Journal of Physics: Conference Series **698** (2016) 012023 doi:10.1088/1742-6596/698/1/012023

Correlation Dimension in Two-Dimensional Disordered Systems with Rashba Spin-Orbit Coupling

E M Hernández, J A Otero and F Delgado

Departamento de Física y Matemáticas, Escuela de Diseño, Ingeniería y Arquitectura, Tecnológico de Monterrey, Campus Estado de México, Atizapán, Estado de México, CP. 52926, México.

E-mail: emcooper@itesm.mx, j.a.otero@itesm.mx, fdelgado@itesm.mx

Abstract. The metal-insulator phase transition that arises in the Integer Quantum Hall Effect has been characterized through the multifractal nature of extended states near the center of the Lowest Landau Level. In this work, we obtain numerical solutions for the one-electron Hamiltonian with disorder, where the correlation dimension of extended states in the first two Landau Levels is obtained, by taking into account the Rashba spin-orbit coupling in the Hamiltonian. Although, spin-orbit coupling at moderate field intensities has been determined experimentally, there is no theoretical evidence for the nature of the transition in this case. The correlation dimension of extended states for the resolved spin levels is obtained, and within the statistical error, it is found that the Rashba Hamiltonian in presence of disorder, belongs to the same universality class of spin unresolved systems.

1. Introduction

After the experimental confirmation of the Integer Quantum Hall Effect (IQHE) by Klaus Von Klitzing [1], the main concern was to find a localization theory for two dimensional electron systems (2DES) in presence of disorder. The general idea was that the origin of localized states, responsible for the plateau regions in the Hall resistance ρ_{xy}, was due to the presence of fixed impurities in the semiconductors used to host the 2DES. Unfortunately, when impurities are included in the physical model, an exact analytic solution becomes extremely difficult. With numerical simulations it is possible to find a solution to the one-electron Hamiltonian with disorder and understand the electronic transport in the 2DES, as a phase transition between localized and extended states. The obtained results from the numerical models proved to be successful, since they were able to predict localization, and find the critical exponents [2, 3] of the localization-delocalization (LD) transition, which were experimentally confirmed in the late 80's [4], and early 90's [5, 6], from temperature and sample size scaling experiments.

There is a major interest to study 2DES with zero field spin splitting, where it is possible to control the spin-orbit coupling (SOC) parameter by tuning the gate voltage. The source of the splitting energy can be attributed to the coupling between the electron's spin magnetic moment with a magnetic field induced by the interfacial electric field. The splitting energies, corresponding to the Rashba SOC, have been observed in several experiments [7–10], by studying

the beating patterns in the Shubnikov de Hass (SdH) oscillations at relatively small magnetic fields. The SOC inside hererostructures, can be tuned by applying different gate voltages; which is very appealing for spintronic applications [11, 12]. From the theoretical point of view, the dependence of the critical exponents in the LD transition and multifractal spectra on the coupling parameter, has not been addressed. In this work, we study the nature of the phase transition on 2DES with Rashba SOC, in presence of disorder. We found extended and localized states within each spin resolved level for different SOC strengths. Even though, the Rashba SOC term in the Hamiltonian, couples adjacent Landau levels; we are able to obtain for different SOC parameters, the correlation dimension of extended states near the center of each spin split level. We apply multifractal analysis to the extended states [13–18], in order to determine the critical behavior of states near the center of each resolved level; and found that, for different SOC strengths, the correlation dimension does not change whithin the statistical error. This work is organized as follows: in section 2 we offer a brief description of the Rashba SOC term and the one-electron Hamiltonian in presence of disorder. In section 3, we introduce the numerical model for disorder and the resulting matrix elements of the Hamiltonian when we take into account the SOC term, in section 4 we present the results obtained from the numerical simulations for several SOC strengths and finally; in section 5, the conclusions are presented.

2. Problem statement

In 2DES formed in inversion layers, when a uniform magnetic field is applied in a direction perpendicular to the interface, the kinetic energy of the electrons is fixed into a series of Landau levels. The two dimensionality and freezing of the kinetic energy due to the magnetic field, play a fundamental role in the electronic transport properties of the system when disorder and electron interactions, are brought into the picture. In the Landau gauge where $\vec{A} = (0, xB)$, the two dimensional Schrödinger equation for the one-electron Hamiltonian, in absence of disorder, can easily be solved. The magnetic field gives rise to Landau levels $E_n = (n + \frac{1}{2})\hbar\omega_c$, fixing the kinetic energy at these values, and giving rise to highly degenerate levels with $N_\phi = A/2\pi l^2$ states, where A is the system size of area $A = L_x L_y$, $\omega_c = eB/m$ is the cyclotron frequency and $l = \sqrt{\hbar/eB}$ is the magnetic length or cyclotron radius.

The Hamiltonian must take into account the interactions between electrons and the impurities within the heterostructure. When disorder is brought into the picture, the problem becomes more complex because many body interactions, between the electron and impurities, must be taken into account. Early numerical models were focused on the microscopic characteristics of disorder; for example, scattering from neutral impurities can be considered with short range interactions [19] in which the cyclotron radius is larger than the range of these potentials. A more general model for disorder was introduced by Huckestein and Kramer [20] where the potential is assumed to have a normal probability distribution:

$$P[V] = Ne^{-\frac{1}{2V_0^2}\int d^2r[V(\vec{r})]^2} \tag{1}$$

where V_0 represents the disorder strength. On highly doped heterostructures, where the mean free path is small enough, the electron-electron interactions can be neglected or treated as a perturbation of the disorder. In this situation, the physical behavior of the system can be described by the one particle Hamiltonian:

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + V(\vec{r}) \tag{2}$$

where the first term, is the kinetic energy of the electron that is being fixed by the applied magnetic field, and the second term represents the disorder potential with the probability

distribution given by equation (1). The effect of disorder is to break the degeneracy of each Landau level, which is observed as a broadening in the density of states (DOS). Equation (2) has been solved numericaly for magnetic fields where $\hbar\omega_c \gg V_0$, which is the lowest Landau level (LLL) approximation. By averaging the results over different realizations of disorder, it is possible to explain the phase transition that takes place between adjacent plateaus in the Hall resistance, through a multifractal analysis of the extended states near the center of the LLL.

In this work, we also consider a SOC term in the Hamiltonian. A surface electron will see in it's rest frame, an effective magnetic field, due to it's relative motion to the electric field at the interface of the inversion layer. On a free 2DES, the interaction energy between the spin of the electron and it's orbital motion expressed in terms of the Pauli matrices; known as the Rashba Hamiltonian, is given by [21]:

$$H_R = \alpha(\vec{p} \times \vec{\sigma}) \cdot \hat{z} \tag{3}$$

where α is linearly proportional to the electric field and provides the spin-coupling parameter which can be tuned through the gate voltage. From the experimental point of view, the zero field splitting energy is determined through extrapolations of the low field behavior of the SdH oscillations. Therefore, one way to study the SOC, is to solve the Rashba Hamiltonian when a external magnetic field is being applied. In this situation, the electron's spin will interact with the external magnetic field as well, giving rise to a Zeeman Hamiltonian which is proportional to the field's intensity. Experiments where the SOC term is dominant, are performed on magnetic field intensities in which the Zeeman splitting can be neglected. In the presence of an external magnetic field, the kinetic momentum changes to $\vec{\Pi} = \vec{p} + q\vec{A}$ and the Hamiltonian transforms to:

$$H = \frac{1}{2m}\vec{\Pi}^2 + \alpha(\vec{\Pi} \times \vec{\sigma}) \cdot \hat{z} + \frac{g}{2}\mu_B B\sigma_z \tag{4}$$

where μ_B is the Bohr's magneton $\mu_B = |e|/2m$. Due to the external magnetic field, we can use creation and annihilation operators to find the solution to equation (4) in the Landau representation [21]:

$$E_0 = \frac{\hbar\omega_c}{2} \quad \text{, and} \quad E_n^{\pm} = \hbar\omega_c n \pm \sqrt{2n\alpha^2 m\hbar\omega_c + \frac{1}{4}(\hbar\omega_c + g\mu_B B)^2} \tag{5}$$

By considering the SOC of the Rashba type in equation (2), we obtain the one-electron Hamiltonian:

$$H = \frac{1}{2m}\vec{\Pi}^2 + \alpha(\vec{\Pi} \times \vec{\sigma}) \cdot \hat{z} + V(\vec{r}) \tag{6}$$

which for moderate magnetic field intensities, the Zeeman term is much smaller than the SOC term, $g \ll \alpha^2 m\hbar$.

3. Numerical model

In our simulations we use a Gaussian White Noise Potential (GWNP) as the source of disorder, by working equation (1) in momentum space:

$$P[V] = \frac{1}{\sqrt{2\pi}s} e^{-\frac{|v(\vec{q})|^2}{2s^2}}, \quad \text{with} \quad \langle v(\vec{q})v(\vec{q}\,')\rangle = s\delta_{\vec{q}+\vec{q}\,'}, \tag{7}$$

where $V_0 = s$. In this case, the potential $V(\vec{r})$ can be calculated as the Fourier transform of $v(\vec{q})$:

$$V(\vec{r}) = \frac{1}{A}\sum_{\vec{q}} v(\vec{q}) e^{\imath\vec{q}\cdot\vec{r}} \tag{8}$$

The key to find the solutions of equation (2), is to use the states in absence of disorder as a basis to obtain the matrix elements of $V(\vec{r})$. This is possible by using a linear combination of the Landau states, in order to build a basis with toroidal boundary conditions (periodic functions in x and y):

$$\phi_{n,m}(x,y) = \frac{1}{L_y l\sqrt{\pi}} \sum_{s=-\infty}^{\infty} e^{\frac{\imath x_{m,s} y}{l^2}} e^{-\frac{1}{2l^2}\bar{x}^2} \chi\left(\frac{\bar{x}}{l}\right) \tag{9}$$

where $x_{m,s} = 2m\pi l^2/L_y + sL_x$, $\bar{x} = x - x_{m,s}$ and a system of units where $\hbar\omega_c = 1$ is being used. The functions $\chi_n(\bar{x}/l) = \mathrm{H}_n(\bar{x}/l)/\sqrt{2^n n!})$, where $\mathrm{H}_n(\bar{x}/l)$ are the Hermite polynomials of degree n. With this basis, and generating uncorrelated complex random variables $v(\vec{q})$, the matrix elements of disorder can be obtained. In the LLL approximation, where $V_0 \ll \hbar\omega_c$, each level can be studied independently. In any LL, the resulting matrix elements of the term $e^{\imath\vec{q}\cdot\vec{r}}$, appearing in equation (8), are given by:

$$\langle m,n|e^{\imath\vec{q}\cdot\vec{r}}|m',n\rangle = \delta_{x_{m',0}-x_{m,0}+q_y l^2,0} \mathrm{L}_n\left(\frac{1}{2}\vec{q}^{\,2}l^2\right) e^{-\frac{1}{4}\vec{q}^{\,2}l^2} e^{\imath x_{m'} q_x} e^{\frac{\imath}{2}q_x q_y l^2} \tag{10}$$

with $q_x = \frac{2\pi n_x}{L_x}$ ($q_y = \frac{2\pi n_y}{L_y}$), where n_x (n_y) are integer numbers, and the functions L_n are the Laguerre polynomials of order n. Using the Rashba Hamiltonian in terms of creation and annihilation operators, the matrix elements of the SOC term in the Landau basis, are given by

$$\langle m,n,\sigma|H_R|m',n',\sigma'\rangle = -\frac{1}{\sqrt{2}}\widetilde{\alpha}\delta_{m,m'}(\sqrt{n'}\langle\sigma|\sigma^-|\sigma'\rangle\delta_{n,n'-1} + \sqrt{n'+1}\langle\sigma|\sigma^+|\sigma'\rangle\delta_{n,n'+1}) \tag{11}$$

The symbol σ represents the index for a state with spin up ($+$) or spin down ($-$) orientation and the operators σ^+ (σ^-) are the creation and annihilation operators in terms of the Pauli matrices σ_x and σ_y. In the system of units where $\hbar\omega_c = 1$, the SOC parameter α can expressed in terms of a dimensionless parameter $\widetilde{\alpha} = \alpha/\omega_c l$. Even though, we are using the LLL approximation, it is evident from equation (11) that the Rashba Hamiltonian is coupling adjacent LL's. Finally, the matrix elements of the one-electron Hamiltonian given by equation (6), can be obtained by applying equations (10) and (11) to the LLL ($n = 0$) and the next LL ($n = 1$).

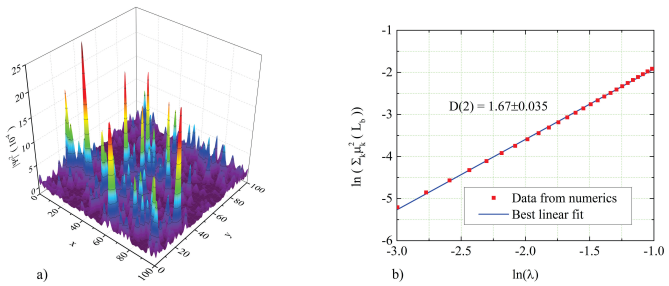

Figure 1. a) Probability density, $|\psi|^2$ (10^{-4}), for an extended state in a system of $L = 100l$, and b) disorder averaged $D(2)$ for states with the smallest IPN in a system of $L = 50l$, which is consistent with that of [14] within the statistical error.

4. Results and discussion

In the first part of this section, we will describe the results obtained by solving the Hamiltonian in equation (2). In the LLL approximation, we only consider the matrix elements of disorder in the LLL, by substituting $n = 0$ in equation (10). The system is solved by diagonalization of

the resulting matrix for N_ϕ states over a square system of size $L = L_x = L_y$. All lengths and energies are expressed as multiples of l and $\hbar\omega_c$, respectively. In Figure 1, we show the most extended state; that is, the state with the smallest inverse participation number (IPN) [22], which is close to the center of the LLL. We apply the multifractal analysis of Mandelbrot [23] to obtain the correlation dimension $D(2)$ of extended states by studying the scale invariance for the second moment of the box probability μ_k, defined as

$$P^2 = \sum_k \mu_k^2(L_b) \sim \lambda^{D(2)} \tag{12}$$

where $\lambda = L_b/L$ and $D(2)$ is the corresponding scaling exponent. For localized states, $D(2) = 0$, since all the probability density falls within one box; however, extended states exhibit a fractal behavior, where the scaling exponent has been calculated for several models of disorder. Aoki and Ando [13] found a value of $D(2) = 1.57 \pm 0.03$ using delta scatterers, Huckestein [14] obtained a value of $D(2) = 1.62 \pm 0.02$ by studying the scaling of equation (12) in a lattice Hamiltonian. In Figure 1, we also show the scaling exponent found by using the GWNP introduced in section 3, for a system of $L = 50l$ over 205 realizations of the random potential.

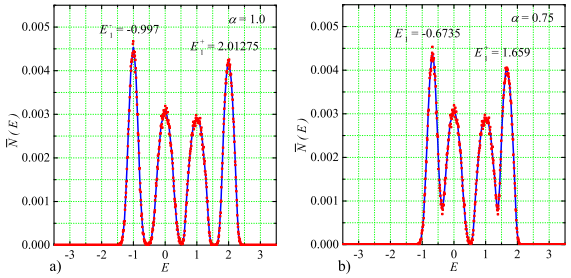

Figure 2. a) Spin resolved DOS for the LLL and first LL with $\widetilde{\alpha} = 1.0$ and b) spin resolved DOS for $\widetilde{\alpha} = 0.75$ for a system of $35l$. The energies shown are obtained numerically and can be verified from equation (5).

In this part of the results, we will discuss the multifractal nature of extended states for spin resolved LLs. We solve the system by diagonalizing the Hamiltonian of equation (6) with the matrix elements obtained in section 3 for different values of $\widetilde{\alpha}$. In Figure 2, is shown the disorder averaged DOS for the lowest and first LLs, taken from 200 different realizations of the random potential on a system of $L = 35l$. Since the Rashba Hamiltonian is coupling adjacent levels, and considering two spin orientations for each level according to equation (11), we need to diagonalize a $4N_\phi \times 4N_\phi$ matrix. The center of each spin resolved level corresponds to the values predicted by equation (5), which are also shown in Figure 2 in our system of units.

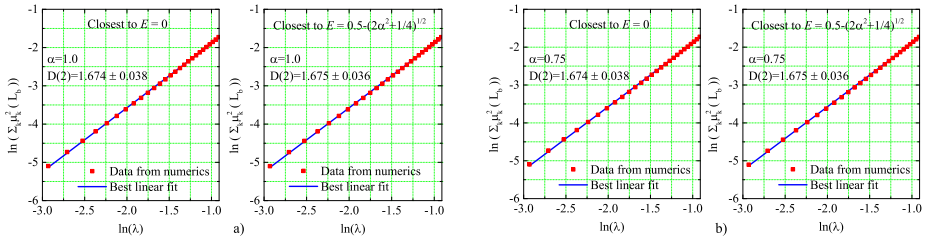

Figure 3. Average $D(2)$ for extended states with the smallest IPN, closest to the center of the LLL and spin down oriented levels for a) $\widetilde{\alpha} = 1.0$ and b) $\widetilde{\alpha} = 0.75$.

From the numerical solutions, we can study the dependence of the correlation dimension $D(2)$ on the SOC parameter for different spin oriented levels. In Figure 3, we show the value of $D(2)$ for extended states with the smallest IPN. The numerical value was obtained by averaging the scaling exponent $D(2)$, over 125 realizations of the random potential on a system of $35l$. As can be observed from Figure 3, within the statistical error, the correlation dimension is practically the same as the value shown in Figure 1 for the spin degenerate case.

5. Conclusions

In this work, it has been shown that extended states are also present near the center of each spin resolved Landau level. By including the Rashba spin-orbit coupling term into the one-electron Hamiltonian with disorder; it has been shown, that extended states belonging to each spin split Landau level are scale invariant, and show multifractality in a similar way to the spin unresolved Hamiltonian. As in the spin degenerate case, we found that the correlation dimension $D(2)$ of extended states near the center of each resolved level, shows universality relative to disorder strength; but more importantly, it is also universal relative to the strength of the spin-orbit coupling parameter. These findings seem to suggest, that disordered systems under moderate magnetic fields, with a Rashba spin-orbit coupling interaction of any strength, belong to the same universality class of the localization-delocalization transition observed in the Integer Quantum Hall Effect.

References

[1] Von Klitzing K, Dorda G and Pepper M 1980 *Phys. Rev. Lett.* **45** 494
[2] Chalker J T and Coddington P D 1988 *J. Phys. C. Solid State* **21** 2665
[3] Huckestein B and Kramer B 1990 *Phys. Rev. Lett.* **64** 1437
[4] Wei H P, Tsui D C, Paalanen M A and Pruisken A M M 1988 *Phys. Rev. Lett.* **61** 1294
[5] Wei H P, Hwang S W, Tsui D C and Pruisken A M M 1990 *Surf. Sci.* **229** 34
[6] Koch S, Haug R J, Von Klitzing K and Ploog K 1991 *Phys. Rev.* B. **43** 6828
[7] Nitta J, Akazaki T and Takayanagi H 1997 *Phys. Rev. Lett.* **78** 1335
[8] Engels G, Lange J, Schapers Th and Luth H 1997 *Phys. Rev.* B. **55** R1958
[9] Sato Y, Kita T, Gozu S and Yamada S 2001 *J. App. Phys.* **89** 8017
[10] Heida J P, Van Wees B J, Kuipers J J and Klapwijk T M 1988 *Phys. Rev.* B. **57** 11911
[11] Sinova J, Valenzuela S, Wunderlich J, Back C H and Jungwirth T 2015 *Rev. Mod. Phys.* **87** 1213
[12] Sinova J, Culcer D, Niu Q, Sinitsyn N A, Jungwirth T and MacDonald A H 2004 *Phys. Rev. Lett.* **92** 126603
[13] Aoki H 1986 *Phys. Rev.* B. **33** 7310
[14] Huckestein B and Schweitzer L 1994 *Phys. Rev. Lett.* **72** 713
[15] Janßen M, Viehweger O, Fastenrath U and Hajdu J 1994 *Introduction to the Theory of the Integer Quantum Hall Effect* (Wiley, Weinheim)
[16] Pook W and Janßen M 1991 *Z. Phys.* B. **82** 295
[17] Huckestein B 1995 *Rev. Mod. Phys.* **67** 357
[18] Huckestein B and Schweitzer L 1992 *Helv. Phys. Acta.* **65** 317
[19] Ando T 1982 *J. Phys. Soc. Jpn.* **52** 1740
[20] Huckestein B and Kramer B 1989 *Solid State Commun.* **71** 445
[21] Schliemann J, Egues J C and Loss D 2003 *Phys. Rev. Lett.* **90** 146801
[22] Hikami S 1986 *Prog. Theor. Phys.* **76** 1210
[23] Mandelbrot B B 1983 *The Fractal Geometry of Nature* (Freeman, San Francisco)