

Non-Inertial Quantization: Truth or Illusion?

S Cruz y Cruz

UPIITA, Instituto Politécnico Nacional
Av. Instituto Politecnico Nacional, 2580 CP 07340 CDMX, Mexico
E-mail: sgcruz@ipn.mx

B Mielnik

Physics Department, Cinvestav, AP 14-740 07000 CDMX, Mexico
E-mail: bogdan@fis.cinvestav.mx

Abstract. The quantum Hamiltonians in non inertial frames, while proper to describe the time evolution, might give a false information about the radiation of physical systems. The formal peculiarities observed for the uniformly rotating and accelerating frames might be caused by an excess of verbal approaches in the present day quantum theories.

1. Introduction

While the relativistic theories create increasingly flexible geometries, their quantum counterparts stick persistently to the same abstract scheme, based on the “crystalline rigid” Hilbert space geometry, with the state evolution maintained linear at the cost of increasing the number of variables (in form of tensor products, in a sense, the modern equivalents of Ptolemy’s epicycles). On more sophisticated level, the branches of the perturbative graphs are reinterpreted as the *virtual quanta*, propagating endlessly in their underworld, the *polarized vacuum* (comparable to *Matmos* of R. Vadim [1], where some creatures wait for an occasion to emerge in the real world). In spite of the well known successes of this allegoric image, the solutions of some elementary problems remain incomplete.

2. Moving frames

Curiously, this includes also the quantum evolution operators $U(t, t_0)$ driven by variable Hamiltonians $H(t)$:

$$i \frac{d}{dt} U(t, t_0) = H(t) U(t, t_0). \quad (1)$$

If $H(t)$ are real, then $U(t_1, t_0)$ are unitary and they can be represented as

$$U(t_1, t_0) = e^{-i(t_1-t_0)F(t_1,t_0)} \quad (2)$$

where the (non unique) self-adjoint $F(t_1, t_0)$ are the *effective Hamiltonians* for the time intervals $[t_0, t_1]$. An important case are the periodic systems with $H(t + T) = H(t)$. If the evolution



within one period $[t_0, t_0 + T]$ is given by $U(T) = U(t_0 + T, t_0) = \exp(-iT F)$, $F = F^\dagger$, its repetitions in the next periodicity intervals define the *Floquet process*

$$U(nT) = U(T)^n = e^{-itF}, \quad t = nT, \quad n \in \mathbb{Z} \quad (3)$$

where the (non unique) $F = F(t_0 + T, t_0)$ is known as the *quasi-energy operator*. The hypothesis of Zeldovich [2] tells that the spectral frequencies emitted/absorbed by a periodic system are given by the differences of F -levels, modulo multiples of $\hbar\omega$ (where $\omega = \frac{2\pi}{T}$):

$$\Delta E = F_n - F_m + l\hbar\omega, \quad l \in \mathbb{Z}. \quad (4)$$

In spite of its naturally, the Zeldovich hypothesis has some physical limits. It does not permit to distinguish the emission and absorption events. It becomes practically useless if the periods T of the Hamiltonians are too large. In fact, if T would be of order of magnitude of days instead of nanoseconds, then how the radiating system will know that within one day the external interactions will be exactly repeated? The same about the trajectories of 2-level states described as *splins* [3]. Suppose, the experimentalist generates an exact, periodic spin trajectory: how the radiating particle will know that he will not change his mind after a while? (Hence, the doubts whether the Zeldovich hypothesis can determine the short term radiative response).

An interesting case are the rotating systems, where the simple unitary transformation reduces the evolution to a constant generator (the “Hamiltonian in the rotating frame”). In the simplest cases, this happens for the initial Hamiltonians $H(t) = H_0 + H_1(t)$, where H_0 is a basic, rotation invariant part and $H_1(t)$ is an interaction term containing some rotating vector (*e.g.*, an external electric field $\mathbf{E} = \mathcal{E}(\cos\omega t, \sin\omega t, 0)$). Assuming that the time dependence of $H_1(t)$ can be expressed by: $H_1(t) = e^{-i\omega(t-t_0)\mathbf{M}\mathbf{s}} H_1(t_0) e^{i\omega(t-t_0)\mathbf{M}\mathbf{s}}$, where \mathbf{M} is the orbital angular momentum, \mathbf{s} a fixed unit vector and the exponentials of $\mathbf{M}\mathbf{s}$ rotate the internal parameters of $H_1(t)$, the whole Hamiltonian $H(t)$ can be written as

$$H(t) = e^{-i\omega(t-t_0)\mathbf{M}\mathbf{s}} H(t_0) e^{i\omega(t-t_0)\mathbf{M}\mathbf{s}}, \quad \omega \in \mathbb{R}. \quad (5)$$

If so, the evolution operator (2) can be conveniently factorized. The substitution

$$U(t, t_0) = e^{-i\omega(t-t_0)\mathbf{M}\mathbf{s}} W(t, t_0) \quad (6)$$

leads to

$$W(t, t_0) = e^{-i(t-t_0)G}, \quad (7)$$

where

$$G = G(t_0) = H(t_0) - \omega\mathbf{M}\mathbf{s}. \quad (8)$$

Since in the time moments $t - t_0 = nT$ the evolution (6) reduces to (7), the generator $G = G(t_0)$ is one of the quasi-energy operators (adequate to test Zeldovich hypothesis [2]); but since (7) holds for all $t \in \mathbb{R}$, G is simultaneously interpretable as the “Hamiltonian in the rotating frame” (with some tempting suggestions about the fundamental quantization problems [4,5]). Note that G does not depend on t , though it depends on the “anchorage moment” t_0 . To fix attention, put $t_0 = 0$ and consider $G = G(0)$. The t -independence of (8) suggests that the system described by (7) obeys the “quantum theory in non inertial frames”, with some formal hints that it should emit/absorb the G -quanta. The idea, though, is not free of dangers.

Some of them are of structural nature. In fact, one can notice that the operators (8) can differ substantially from the traditional Hamiltonians. Thus *e.g.* if the initial $H(0)$ is a positive quadratic form of the canonical variables $\mathbf{q} = (x, y, z)$ and $\mathbf{p} = (p_x, p_y, p_z)$, it might easily happen

that G has no lower bound. This was indeed detected for the charged Schrödinger particle in rotating magnetic fields [6, 7] where G becomes a non-positive combination

$$G = \omega_0 H_0 - \omega_1 H_1 + \omega_2 H_2 \quad (9)$$

of 1D oscillators H_0, H_1, H_2 (with $\omega_j \geq 0, j = 0, 1, 2$). The analogous results hold for the rotating electric fields, rotating oscillators [8–10] etc. Independently of any deeper problems, the non-positive character of (9) can awake a lot of doubts against too literal understanding of G as a “true Hamiltonian”, i.e., as the energy analogue in the rotating frame.

Yet, an attempt of incorporating the indefinite structures (9) into the radiation theory has been carried in the well known papers on the atomic or molecular states analogous to the “Trojan asteroids” circulating around the Lagrange points of the rotating potentials [8–12]. The indefinite evolution generators G in the rotating frames were then introduced into the spontaneous emission formulae (as it seems, in hope to reduce the problem of the radiation theory to the time-independent Hamiltonians). Yet, some doubts did not disappear. While the “Hamiltonian” (8) explains reasonably well the motion of the rotating states, the problem remains whether one can extrapolate still further, treating G as a key to determine the quantum jumps, emission rates, lifetimes, etc [6, 7, 13–15].

The formal experiment adopting this idea turned even more interesting than intended, since it was a step to test the Quantum Field Theory (QFT) “in the rotating frames” [8, 10, 12]. The results at this stage were not yet conclusive, since the quadratic forms of the Hamiltonians around the “Lagrange wells” were just an approximation, so the effects of the “inverted oscillators” could not be checked for arbitrary field intensities. Indeed, the rotating field \mathcal{E} in the “Trojan models” can be neither too strong nor too weak, otherwise the system goes out of the stability zone and the bound states disappear. Moreover, even for adequate \mathcal{E} the tunnel effect presents a notable competition, making it difficult to check the exact contribution of G to the supposed radiative decay. However, the problem admits some simple toy models, which facilitate conclusions.

To avoid complications, we applied the same methods [8–10, 12] to a more elementary case which shares some properties of Trojans [10] but is free of the ionization and tunnel effects. We refer simply to the 2D harmonic oscillator in the rotating electric field

$$H(t) = H_0 - e\mathcal{E}\mathbf{n}(t)\mathbf{x}, \quad (10)$$

where $H_0 = \frac{\mathbf{p}^2}{2} + \nu^2 \frac{\mathbf{x}^2}{2}$ and $\mathbf{n}(t) = (\cos \omega t, \sin \omega t, 0)$ is a unit vector rotating in the x, y -plane. Quite obviously, $H(t)$ has the form (5), so if $t_0 = 0$, the transformation to the rotating frame leads to a constant generator:

$$G = H_0 - e\mathcal{E}x - \omega(xp_y - yp_x). \quad (11)$$

The peculiarity of (11) shows up already at this point. Indeed, an elementary algebraic argument (c.f. [6]) reduces G to a pair of 1D oscillators:

$$G = \omega_+ A_+^\dagger A_+ + \omega_- A_-^\dagger A_- + \nu + \frac{e^2 \mathcal{E}^2}{2(\omega^2 - \nu^2)}, \quad (12)$$

where $\omega_\pm = \nu \pm \omega$ and A_\pm are given by

$$A_\pm = \frac{1}{2\sqrt{\nu}} \left(\nu x \pm i\nu y + ip_x \mp p_y \mp \frac{e\mathcal{E}}{\omega \pm \nu} \right), \quad (13)$$

satisfying the commutation rules

$$[A_-, A_+] = [A_-^\dagger, A_+] = 0, \quad [A_\pm, A_\pm^\dagger] = 1, \quad (14)$$

(compare [6, 10, 12]). The generator G is singular at $\omega = \nu$, due to the interaction between the rotating frame and quantum trajectories rotating in two opposite directions. Suppose, we use it for $\omega < \nu$ and $\omega > \nu$. For $\omega < \nu$, both ladders are just softly modified. However, if $\omega > \nu$, then $\omega_- < 0$ and the second oscillator is turned upside down, producing the top state instead of the ground state, together with an infinite ladder of negative eigenvalues. By adopting G as a criterion for the radiation (see [8, 9, 12, 16]) and the standard rules, one might be tempted to infer that the *ground-top* state of (12) is unstable and must fall down spontaneously to the first negative level. Such is indeed the conclusion in the works on the rotating systems [8, 10, 11]. Yet, the story does not end up so easily. If the argument is right, then the spontaneous transitions to the subsequent negative levels should occur, producing a radiative avalanche described humorously by Roy Glauber: “*To be in actual possession of an inverted oscillator would be a fantastic thing: it would solve world energy problem*” [17].

A tempting hypothesis is that (10-13) can indeed produce a radiative cascade at the cost of the rotating field (10) pumping the energy to the system (compare [18]). However, the idea does not apply. To show this it is enough to take the limit $\mathcal{E} \rightarrow 0$ (a test counterproductive in the “Trojan case” [8, 10], but relevant in ours). One therefore has

$$A_{\pm} = \frac{1}{2\sqrt{\nu}} (\nu x \pm i\nu y + ip_x \mp p_y). \quad (15)$$

So, after an elementary calculation

$$A_+^\dagger A_+ = \frac{1}{2\nu} (H_0 - \nu) - \frac{1}{2} M_z, \quad (16)$$

and

$$A_-^\dagger A_- = \frac{1}{2\nu} (H_0 - \nu) + \frac{1}{2} M_z. \quad (17)$$

Hence

$$G = \omega_+ A_+^\dagger A_+ + \omega_- A_-^\dagger A_- = (H_0 - \nu) - \omega M_z. \quad (18)$$

Here the singularity no longer exists, but if $\omega > \nu$, one of the oscillators is still inverted, with the descending ladder of eigenvalues suggesting the spontaneous system jumps to $-\infty$. If treating seriously the G -levels, it would mean: *NOTHING, if rotates fast enough, produces radiation!*

However, the supposed effect cannot be the result of the energy pumping (no energy can be pumped by vanishing \mathcal{E} !) but rather an illusion caused by over-interpreting G .

This is indeed one of few cases where the “formal story” of QFT becomes visibly unreal. In fact, the puzzle disappears after a simple redefinition. It is enough to notice that for $\mathcal{E} = 0$ the eigenstates of the “negative ladder” are simultaneously the eigenstates of the initial H_0 with the increasing, positive energies in eyes of the inertial observer. In fact, the simple calculation for $\mathcal{E} = 0$ with A_{\pm} given by (15-17) yields

$$H_0 - \nu = \nu (A_+^\dagger A_+ + A_-^\dagger A_-). \quad (19)$$

So, G and H_0 commute and the “positive” and “negative” eigenstates of G are just a part of the ordinary, positive state ladder of H_0 . More explicitly, if

$$|n_+, n_-\rangle = \frac{1}{\sqrt{n_+! n_-!}} (A_+^\dagger)^{n_+} (A_-^\dagger)^{n_-} |0, 0\rangle, \quad (20)$$

then one has

$$G|n_+, n_-\rangle = (n_+\omega_+ + n_-\omega_-) |n_+, n_-\rangle, \quad (21)$$

but simultaneously

$$(H_0 - \nu)|n_+, n_-\rangle = \nu(n_+ + n_-)|n_+, n_-\rangle. \quad (22)$$

The question arises up to what extend one can still adopt the concept of the *rotational frequency shift* [16], supposing that while for the inertial observer (19) the system states fall down to the oscillator ground state, the same process in eyes of the rotating observer looks as the system descent down the negative ladder emitting some modified frequencies? Such seems the hypothesis in [16], where the transitions down the negative ladder is considered as one of stability challenges.

This is certainly not the case in our oscillator model (15-17) where the same process can be observed simultaneously from the inertial and rotating frames for the same (though rearranged) ladders. For the inertial observer all oscillator states must *shrink* falling to the ground state. So, can the same states, in eyes of the rotating observer, expand in one of their canonical pair of variables, when descending down to $-\infty$? This turns impossible: the space extension of the atomic states are exactly the same in the rotating and in the inertial frames! Hence, the quantum states in the rotating frame cannot expand going down, but must also shrink *going up*. This is what indeed happens, as the “negative ladder” is an inverted part of the positive one. This inverts also the conclusion of [16], where the spontaneous transition from the most compact state ψ_0 to the less localized ψ_1 (cf. [16], Eq. (22), p.56), is considered as one of stability dangers. Our hypothesis is that the “negative levels” are not the stability danger but inversely, offer some little help to the system stability.

We heard some comments that the QFT in the rotating frame remains valid for $\omega > \nu$, but then the quantum states jumping on the negative ladder would get rid of their negative energies, by *emitting* the negative energy quanta $E = \nu - \omega$, meaning that they indeed go up, and everything is correct!

Yet, the *negative energy photons* are a significant sacrifice of the common sense, just to preserve the formal shape of the theory. Worse, because if one wants to use the Hamiltonian G for $\mathcal{E} = 0$ and $\omega = \nu$, one thus obtains the rotating frame plagued by zero-energy quanta (a strange concept, tempting the peculiar questions, e.g., how many *0-frequency photons* do we need to describe the static electric field?). It might mean that the QFT in non-inertial frames is only a sort of verbal exercise whose correct understanding requires always the return to the inertial frames. This was indeed noticed by Delande and Zakrzewski [12]. They write: “*Hence, emission of the spontaneous σ^+ polarized photon increases the energy in the rotating frame*” [12], p. 473, col. 2, lines 3-5 from bottom. With our only comment, that by “energy” they mean the G -eigenvalue.

Of course, our toy model is non-relativistic. Already in special relativity the rotating coordinates would loose their sense beyond $2\pi r \geq \frac{c}{\omega}$, see the discussions by Padmanabhan [19] and Earman [20]. However, it is precisely our oversimplified case, permitting to observe simultaneously the behaviour of quantum system in the inertial and non-inertial frames, which may illustrate some other interpretational problems.

3. Rotating \mathcal{E} as seen in the inertial frame

Hence, what about the rotating $\mathcal{E} \neq 0$ observed from the inertial frame? Basically, it is a member of a wide family of the quadratic systems described by the Floquet Hamiltonians [21]. Yet, in our case, the evolution equation (1) can be exactly solved for any time limits, giving explicitly the unitary operators $U(t_1, t_0)$ and the effective Hamiltonians $F(t_1, t_0)$. This can be conveniently done in the interaction instead of rotating frame:

$$U(t_1, t_0) = e^{-i(t_1-t_0)H_0}V(t_1, t_0) \quad (23)$$

obtaining $V(t_1, t_0) = \exp[-iK(t_1, t_0)]$, where $K(t_1, t_2)$ is linear in \mathbf{x}, \mathbf{p} -operators, with coefficients expressed through elementary trigonometric integrals. Now, if $\omega \neq \nu$, the traditional

Baker-Campbell-Hausdorff formula with the “polarization derivative” [22], or else, the techniques of Gilmore [23] permit to write down $U(t_1, t_0)$ explicitly in form (2) with $F(t_1, t_0)$ given in terms of $\tau = (t_1 + t_0)/2$ and $\delta t = t_1 - t_0$ [24]:

$$F(t_1, t_0) = \frac{\mathbf{p}^2}{2} + \nu^2 \frac{\mathbf{x}^2}{2} - \frac{\xi}{\sin \frac{\nu}{2} \delta t} [a\nu(\cos \omega \tau x + \sin \omega \tau y) + b(\sin \omega \tau p_x - \cos \omega \tau p_y)] \quad (24)$$

where $\xi = e\mathcal{E}/(\omega^2 - \nu^2)$ and

$$a = \nu \sin \frac{\nu}{2} \delta t \cos \frac{\omega}{2} \delta t - \omega \cos \frac{\nu}{2} \delta t \sin \frac{\omega}{2} \delta t, \quad b = \nu \cos \frac{\nu}{2} \delta t \sin \frac{\omega}{2} \delta t - \omega \sin \frac{\nu}{2} \delta t \cos \frac{\omega}{2} \delta t.$$

For $\delta t \rightarrow 0$, (24) reproduces the instantaneous Hamiltonian $H(\tau)$, while for $\delta t = T$, it yields the effective Hamiltonian for the entire period T :

$$F(t_0) = \frac{1}{2} [(p_x + \omega \xi \sin \omega t_0)^2 + (p_y - \omega \xi \cos \omega t_0)^2 + \nu^2 ((x - \xi \cos \omega t_0)^2 + (y - \xi \sin \omega t_0)^2) - f_0], \quad (25)$$

with $f_0 = \xi^2(\omega^2 + \nu^2)$. Differently than $G(t_0)$, the $F(t_0)$ is a *continuous extension* of $H(t)$. The divergence of ξ for $\mathcal{E} \neq 0$ and $\omega \rightarrow \nu$ reflects the fact that the electric field rotating with the oscillator frequency ν must cause the resonance of the quantum mechanical system. Besides, all illusions disappear. There are no negative oscillators, no ladders of eigenvalues descending to $-\infty$, no radiative avalanches precipitated by violently rotating but vanishing fields. Note that the motion integral (24) was known for the forced oscillators (see Brewer and Holthouse [25]), but apparently, without appreciating its possibilities to construct the alternative Floquet Hamiltonians for the rotating case.

It seems to confirm the hypothesis that for the non inertial frames some concepts split: while the “non inertial Hamiltonians” describe well the unitary evolution, they might falsify the radiation effects which require different generators, in our case (25) rather than (12-13); bad news for too formal radiation theories though good news for the stability of the trapped states.

In fact if such problems exist even for the uniformly rotating frames, this implies a warning about more general cases of quantum theory in non-inertial frames and/or relativistic space-times. One of them is still intensely discussed.

4. Unruh radiation

In his inspired 1973 study Fulling [5] wondered about the unclearly defined concepts of QFT leading to the peculiar forms of the creation and annihilation operators in non-inertial reference frames. If one deduces the annihilation and creation operators of quantum fields from the positive and negative frequency components of their classical equivalents (the definition automatically adopted in all QFT versions), then the creation/annihilation operators in the non-inertial frame do not coincide with those of the inertial Minkowski frame, but are altered by linear transformations of Bogolubov type. As it seems it exhibited one of weak points of the too verbal formulation of QFT. Indeed, why the “quantization” should follow the calligraphic formulae of field energy in terms of “frequencies”? Moreover, why the physical observer whose space-time trajectory is not temporal straight line, comparing the frequencies of the hypothetical external fields to his proper time, should feel surrounded by the clouds of real (unreal?) quanta?

The authors interested in [5] started to analyse the (physically peculiar) case of quantum systems undergoing a constant, never-ending acceleration with trajectories limited to one of Rindler sectors in the Minkowski space-time. In his interesting paper Davies [26] wondered about the capacity of the mirror in this universe to produce the real quanta and about the

questionable aspects of Hawking theory on the black hole evaporation. However, it seems that the initial doubts almost disappeared in an intense trend of publications.

In his 1976 study Unruh adopts *bona fide* the non inertial quantization in Rindler coordinates [27]. While considering his elementary example of an endlessly accelerated detector, c.f. [27], Eq. (3.1a), he presents a kind of allegoric picture (see [27], p. 885):

“What the detector regards as the detection (and thus absorption) of a Φ -quantum, the Minkowski observer sees as the emission by the detector of such a quantum. The energy of this emission (...) comes from the external field accelerating the detector”.

In his further arguments, assuming a coupling constant ϵ between the quantum detector and the massless field Φ in the polarized Minkowski vacuum, Unruh concludes that the result is *what one would expect of a detector in a thermal bath of photons of the temperature $\frac{a}{2\pi}$* . The authors interested in the idea (see, e.g. [28, 29]) expressed it in a more complete form

$$T = \frac{a\hbar}{2\pi ck}. \quad (26)$$

The consistency of units on both sides of (26) is notable! However, the provocative argument about the absorption–emission equivalence in different frames, even if shaking, is not entirely clear. At least, some elements of the hypothesis are absent and they seem essential for the precise understanding of the assumed phenomenon.

In the collection of papers, interpreting (26) a lot of authors simplify the problem introducing a hypothetical point-like Unruh-De Witt detector (in the simplest case, just a spin $\frac{1}{2}$ particle). Sharing the accelerated motion, it “perceives” the bath of virtual quanta of the polarized Minkowski vacuum invisible for any inertial observer. In what follows, we shall be interested not so much in the thermal effect (which can occur for various reasons, without excluding the macroscopic propulsion), but in the first place, in the specific doctrine of the accelerated detector which “sees” (and is bombarded by) some virtual quanta of Minkowski vacuum. Here, some details are still missing. We shall make a preliminary list, starting from apparently minor points.

(1) Can the detector be truly microscopic (e.g. a micro particle which should “click”, consistently with Unruh-De Witt idea)? An idealization seems excessive, even if it might avoid some troubles. Indeed, *what does it mean* that the point-like particle “clicks”? The assumption seems to ignore the nature of quantum detection acts. We do not think that the existing QM interpretation is sacred, but if one tries to predict some effects in *frames of the existing theory* then one should respect its assumptions. They tell us that in order to detect anything, the detector should produce some macroscopic (or at least mesoscopic) effects. So, it should have some macroscopic or mesoscopic parts. If not, the detector will never click. It will just continuously evolve, obeying the quantum mechanical unitary propagators.

(2) Yet, the avalanche of papers is dominated by the maximally simplified model of the point-like detectors. Is this unavoidable? Can the supposed Unruh radiation affect the macroscopic objects? Assuming for a moment that such radiation indeed exists, the answer must be: yes!. A shortcut statement was offered by Unruh himself: “it is real enough to roast a steak” (see also the polemic discussions of Scully et al [30–32]) So, the steak (or at least its molecules) is the correct detector of the hypothetical Unruh bath. All this does not yet explain everything about the macroscopic detectors; compare the description of the “thermometer” in [28, 29], or else, the critical study of Narozhny et al [33–35] who write: “...its treatment in the literature has led to contradictory results. (...) This is because different points of a finite size body rigid with respect to Rindler coordinate frame, move actually with different accelerations in MS”. So, was it the reason to restrict the story to the point-like detectors?.

Suppose, however, one is ready to accept some slight errors of the acceleration a in (26). The discussion, anyhow, cannot escape the next questions.

(3) One of the original Unruh models of an extended detector is a cabin moving with a constant, linear acceleration. But if cabin, then (forgive us this extreme vision) why not the spaceship? Moreover, to protect the story from being too narrow, one should also ask, whether the detection must consist in particle *absorption*? In the real labs (inertial or not) it is certainly not so. The particle can run in some sensitive medium like the bubble chamber or a block of photographic emulsion, leaving a visible trace but without being absorbed. Suppose then that a cabin of an accelerated spaceship (the laboratory) contains a sensitive blocks of photographic emulsion. Is it forbidden to expect that the particles of the Unruh bath (which can roast the steak) will mark some traces in the block of emulsion? If not forbidden, then some more questions inevitably follow.

(4) In fact, suppose in an accelerated lab there is one detector ready to *absorb* particles, inside of a block of photographic emulsion, ready to *register* the trajectories. Suppose now, there are two observers watching the same phenomenon. One is a physicist of the laboratory, sharing the accelerated motion. Another one is an inertial observer in the second spaceship accompanying the accelerating one in a little fragment of its trajectory. Suppose moreover, that both spaceships have the transparent windows: so the inertial observer, (who cannot see directly the virtual Minkowski quanta), will anyhow notice the macroscopic traces in the accelerated laboratory. According to the doctrine, he must see them running in the opposite direction: not *absorbed* but *emitted*, not *to* but *out* of the detector! This ambivalence, though, is not easy to sustain. It could be true only if the *time* of an accelerated system was running backward with respect to the inertial time, which is not the case neither for the rotating nor for accelerating frames.

A chance to mask the difficulty (saving the inspiration of [27]) would be to assume that the photographic emulsion shares the fate of the beefsteak, absorbing such an avalanche of the bath particles that any “trajectories” disappear in the radiation chaos. However, we do not think that this is a good answer. The formula (26) shows an extremely delicate effect, with bath particle seldom achieving high energies sufficient to paint the macroscopic traces, so should such effect occur, it is difficult to assume that it will be masked by some general trajectory chaos.

(5) Moreover, must the Unruh particles be massless? The original work of Fulling [5] on Klein-Gordon fields does not exclude particles with non-vanishing rest mass. Now, while in our present day experiments there may be still some doubts about the emission moments of some very tiny particles (e.g. neutrinos) in our earthly (non-inertial!) laboratories there seem to be no danger of mistaking the emission and absorption acts for heavy bodies. Yet, one of characteristic aspects of the publication trend on Unruh radiation is that (with some exceptions) it focuses attention on the massless scalar fields. May it be some (subconscious?) resistance against slow and massive particles which do not obey the image of delicate, almost not registrable trajectories? If so, even this cannot help, since the high energy photons mark also the macroscopic traces! Moreover, on p. 886, top of col. 2, in his paper [27] Unruh writes: “...one could also say that the accelerated proton has detected one of many high energy neutrinos which are present in the Minkowski vacuum in the proton’s accelerated frame of reference”. The model of particle bath containing neutrinos is repeated in all papers explaining proton decay observed in the accelerated frame. However, if neutrinos, then why not other massive particles, like nucleons, atomic nuclei, or even tennis balls? Can their flight direction be opposite in different frames? Certainly not in the Rindler’s frame used to formulate the bath hypothesis.

(6) We must conclude, that the supposed change of the flight direction of the hypothetical Unruh particles under the transition from the inertial to non-inertial frame might be allegoric but not real. It is simply not possible that the bath particle falls *into* the detector in eyes of one observer, but is *emitted* in eyes of another. So, if the detector reacts at all, both observers must either see the absorption, or the emission of the detected particle. Now, since for the inertial observer the virtual particle of Minkowski vacuum are undetectable, both can only

detect the same particle emission by the accelerated detector, though looking slightly different in coordinate of both observers. (Similarly, as in our naive example of the rotating frames, to distinguish between *emission* and *absorption*, only the inertial observer has a good key) As it seems, this simple argument makes stronger the conclusion of Earman, who writes carefully: “*Of three approaches to the Unruh effect (...) the least helpful is (...) of thermal flux of Fulling quanta*”, see [20], Ch.8, p.93. See also the transparent analysis by Padmanabhan [19, 36].

However, in some works the subject acquires an almost esoteric sense. So, the caloric bath particles are no longer virtual Minkowski quanta (as initially suggested by Unruh), but they are something else (Fulling particles?). If massive, they would be anyhow invisible, since they remain sharply localized, below the Planck distance, etc... However, a careful analysis of some interpretational dangers typical for quantum theory in sub-manifolds of Minkowski spacetime were already discussed by Padmanabhan who notices: “*...we extend our analysis to non uniformly accelerated frames (...) Here we reach the surprising conclusion that no ambiguity arises in the particle definition. Thus, physically realizable detectors do respond to the Minkowski definition of particles*” [36], p.248. See also the short and transparent notes in [37]. As if it was not enough, some descriptions of the supposed “thermal bath” reveal that the detector will be affected by an infinite numbers of 0-energy photons. In spite of all efforts, this is a sign that the formalism entered into an illusionary domain. The applied “regularization” deserves perhaps some artistic prize, but we doubt that it will convince the detector! (reported in [38], p. 20).

5. The thermal effect?

Our critical remarks still do not explain the sense of the formula (26). In the stream of publications an implicit idea seems to be that if only the existence of (an extremely weak) thermal effect in the accelerating bodies can be experimentally proved, it will justify the Unruh discovery, even if not interpreted exactly according to his hypothesis. Indeed, as one can notice, the two parts of the original Unruh argument [27] “*What the detector regards as the detection (and thus absorption) of a Φ -quantum, the Minkowski observer sees as the emission by the detector of such a quantum*”, and “*The energy of this emission (...) comes from the external field accelerating the detector*” are rather in narrative than logical connection. They express a kind of provocative idea, which made the hypothesis so widely discussed, but... it can still happen, that a questionable theory might help to guess the true result! In what follows, we henceforth try to review various points of view about the caloric effect (26) without rejecting a priori the “detector mysteries”, as the unknown coupling with the vacuum scalar field Φ or with forces maintaining the acceleration.

As far as we could find, the hypothesis about the thermal effect appeared originally for the physical bodies moving with a constant linear acceleration, on the trajectories restricted to one of Rindler wedges in the Minkowski space-time. The information signals available on such trajectory are as well restricted by the Rindler horizons. The most natural hypothesis was that the response of the detector depends on its internal structure. The simplest assumption about an interaction between the detector with an outside field Φ was described in the first Unruh paper [27], in terms of a coupling constant ϵ , which after the 2nd order perturbation steps appears in the expressions for the transition probabilities (see e.g. formulae (3.2-4) on p. 884, then (3.10), (3.12), (3.14) p.p. 885-6). As stated by Unruh, if $\epsilon = 0$, the detector is not coupled with Φ and the caloric effect will not materialize. However, Unruh skips some details. He only explains that (if $\epsilon \neq 0$), the detector reacts “as if it was submerged in the particle bath” of temperature T . Quite similarly, Grove and Ottewill introduce the coupling constant λ [39]. Yet, they do not explain why the final thermal effect (26) does not depend on the coupling constants.

The only explanation one can imagine is that the temperature (26) is independent of any detector, it is an abstract aspect of the Minkowski polarized vacuum limited to the Rindler wedge, though the consequences will materialize only when the detector persists moving with a

constant acceleration a and will finally achieve the thermodynamical equilibrium state, which may happen after shorter or longer time depending on the value of ϵ or λ (see also Ford and Connel [40]). In some cases, though, the calculation reveals that the equilibrium requires the time comparable with the age of the universe (see [38], p. 823, col. 2, down). How nice: so, shall we have a chance to observe the detector still before the thermal equilibrium?

This explanation was critically analysed in ample studies by Earman [20], and by Narozny et al [33, 35]. What cannot be understood is that a detector in any instant of its existence, with an acceleration a (which at any moment can change), should “feel” the properties of the abstract *polarized vacuum* in an entire, infinite Rindler zone. This may look more natural, if the detector follows forever one of killing trajectories from $-\infty$ to $+\infty$. Even so, it is incomprehensible, how in any moment of its existence, it can know, that his acceleration will not change in the future? As humorously expressed by Earman, it is an assumption “...which requires in turn a knowledge of entire past and future of the observer; but barring backward causation, no laboratory registration taken at a finite time is sensitive to what the instrument does in the future”. Quite obviously, the assumption about a detector moving with a constant, never-ending acceleration is of Science-Fiction type, and *cannot be realized as a trajectory of any physical object* [33].

Some attempts of obtaining more credible scenario were made by assuming that the detector was switched on in the past and switched off in the future. If switched on/off violently, the discontinuous jump was falsifying the expected result (26) (see ample reports in [38]). The desired formula (26) for T was obtained in the papers assuming the very slow, adiabatic switching on the detector in the distant past and switching off in the distant future. However, if the “detector” is an electron, can somebody explain, e.g., what does it mean that the electron is switched off adiabatically? Does it mean that its coupling constants ϵ , λ , etc, will die out? And again, how the electron will know that it will be switched off, not so soon, but in a distant future? As it seems, what can be switched is not the detector itself (electron?) but simply the accelerating mechanism. This is, however, a different story, which leads us naturally to the spaceship model.

6. The cosmic travel?

A realistic trajectory of a space-ship in the Minkowski spacetime can have indeed some intervals of constant acceleration but they do not need and cannot continue indefinitely. In some moments the traveling laboratory might change its acceleration or even recover an inertial (straight) world line e.g. to save the fuel (we apologize for using so elementary argument!) The lines $x = \pm ct$ then cease to be horizons, there is nothing special in the coordinate center $x = t = 0$ and the fine mathematical problems in the Rindler wedge no longer matter. Hence, the collections of papers with attention focused on the “distinguished point”, inviolable horizons, etc, seem to be trapped in too narrow and unphysical visions. In the ample literature on the subject one can also find some efforts to define the correct “asymptotic conditions” in the past and in the future, under which the conclusion about acceleration heating could become correct. However, why the effect (if real) should depend on any asymptotes in the future?

Of course, the travelling detector can change his trajectory in quite capricious way. One may thus wonder whether the supposed Unruh radiation, if any, should not produce some strictly local phenomena, bursts of radiation from the “detector’s eye” caused by instants of accelerating force, having not much to do with the asymptotic behaviour, and neither responding to the universal “radiation bath”?

A mathematically elegant attempt to describe the finite-time analogue of Unruh phenomenon was undertaken by Rovelli and Martinetti [41] but apparently, succeeded only to stress the difficulty. The authors describe a hypothetical detector conserving his constant acceleration in a certain final interval of its proper time. By applying a global view, they describe the “detector” perceptions during his entire *lifetime* (meaning the constant acceleration interval between certain

time moments τ_0 and τ_1). Using the theory of local observables [42] they assume that the detector, in principle, interacts with the events localized in a “diamond”, i.e. the spacetime region (τ_0, τ_1) between the future cone of the initial point $P(\tau_0)$ and the past cone of the end point $P(\tau_1)$.

As the result, they obtain the graphic of the “inverse temperature” $\beta(\tau)$, symmetric with respect to the center of the diamond’s time. Suppose, we take the derivation *bona fide*. The value of the parameter $\beta(\tau)$ is symmetric and maximal (corresponding to the Unruh effect) in the center of the interval, i.e. in the middle of the accelerated trajectory (see [41], Figure 1). Let us suppose that β is measurable. The problem with causality is then quite evident. Even if not reading exactly his future, the intelligent detector approaching the middle of the $\beta(\tau)$ -graphic could discriminate at least some *scenarios* which can never happen: for instance, that the acceleration of his vehicle just in the next few seconds will come to an abrupt end, e.g., due to an accidental defect of the accelerating motor. This is not everything, however.

In all papers on the non-inertial detectors registering the virtual quanta invisible in the inertial frame, we never noticed an inverse problem: that this can happen only at the cost of overlooking some *real quanta*, visible for the inertial observers. In fact, if the traditional annihilation operators a_j are given by the linear (Bogolubov) transformations of their Rindler equivalents, i.e.,

$$a_j = \int_{-\infty}^{\infty} dk U(j, k) b_k + \int_{-\infty}^{\infty} dk V(j, k) b_k^\dagger$$

see [5], formulae (30) and (40)), where $V(j, k)$ is a non-trivial function, describing the probability of observing the Minkowski vacuum as the bath of Unruh particles. Then in the equivalent formula

$$a_j^\dagger = \int_{-\infty}^{\infty} dk \bar{U}(j, k) b_k^\dagger + \int_{-\infty}^{\infty} dk \bar{V}(j, k) b_k$$

the same function $\bar{V}(j, k)$ will express the probability of perceiving the physical $a^\dagger|0\rangle$ particle as the physical vacuum. So, should the detector be bombarded by a sequence of real particles with the creation operators a_j^\dagger there will be always the probability (however little) that they will all be unperceived by an accelerating detector! However, could such a chance exist also if the detector (spaceship) collides with massive beam of heavy, charged particles (e.g. protons), which must destroy it in eyes of of any inertial observer... but could it remains invisible in the accelerated frame? What about the macroscopic charge and mass values under the transition from an inertial to the Rindler frame?

All this may indicate that, in spite of so many successes (the anomalous magnetic moments, etc.), the “verbal saga” of QFT, at least this time, is overdone. The “non inertial Hamiltonians”, while offering a part of the truth, create also some mythologies which cannot be literately taken. Though we are outsiders in the subject, we cannot abstain from citing a fragment of the original Fulling work [5] (even if Fulling himself changed later his point of view). He writes about the supposed particle bath: “... *One must accept the fact, however, that the quanta of this Fock representation do not correspond to physical particles. One must find something else to play the role of the basic observables of the theory...*” ([5], p. 2857, down the col. 1). Indeed, if there is any vacuum friction opposing the acceleration of microsystems, then even the Nernst ideas of the “zero point radiation” [40, 43–45], in spite of all doubts, are easier to accept than the vision of the particle bath, real in one reference frame and absent in another. Does it mean that an ample trend of inspired publications, indeed an almost superhuman effort of so many authors was on vane? Certainly not. Even if questionable, it has an enormous success, precisely by approaching the applicability limit of the present day theories. After all, Cristobal Columbus did not found his dreamed way to India. Yet, he achieved something else. He just discovered an obstacle.

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