{\alpha}{\tau}, \ S_e = \frac{\beta}{\tau}, \ S_i = \frac{\gamma}{\tau}. \tag{7}$$

Here τ is effective time between two consecutive creations of a charged particle, while α, β and γ give the number of created particles of respective species in unit volume. Note that α, β, γ, and τ are all positive constants and their values are such that the subtraction of annihilated particles from the created particles has already been taken into account. Steady state is assumed, so all partial derivatives over time are neglected. It is assumed that on one side the plasma is bounded by a large planar electrode (collector), which is perpendicular to the x axis. The model is one-dimensional, so gradient and Laplace operators are replaced by

$$\nabla \to \frac{d}{dx}, \ \nabla^2 \to \frac{d^2}{dx^2}, \tag{8}$$

and the electric field has one component only given by:

$$E = -\frac{d\Phi}{dx}. \tag{9}$$

When (6) – (9) are taken into account, and the collision terms $\mathbf{A}_i$, $\mathbf{A}_e$ and $\mathbf{A}_2$ are neglected, equations (1) – (5) are written in the following form:

$$\frac{d}{dX}\left(N_i V_i \right) = \varepsilon s_i, \ \frac{d}{dX}\left(N_e V_e \right) = \varepsilon s_e, \ \frac{d}{dX}\left(N_2 V_2 \right) = \varepsilon s_2, \tag{10}$$

$$N_i V_i \frac{dV_i}{dX} = -N_i \frac{d\Psi}{dX} - \Theta \frac{dN_i}{dX} - \varepsilon V_i s_i, \tag{11}$$

$$N_e V_e \frac{dV_e}{dX} = \frac{N_e}{\mu} \frac{d\Psi}{dX} - \frac{1}{\mu} \frac{dN_e}{dX} - \varepsilon V_e s_e, \tag{12}$$

$$N_2 V_2 \frac{dV_2}{dX} = \frac{N_2}{\xi}\frac{d\Psi}{dX} - \frac{\Omega}{\xi}\frac{dN_2}{dX} - \varepsilon V_2 s_2, \tag{13}$$

$$\frac{d^2\Psi}{dX^2} = N_e(X) + N_2(X) - N_i(X). \tag{14}$$

If one is interested only in the quasi-neutral pre-sheath region, the Poisson equation (14) is replaced by the following neutrality condition:

$$N_e(X) + N_2(X) - N_i(X) = 0. \tag{15}$$

The following variables have been introduced:

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT_e}{n_0 e_0^2}},\ c_0 = \sqrt{\frac{kT_e}{m_i}},\ L = c_0\tau,\ \varepsilon = \frac{\lambda_D}{L},\ \mu = \frac{m_e}{m_i},\ \xi = \frac{m_2}{m_i}, \tag{16}$$

$$N_i = \frac{n_i}{n_0},\ N_e = \frac{n_e}{n_0},\ N_2 = \frac{n_2}{n_0},\ s_i = \frac{\alpha}{n_0},\ s_e = \frac{\beta}{n_0},\ s_2 = \frac{\gamma}{n_0},\ \Psi = \frac{e_0\Phi}{kT_e}, \tag{17}$$

$$V_i = \frac{u_i}{c_0},\ V_e = \frac{u_e}{c_0},\ V_2 = \frac{u_2}{c_0},\ \Theta = \frac{T_i}{T_e},\ \Omega = \frac{T_2}{T_e},\ X = \frac{x}{\lambda_D}. \tag{18}$$

Here n_0 is the plasma density in the unperturbed region far from the collector and c_0 is called the normalizing velocity; it is not the same as the ion sound velocity c_S, which is in normalized units (16) – (18) given by [1]:

$$V_S = \frac{c_S}{c_0} = \sqrt{\vartheta + \Theta}, \tag{19}$$

where ϑ is the "screening temperature" normalized to T_e and is given by [3,1]:

$$\vartheta = \frac{N_e + N_2}{\frac{dN_e}{d\Psi} + \frac{dN_2}{d\Psi}}. \tag{20}$$

3. Results

In the top graphs of figure 1, solutions of the system (10) – (14) are shown for the following parameters: $\mu = \xi = 1/1836$ (protons), $\Omega = 25$, $\Theta = 0$, $s_i = s_2 = 1$, $s_e = 0$, $\varepsilon = 10^{-5}$; and boundary conditions, given at $X = 0$, $N_i(0) = 1$, $N_1(0) = 0.736$, $N_2(0) = 0.264$, $V_i(0) = 10^{-7}$, $V_e(0) = V_2(0) = 0$, $\Psi(0) = 0$ and $d\Psi/dX(0) = 0$. The selection of the parameters is not motivated so much by some specific physical situation, but rather by our intention to illustrate some properties of the model. For example, in a recent paper [4], Langmuir probe measurements in COMPASS tokamak and TJ-II stellarator have been presented, where it has been shown that in both machines in the vicinity of the last closed flux surface the electron distribution function is bi-Maxwellian with the higher electron temperature 3 – 5 times larger than the lower one and the density of the lower electron temperature population several times larger than the density of the high temperature electron population. The system (10) – (14) is integrated in the positive X direction towards the collector. The system (10) – (14) has three singularities [5]. The integration of the system (10) – (14) breaks down when either $V_i(X)$ drops below $\sqrt{\Theta}$, or $V_e(X)$ exceeds $\mu^{-1/2}$, or $V_2(X)$ exceeds $\sqrt{\Omega/\xi}$. In the case shown in Fig. 1, the last condition is fulfilled. In plot (*a*) the potential profile $\Psi(X)$ is shown, in graph (*b*) density profiles $N_i(X)$, $N_1(X)$ and $N_2(X)$ are displayed, in graph (*c*) the profile of the screening temperature $\vartheta(X)$ is presented and in graph (*d*) the profiles of the ion velocity $V_i(X)$ and ion sound velocity $V_S(X)$ are shown. All the profiles exhibit oscillations. Because of this, the ion velocity $V_i(X)$ and ion sound velocity $V_S(X)$ profiles have many intersections and it is not possible to say which of these points represents the sheath edge. Thus, the system (10) – (13) and (15) is solved with the same parameters

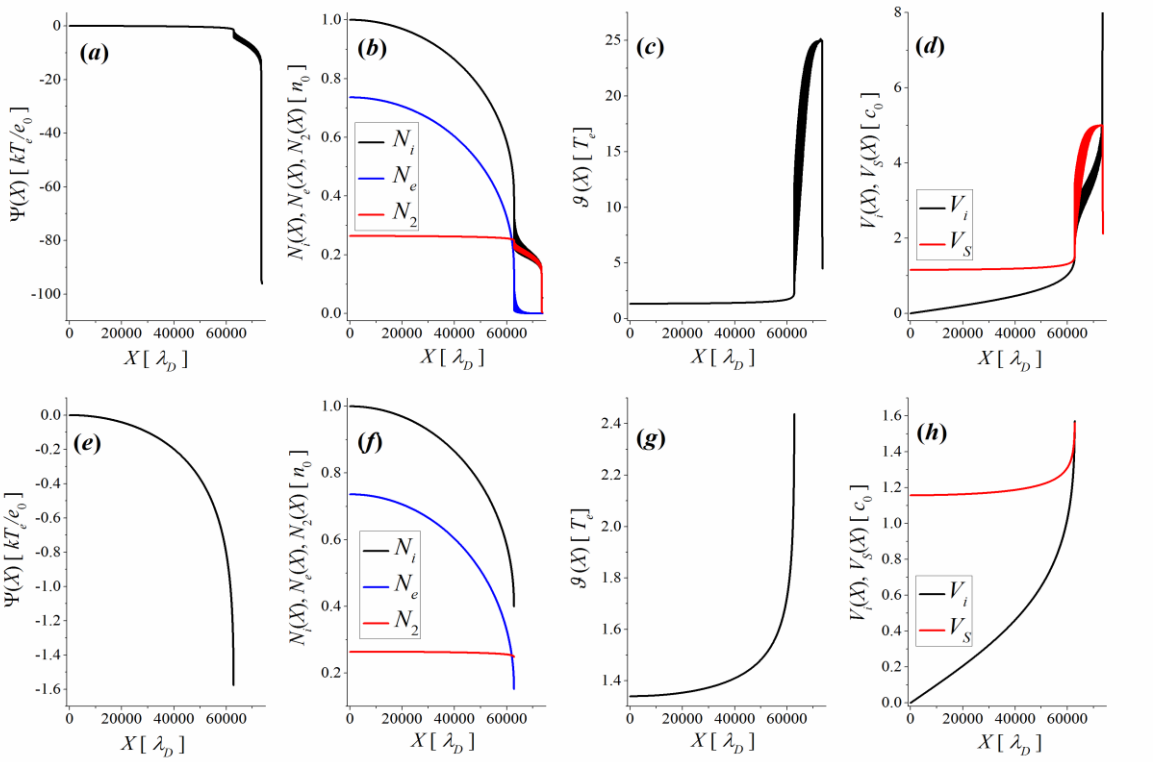

Figure 1. Solutions of the system (10) – (14) in the top graphs and of the system (10) – (13), (15) in the bottom graphs. Parameters and boundary conditions are the same in both cases: $\mu = \xi = 1/1836$, $\Omega = 25$, $\Theta = 0$, $s_i = s_2 = 1$, $s_e = 0$, $\varepsilon = 10^{-5}$, $N_i(0) = 1$, $N_1(0) = 0.736$, $N_2(0) = 0.264$, $V_i(0) = 10^{-7}$, $V_e(0) = V_2(0) = 0$, $\Psi(0) = 0$ and $d\Psi/dX(0) = 0$.

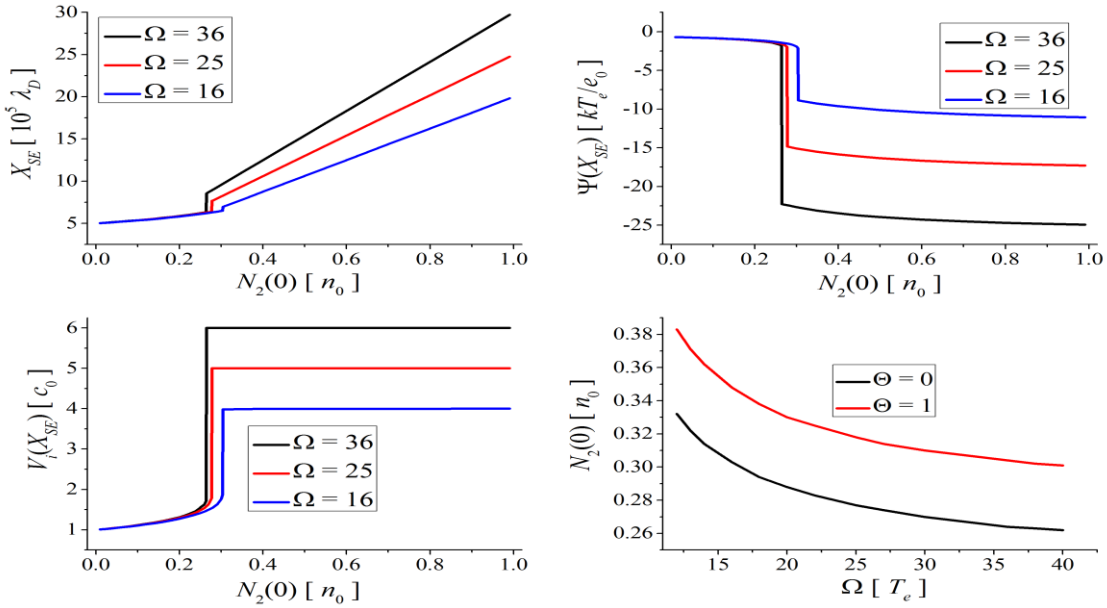

Figure 2. The sheath edge position X_{SE}, the respective potential $\Psi(X_{SE})$ and the ion velocity at the sheath edge $V_i(X_{SE})$, obtained from the system (10) – (13) and (15), plotted versus $N_2(0)$ at three different values of Ω. In the bottom right plot $N_2(0)$, where the jump between the low and the high solution occurs, is plotted versus Ω for two different ion temperatures Θ.

and boundary conditions – bottom plots (*e*) – (*h*). The system (10) – (13) and (15) also has several singular points. Their analysis is beyond the scope of this work. Here we are only interested in the singularity of the system (10) – (13) and (15), which occurs when the Bohm criterion [6], $V_i = V_S$, is fulfilled – plot (*h*). The position, where this occurs, is the sheath edge and is labeled by X_{SE}.

Figure 2 presents the sheath edge position X_{SE}, the respective potential $\Psi(X_{SE})$ and the ion velocity at the sheath edge $V_i(X_{SE})$, obtained from the system (10) – (13) and (15), plotted versus $N_2(0)$ at three different values of Ω. The other parameters and boundary conditions are: $\mu = \xi = 1/1836$, $\Theta = 0$, $s_i = s_2 = 0.1$, $s_e = 0$, $\varepsilon = 10^{-5}$, $N_i(0) = 1$, $N_1(0) = 1 - N_2(0)$, $V_i(0) = 10^{-7}$, $V_e(0) = V_2(0) = 0$, $\Psi(0) = 0$ and $d\Psi/dX(0) = 0$. All the curves exhibit sharp jumps at the transition between the "low" and the "high" solution. In the bottom right plot $N_2(0)$, where the jump between the low and the high solution occurs, is plotted versus Ω for two different ion temperatures Θ.

4. Conclusions

A three-fluid one-dimensional model of the plasma-wall transition in front of a planar negative electrode immersed in plasma that contains two species of negatively biased particles is presented. The case with two-temperature electrons is analyzed. Depending on the ratio between the densities and temperatures of the two groups of electrons, the system of equations (10) – (14) can predict multivalued Bohm criterion, i.e. several positions and potentials of the sheath edge. But if the Poisson equation (14) is replaced by the neutrality condition (15), the integration of the system (10) – (13) with (15) in most cases breaks down at the sheath edge, where the plasma neutrality breaks down. The model can also be used for the analysis of plasma that contains electrons and singly charged negative ions.

Acknowledgements

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