Non-equilibrium slave bosons approach to quantum pumping in interacting quantum dots

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Abstract. We review a time-dependent slave bosons approach within the non-equilibrium Green's function technique to analyze the charge and spin pumping in a strongly interacting quantum dot. We study the pumped current as a function of the pumping phase and of the dot energy level and show that a parasitic current arises, beyond the pure pumping one, as an effect of the dynamical constraints. We finally illustrate an all-electrical mean for spin-pumping and discuss its relevance for spintronics applications.

1. Introduction

The idea of parametric quantum pumping regards a nanostructure (scattering region) connected to two external leads (reservoirs) in which a dc current is produced in absence of bias voltage by time periodic modulation of two system parameters. If the parameters change slowly as compared to all internal time scales of the system, the pumping is *adiabatic*, and the average charge per period does not depend on the details of the time dependence of the pumping parameters $X_i(t)$[1]. Using the concept of emissivity proposed by Büttiker et al.[3], Brouwer[4] related the charge pumped in a period to the derivatives of the instantaneous scattering matrix of the nanostructure with respect to the time-varying parameters. Since then, a general framework to compute the pumped charge through a conductor has been developed for noninteracting electrons[5]. The interest in the pumping phenomenon has shifted then to the experimental investigations of confined nanostructures, as quantum dots, where the realization of the periodic time-dependent potential can be achieved by modulating gate voltages applied to the structure[6]. In case of interacting electrons the computation of the pumped charge becomes rather involved and few works have addressed this issue for different systems and in specific regimes[7]. As for the case of interacting quantum dots, the pumped charge in a period was calculated by Aono[8] by exploiting the zero-temperature mapping of the Kondo problem within a non-equilibrium Green's function approach (NEGF). A very general formalism was developed in Ref.[9] where an adiabatic expansion of the self-energy based on the average-time approximation was used to calculate the dot Green's function, while a linear response scheme was employed in Ref.[10]. Alternatively to the NEGF approach, a powerful diagrammatic technique to treat the interacting quantum pumping based on a generalized-master equation method was instead proposed in Ref.[11]. In more recent years the interest on quantum pumping in interacting quantum dots has moved to the comprehension of the role of spin-orbit interaction[12] for achieving spin-current, and to non-adiabatic effects[13]. While the role of coupling of the quantum dot to elastic deformations was analyzed in various papers in the limit of interaction going to zero[14, 15].

Here we review a non-equilibrium slave boson approach for an interacting quantum dot within the NEGF method to study the quantum pumping effects and which was introduced in Ref.[16]. In particular we focus on the strongly interacting regime $U \to \infty$ and derive first, a time-dependent mean field equation for the slave boson and the constraint in the presence of a time-dependent tunnel barrier then, we derive the expression of the pumped current in the leads in terms of the Keldysh Green's function of the dot which depends on the dynamical constraint. In our treatment we are able to obtain an expression of the pumped current beyond the adiabatic (i.e. $\omega \to 0$) approximation, i.e. at finite frequency ω of the pump. Our main result is that the effect of the dynamical constraint generates a dynamical phase which couples to the phase of the external parameters φ giving rise to a parasitic current beyond the genuine pumping one. Finally, we discuss the proposal of a quantum pump able to produce spin current without polarized electrodes in the presence of magnetic tunnel barriers. Let us note that in the literature only few generalization of the slave-boson approach to a time-dependent case are present. Among them there is the generalization of slave-boson mean-field theory for t-J model to the time-dependent regime[19] to study the Landau quasiparticle transport and the study of nonlinear transport in QD connected to reservoirs with arbitrary strength of the Coulomb interaction under finite magnetic fields[20].

The organization of the paper is the following: In Sec.2, we introduce the model Hamiltonian and derive the effective Hamiltonian by means of the slave boson approach in the strongly interacting regime. In Sec.3 we give the dot Green's functions within the Keldysh approach and the expression of the non-equilibrium pumped current, focussing on the single photon approximation. Based on an approximate solution of the constraints equation, in Sec.4 we discuss the results obtained for the pumped current both in the case of a charge quantum pump and in the case of a spin quantum pump. The conclusions are given in Sec.5.

2. The interacting quantum dot model

We consider an interacting quantum dot (QD) coupled to noninteracting leads in which the current is generated by means of the temporal modulation of two out-of-phase gate voltages controlling the transparency of the tunneling barriers.
The Hamiltonian of the system is:

$$H = \sum_{k\sigma\alpha} \varepsilon_k c^\dagger_{k\sigma\alpha} c_{k\sigma\alpha} + \sum_\sigma \epsilon\, d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow + \sum_{k\sigma\alpha} (V_{k\alpha\sigma}(t) c^\dagger_{k\sigma\alpha} d_\sigma + h.c.), \tag{1}$$

where the operator $c^\dagger_{k\sigma\alpha}$ creates an electron of momentum k and spin σ in the lead α, while $d^\dagger$ is the creation operator of an electron state on the interacting quantum dot, ϵ being the energy of a single occupied electron state. The third term is the electron-electron (e-e) interaction energy U which comes into play when the dot screening length λ_s is bigger than the typical size of the quantum dot. The last term describes the tunneling between the quantum dot and the leads.
We focus on the strongly interacting limit ($U \to \infty$) where the isolated QD can be only in the empty and in the singly occupied states $\{|0\rangle, |\sigma\rangle\}$ (where $\sigma =\uparrow,\downarrow$) while double occupancy is completely suppressed. By introducing the reference state $|\Omega\rangle$ and the operators b (slave-boson) and f_σ (pseudofermion) we may define: $|0\rangle = b^\dagger|\Omega\rangle$ and $|\sigma\rangle = f^\dagger_\sigma|\Omega\rangle$. Thus the creation/annihilation original operators can be written in terms of bosonic (i.e. b) and quasi-fermionic (i.e. f_σ) operators[17, 18] as: $d_\sigma \to b^\dagger f_\sigma$, $d^\dagger_\sigma \to f^\dagger_\sigma b$ with the extra constraint on the single occupation of the dot which can be introduced by a Lagrange multiplier λ.

Within the slave boson representation the Hamiltonian becomes:

$$H_{SB} = \sum_{k\sigma\alpha} \varepsilon_k c^\dagger_{k\sigma\alpha} c_{k\sigma\alpha} + \sum_\sigma \epsilon f^\dagger_\sigma f_\sigma + \sum_{k\sigma\alpha} (V_{k\alpha\sigma}(t) c^\dagger_{k\sigma\alpha} b^\dagger f_\sigma + h.c.) + \lambda(b^\dagger b + \sum_\sigma f^\dagger_\sigma f_\sigma - 1). \quad (2)$$

Here the Lagrange multiplier λ is fixed by the equation:

$$\partial_\lambda \langle H^{dot}_{SB} \rangle = 0 \to \sum_\sigma \langle f^\dagger_\sigma f_\sigma \rangle + \langle b^\dagger b \rangle - 1 = 0, \quad (3)$$

while the slave boson operator b evolves in time according to the equation of motion $i\hbar\partial_t b = [b, H_{SB}]$[21]:

$$i\hbar\partial_t b = \lambda b + \sum_{k\sigma\alpha} V_{k\alpha\sigma}(t) c^\dagger_{k\sigma\alpha} f_\sigma. \quad (4)$$

We treat the slave boson operator b within the mean field approximation ($\langle b \rangle = \mathcal{B}$, $\langle b^\dagger \rangle = \mathcal{B}^*$) and the original problem containing strong correlations is replaced by a constrained free-fermions-like theory whose dynamics is completely described by the following Hamiltonian:

$$\begin{aligned} H_{SBMF} &= H_{leads} + \sum_\sigma (\epsilon + \lambda(t)) f^\dagger_\sigma f_\sigma + \sum_{k\sigma\alpha} (V_{k\alpha\sigma}(t) \mathcal{B}^*(t) c^\dagger_{k\sigma\alpha} f_\sigma + h.c.) + \\ &+ \lambda(t)(|\mathcal{B}(t)|^2 - 1), \end{aligned} \quad (5)$$

with the dynamical equation for the constraints:

$$i\hbar\partial_t \mathcal{B}(t) = \lambda(t)\mathcal{B}(t) + \sum_{k\sigma\alpha} V_{k\alpha\sigma}(t) \langle c^\dagger_{k\sigma\alpha} f_\sigma \rangle \quad (6)$$

$$\sum_\sigma \langle f^\dagger_\sigma f_\sigma \rangle + |\mathcal{B}(t)|^2 - 1 = 0. \quad (7)$$

As shown in (5) the interaction renormalizes both the QD energy level ($\epsilon \to \epsilon + \lambda(t)$) and the tunneling amplitudes ($\bar{V}_{k\alpha\sigma}(t) \to V_{k\alpha\sigma}(t)\mathcal{B}^*(t)$). Let us note that due to the time evolution of the slave boson field, which is governed by the equation (6), a dynamical phase shift with respect to the phase of the external driving signals will appear.

3. Non-equilibrium Green's function method

3.1. The quantum dot Green's function

To derive the current flowing through the system via a quantum pumping mechanism we employ the non-equilibrium Green's functions (NEGF) formalism[22]. In our scheme of pumping we modulate the tunneling rates $\Gamma^\alpha_\sigma(t) = \Gamma^\alpha_{\sigma,0} + \Gamma^\alpha_{\sigma,\omega} \sin(\omega t + \varphi_\alpha)$ defined by $\Gamma^\alpha_\sigma(t) = 2\pi\rho_\alpha |V_{k_F\alpha\sigma}(t)|^2$. The retarded GF of the quantum dot (QD) uncoupled from the external leads is ($\hbar = 1$):

$$g^r_{sp}(t, t') = -i\delta_{sp}\theta(t - t') \exp\{-i \int_{t'}^t dt_1 (\epsilon + \lambda(t_1))\}, \quad (8)$$

being $s, p \in \{\uparrow, \downarrow\}$ the spin index. It has a form similar to that of the non-interacting system except for the appearance of the Lagrange multiplier $\lambda(t_1)$ in the exponential. When the QD is coupled to the leads, we must take into account the transition rate of an electron through the system and thus:

$$G^r_{sp}(t, t') = g^r_{sp}(t, t') \exp\{-\int_{t'}^t dt_1 \sum_{\alpha=l,r} \frac{\Gamma^\alpha_s(t_1)}{2} |\mathcal{B}(t_1)|^2\}, \quad (9)$$

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where the exponential comes from the retarded self-energy:

$$\Sigma^r_{sp}(t_1,t_2) = -i\delta_{sp}\delta(t_1-t_2)\sum_{\alpha=l,r}\frac{\Gamma^\alpha_s(t_1)}{2}|\mathcal{B}(t_1)|^2, \tag{10}$$

which depends on the renormalization factor $|\mathcal{B}(t)|^2$. The corresponding advanced quantities, i.e. G^a and Σ^a, are computed using the symmetry properties of the two-times Green's function and are obtained directly from the relation $\chi^a_{sp}(t_1,t_2) = \chi^r_{ps}(t_2,t_1)^*$, χ being G or Σ. Finally, the Langreth rules[23] can be employed to compute the *lesser* self-energy as

$$\Sigma^<_{sp}(t_1,t_2) = i\delta_{sp}f(t_1-t_2)\sum_{\alpha=l,r}\Gamma^\alpha_s(t_1)|\mathcal{B}(t_1)|^2, \tag{11}$$

where $f(t_1-t_2)$ is the anti-Fourier transform of the Fermi function $f(E)$ (notice that the chemical potential is the same for both the leads).

In order to calculate the current flowing through the QD we need to calculate the *lesser* GF $G^<_{sp}(t_1,t_2)$ of the QD exploiting the Keldysh equation[24] and by using $G^{r/a}$ and $\Sigma^<$. Together with the single particle Green's function we need yet to solve the constraint equation. Explicitly we can rewrite Eq.(6) and (7) as:

$$\begin{aligned} i\partial_t\mathcal{B}(t) &= \Big[\lambda(t)+\sum_\sigma G^<_{\sigma\sigma}(t,t)\frac{\Gamma_\sigma(t)}{2}\Big]\mathcal{B}(t)+ \qquad (12)\\ &+ \sum_\sigma\int dt_1 G^r_{\sigma\sigma}(t,t_1)\mathcal{B}(t_1)\Gamma_\sigma(t_1)f(t_1-t)\\ |\mathcal{B}(t)|^2 &= 1+i\sum_\sigma G^<_{\sigma\sigma}(t,t). \qquad (13)\end{aligned}$$

Previous equations along with the H_{SBMF} completely describe the physics of the interacting quantum dot in the infinite-U limit.

3.2. Non-equilibrium currents

Within the non-equilibrium theory the spin resolved current pumped in the lead α can be written in terms of the QD Green's function in the following form:

$$\begin{aligned} I^\sigma_\alpha(t) &= \frac{2e}{\hbar}\sum_{\sigma'}Re\Big\{\int\frac{dE_1dE_2dE_3}{(2\pi)^3}e^{i(E_3-E_1)t}[G^r_{\sigma\sigma'}(E_1,E_2)\Sigma^<_{\alpha,\sigma'\sigma}(E_2,E_3)+ \qquad (14)\\ &+ G^<_{\sigma\sigma'}(E_1,E_2)\Sigma^a_{\alpha,\sigma'\sigma}(E_2,E_3)]\Big\},\end{aligned}$$

where the two-times Fourier transform of the GF are needed in a non-equilibrium situation since translational time invariance is broken. Here we are interested in deriving the pumping current in an almost adiabatic situation, i.e. at small ($\omega \ll U/\hbar$) but finite frequency of the pump, thus we expand all the relevant time dependent quantities by considering only the so called *single-photon* contributions as follows:

$$\begin{aligned} \lambda(t) &= \sum_{n=0,\pm1}\lambda_n\exp\{-in\omega t\} \qquad (15)\\ \Gamma^\alpha_\sigma(t) &= \sum_{n=0,\pm1}\Gamma^\alpha_{\sigma;n}\exp\{-in\omega t\}\\ \mathcal{B}(t) &= \sum_{n=0,\pm1}\mathcal{B}_n\exp\{-in\omega t\},\end{aligned}$$

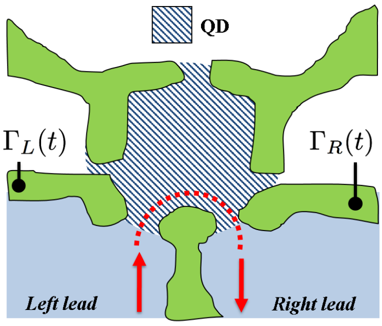

Figure 1. The system described in the main text: A Quantum dot (dashed region) coupled to external leads via tunnel barriers whose transparency can be modulated in time via top gates.

while the Green's function and the self-energy become of the form:

$$\chi^j_{sp}(E_1, E_2) = \delta_{sp} \sum_{\eta=0,\pm1} \chi^j_{s;\eta}(E_1)\delta(E_1 - E_2 + \eta\omega), \tag{16}$$

where χ is G or Σ, $j =<, r, a$. As will be carefully explained in Sec. 3.4, the single photon approximation is well justified under the assumptions of (a) adiabatic ($\omega \ll U/\hbar$) driving signals and (b) small modulation amplitudes (weak pumping). Assuming simultaneous validity of (a) and (b), the structure of the effective Hilbert space, lacking of the double occupied state of the quantum dot, is suitable to get a correct description of the time-dependent case. Within the single-photon approximation, the spin resolved dc current $\bar{I}^\sigma_\alpha$ pumped in the lead α takes the form:

$$\bar{I}^\sigma_\alpha = (2e/\hbar) \sum_{\eta=\pm1} Re[\int \frac{dE_1}{(2\pi)^3}[G^r_{\sigma;\eta}(E_1)\Sigma^<_{\alpha\sigma;-\eta}(E_1+\eta\omega) + G^<_{\sigma;\eta}(E_1)\Sigma^a_{\alpha\sigma;-\eta}(E_1+\eta\omega)], \tag{17}$$

which contains information about the pumping cycle and the absortion/emission processes of one photon. Within the single-photon approximation the behavior of the pumped current is mainly determined by integrals of products of the function:

$$\mathcal{D}^\sigma_\eta(E) = [E + \eta\omega - (\epsilon + \lambda_0) + i(\gamma_\sigma/2)]^{-1}, \tag{18}$$

where the $\eta = 0, \pm1$, λ_0 is the static part of the Lagrange multiplier and γ_σ is a renormalized linewidth depending on the slave boson field:

$$\gamma_\sigma = \sum_\alpha \Big[|\mathcal{B}_0|^2\Gamma^\alpha_{\sigma;0} + 2Re\{|\mathcal{B}_0|^2\Gamma^\alpha_{\sigma;1} + \Gamma^\alpha_{\sigma;0}(\mathcal{B}^*_0\mathcal{B}_1 + \mathcal{B}_0\mathcal{B}^*_{-1})\}\Big]. \tag{19}$$

As shown in (19) the finite frequency effects modifies the mean lifetime of the electron on the QD due to the interference effects originated by the slave boson field $\mathcal{B}_{\pm1}$. Moreover, let us note that the Fourier components of the slave boson field are a complex quantities, $\mathcal{B}_{\pm1,0} = |\mathcal{B}_{\pm1,0}|\exp\{i\phi^{\pm,0}_b\}$, i.e. they can be expressed in term of a phase, thus interference terms of the form $\cos(\phi^\pm_b - \phi^0_b)$ appear in $\mathcal{D}^\sigma_\eta(E)$. The existence of such dynamical phases (and the corresponding interference terms) provides a decoherence source for the electron transport. Furthermore, since $\Gamma^\alpha_{\sigma;\pm1}$ picks up a phase due to the external driving signals $\Gamma^\alpha_{\sigma;\pm1} = \pm i\frac{\Gamma^\alpha_{\sigma,\omega}}{2}\exp\{\mp i\varphi_\alpha\}$, the GF of the problem depends separately, on $\varphi_{L,R}$ and not on the phase difference $\varphi = \varphi_R - \varphi_L$. This will imply the presence of *parasitic* currents, as we will see below.

3.3. Constraint equations within the single photon approximation

As we have seen above, the current pumped through the quantum dot can be calculated from the Green's function. Since it depends parametrically on $\{\lambda_0, \lambda_{\pm 1}; \mathcal{B}_0, \mathcal{B}_{\pm 1}\}$ one has first to solve the constraint equations. By exploiting the Fourier expansions of Eqs.(12)-(13) within the one-photon approximation one gets a set of coupled equations that can be approximately solved. In particular, one considers the corrections beyond the adiabatic limit. From the analysis of the parametric expression of the current pumped one observes that the relevant quantities to be computed are: λ_0, λ_1, $|\mathcal{B}_0|^2$ and $\mathcal{B}_0^*\mathcal{B}_1 + \mathcal{B}_0\mathcal{B}_{-1}^*$. Since we are modulating the tunneling amplitudes via top gates, we can derive an approximate expression of $\lambda_{\pm 1}$ and $\mathcal{B}_0^*\mathcal{B}_1 + \mathcal{B}_0\mathcal{B}_{-1}^*$ to the first order in $\Gamma_{\sigma;1}$. The constraint equations to the lowest order are:

$$\lambda_0 = -\sum_\sigma \Gamma_{\sigma;0} \int \frac{dE}{2\pi} f(E) Re\{\mathcal{D}_0^\sigma(E)\} \tag{20}$$

$$|\mathcal{B}_0|^2 = 1 - \sum_\sigma |\mathcal{B}_0|^2 \Gamma_{\sigma;0} \int \frac{dE}{2\pi} f(E) |\mathcal{D}_0^\sigma(E)|^2.$$

These equations have the same structure as in the adiabatic case[9] but implicitly depend on $\mathcal{B}_{\pm 1}$ contained in $\mathcal{D}_0^\sigma(E)$, thus higher order constraints have to be considered in order to solve the problem. In particular, the equation which needs to be considered is the one for $\mathcal{B}_0^*\mathcal{B}_1 + \mathcal{B}_0\mathcal{B}_{-1}^*$ whose formal solution is[16]:

$$\mathcal{B}_0^*\mathcal{B}_1 + \mathcal{B}_0\mathcal{B}_{-1}^* = \frac{-\sum_\sigma |\mathcal{B}_0|^2 \Gamma_{\sigma;1} \int \frac{dE}{2\pi} \mathcal{D}_0^\sigma(E) \mathcal{D}_{-1}^{\sigma *}(E) f^{(-)}(E)}{1 + \sum_\sigma \Gamma_{\sigma;0} \int \frac{dE}{2\pi} \mathcal{D}_0^\sigma(E) \mathcal{D}_{-1}^{\sigma *}(E) f^{(-)}(E)}, \tag{21}$$

where $f^{(\pm)}(E) = f(E \pm \omega)$. Expanding the r.h.s. of Eq.(21) up to the first order in $\Gamma_{\sigma;1}$ and taking the limit $\omega \to 0$, the dependence on $\mathcal{B}_{\pm 1}$ disappears and thus Eqs.(20)-(21) can be solved independently from the other equations. Following a similar procedure, one can also solve the equation for λ_1 whose solution is:

$$\lambda_1 \approx \frac{\lambda_0}{2} \sum_\sigma |\mathcal{B}_0|^2 \Gamma_{\sigma;1} \int \frac{dE}{2\pi} |\mathcal{D}_0^\sigma(E)|^2 f(E) \Big|_{\mathcal{B}_{\pm 1} \to 0}. \tag{22}$$

The solution for the constraints are then plugged in the expression (17) to numerically obtain the current.

3.4. Validity of the non-equilibrium slave-boson approximation

Before presenting the results for the pumped current, let us comment on the validity of the time-dependent slave boson approximation. It is well established that the slave boson method works well in the strong U-limit for a static problem. As for the time-dependent case, we consider the weak-modulation (small amplitude of the pumping parameter) and almost adiabatic pumping limit which ensure that the double occupied quantum dot state remains empty, while the net electron flow is driven from one reservoir to another by absorbing or emitting an energy quantum $\hbar\omega \ll U$ from the driving signals. In general, an arbitrary time-dependent driving signal allows multiple photon-assisted tunneling (mPAT) processes where the absorption/emission of n photons takes place, with an absorbed/emitted energy $n\hbar\omega$. When $n\hbar\omega$ is comparable to U, the quantum dot double occupation becomes allowed and the slave boson picture described in our work is not applicable. However, under the simultaneous assumptions of: (a) $\hbar\omega \ll U$ and (b) small amplitude modulation of the pumping signals the time-dependent slave boson procedure should provide a (non-trivial) perturbation of the static slave boson theory. Here, the assumption (b) ensures that multi-photon processes are less probable compared to the single-photon absorption/emission. It's also interesting to notice that for a small quantum dot (tens of nanometers in length) the charging energy U takes values in the meV range. This implies that $U/\hbar$ takes value 1.5 THz (assuming U= 1 meV) and thus pumping frequencies ω up to the GHz range (i.e. the relevant experimental range) can be safely considered as adiabatic. Within this framework the single photon approximation is well justified and the structure of the Hilbert space, lacking of the double occupied quantum dot state, remains similar to that of the static case. In this respect, the time-dependent slave boson theory presented here provides small, but relevant, perturbations to the well established slave-boson static method. In general, the method is not applicable to the generic time-dependent case (strong amplitude pumping, non-adiabatic signals).

4. The pumped current

4.1. The charge current

Here we discuss the results of the current pumped in the right-lead by assuming $\Gamma_0 = 10\ \mu eV$ as unit of energy, while the current is normalized by $i_0 = -e\Gamma_0/\hbar$ ($i_0 \approx 2.56$ nA) and the temperature is fixed to zero.

In Fig.2 we show in the panel (d) the current pumped in the right lead I_R (continuous line) as a function of the position of the bare level of the dot ϵ and by setting the remaining parameters as follows: $\Gamma_{\sigma;0}^{\alpha=L/R} = 1.5$, $\Gamma_{\sigma;\omega}^{\alpha=L/R} = 0.05$, $\omega = 0.05$, $\varphi_L = 0$, $\varphi_R = \pi/4$. Moreover, the tunneling does not depend on the spin, and thus $I_R = I_R^\sigma$, i.e. the current is spin-independent. The lower curve in panel (d) of Fig.2 represents the contribution to the current due to the modulation of the constraint $\lambda(t)$, while the upper curve is the current pumped by the modulation of the tunnel barriers. As shown when $\epsilon \gg \mu$ the current induced by $\lambda(t)$ is vanishing and the total current is dominated by the contribution of the tunnel amplitudes; on the other hand, when the energy level of the QD ϵ is much lower then the Fermi energy, the current induced by the modulation of $\lambda(t)$ is the dominating one. The behavior of the current shown in Fig.2 can be explained when looking at the behavior of the slave boson parameters as a function of $(\epsilon-\mu)/\Gamma_0$. Panel (a) of Fig.2 shows $|\mathcal{B}_0|^2$ *vs* $(\epsilon - \mu)/\Gamma_0$ which is proportional to $(1 - n)$ where n is the average occupation on the QD. The average occupation is going to zero when the energy level ϵ is well above the Fermi energy and the tunneling amplitudes is weakly renormalized, while a strong renormalization is obtained in presence of a non-vanishing electron density on the QD, i.e. when its occupation is close to one. A similar effect is evident in panel (b) of Fig.2 where λ_0 is plotted as a function of $(\epsilon - \mu)/\Gamma_0$. When $\epsilon \leq \mu$ the effect of the constraints becomes dominant. Another effect of the constraints becomes clear when we look at the absolute value of the oscillation amplitude of $\lambda(t)$, i.e. $|\lambda_1|$ (see panel (c)). As shown, below the Fermi energy, $|\lambda_1|$ becomes comparable to the oscillation amplitudes of the tunneling rates Γ_σ^α ($\approx 0.05\Gamma_0$) and thus the pump starts to be affected by the dynamics of the constraints. Indeed, comparing panel (c) and (d) one observes that the λ-induced current (red curve) is enhanced when $|\lambda_1|$ is close to its maximum value.

In Fig. 3 the pumped charge current in the left and right lead is shown as a function of the pumping phase φ_R. Differently from the adiabatic ($\omega \to 0$) and noninteracting case, the external phase difference $\varphi_R - \varphi_L$ is not the only relevant phase. Actually, in the strongly interacting limit terms of the form $\sin(\varphi_\alpha \pm (\phi_b^\pm - \phi_b^0))$ appear in the charge current. These interaction-induced terms are of the same order of magnitude of the perturbation induced by the external driving signals and thus strongly affect the usual $\sin(\varphi)$-behavior. In particular, the modification of the current-phase dependence due to the interaction produces rectification contributions ($I_{rect}(\varphi_L = 0, \varphi_R = 0) \neq 0$) similar to those reported in a different context in Ref. [25]. Here we do observe that at varying the bare QD energy level ϵ the system is guided through the resonance $(\epsilon+\lambda_0)$ and the current changes its sign (see lower red dashed line). Thus in the strongly interacting case the presence of new phase differences in the pumped current is the main modification compared to the free-fermion case.

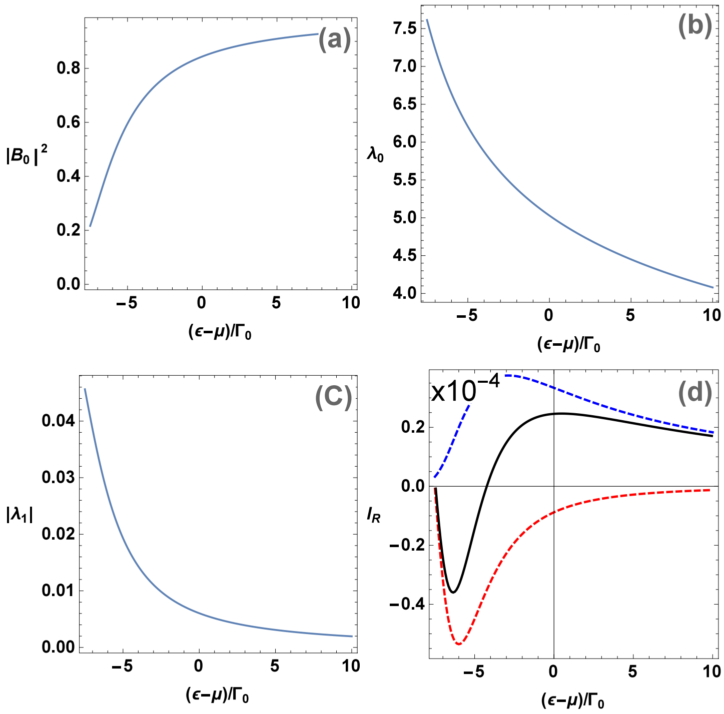

Figure 2. The top-panels (a)-(b) show the holes density $|\mathcal{B}_0|^2$, and λ_0 as a function of $(\epsilon-\mu)/\Gamma_0$, while the lower panels (c)-(d) show $|\lambda_1|$ and the total charge current I_R (full line) as a function of $(\epsilon-\mu)/\Gamma_0$ computed setting the remaining parameters as follows: $\Gamma_{\sigma;0}^{\alpha=L/R} = 1.5$, $\Gamma_{\sigma;\omega}^{\alpha=L/R} = 0.05$, $\omega = 0.05$, $\varphi_L = 0$, $\varphi_R = \pi/4$. The dashed lower curve (red) in panel (d) represents the current induced by the modulation of λ, while the upper curve (dashed-blue curve) depends only on the modulation of the tunneling amplitudes.

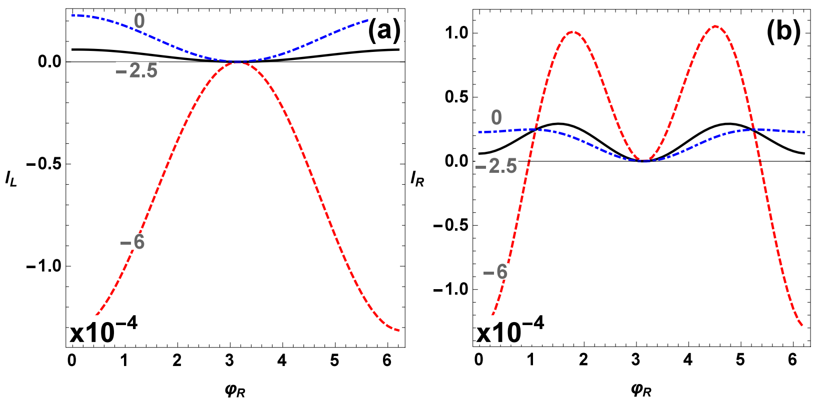

Figure 3. Charge current I_L (left panel) and I_R (right panel) as a function of the pumping phase φ_R computed fixing the remaining parameters as follows: $\Gamma_{\sigma;0}^{\alpha=L/R} = 1.5$, $\Gamma_{\sigma;\omega}^{\alpha=L/R} = 0.05$, $\omega = 0.05$, $\varphi_L = 0$. For both panels the curves are obtained for different values of the QD energy level $(\epsilon-\mu)/\Gamma_0$ as indicated in the figure labels. Data are rescaled by a factor 10^{-4}.

4.2. Spin pumping
A spin-pump can be obtained when the tunneling amplitudes depend explicitly on the spin e.g. as: $\Gamma^{\alpha}_{\sigma}(t) = \Gamma^{\alpha}(t)(1+p\sigma)$, where p is a polarization. This situation can be realized when the QD is connected to the external non-magnetic leads via thin magnetic tunnel barriers[26, 27]. The magnetic barriers, in fact, work as a spin-filter (SF) whose effect is controlled by the polarization $p \in [0,1]$ of the barriers. We assume that the polarization p is independent on time, while the tunneling amplitude $\Gamma^{\alpha}(t)$ is taken of the form:

$$\Gamma^{\alpha}(t) = \Gamma^{\alpha}_0 + \Big[\frac{\Gamma^{\alpha}_{\omega}\exp(i\varphi_{\alpha})}{2i}\exp(i\omega t) + c.c.\Big]. \tag{23}$$

In the presence of the ac modulations of the external gates, along with the spin-selective action of the barriers, a spin current $I_s = \sum_{\sigma}\sigma I_{\sigma}$ is generated beyond charge current $I_c = \sum_{\sigma} I_{\sigma}$. In Fig.4 we study the charge (full line) and spin (dashed-dotted line) currents as a function of the bare energy level ϵ of the QD setting the remaining parameters as in the figure caption. For non-vanishing polarizations (i.e. $p = 0.4$) of the barriers a spin current is observed. Furthermore when $\epsilon \approx \mu - 5\Gamma_0$ the charge current takes negligible values and the pump works as a pure spin current pump (Fig.4a). Let us note that the pure spin current obtained here is generated in absence of polarized leads thus avoiding the problem of the spin injection. Our analysis shows that an all-electrical control of the spin current is possible and is potentially useful in spintronics devices.

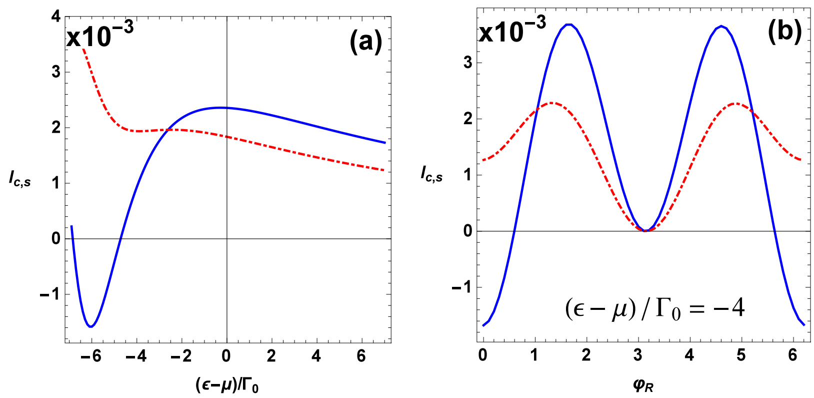

Figure 4. Charge current I_c (full line) and spin current I_s (dashed-dotted line) pumped in the right lead as a function of $(\epsilon-\mu)/\Gamma_0$ (left panel) and of the phase φ_R (right panel) by setting the remaining parameters as follows: $\Gamma_0^{\alpha=L} = 1.5$, $\Gamma_0^{\alpha=R} = 1.3$, $\Gamma_{\omega}^{\alpha=L/R} = 0.3$, $\omega = 0.05$, $p = 0.4$, $\varphi_L = 0$. In the left panel the phase bias is fixed as $\varphi_R = \pi/4$. Data are rescaled by a factor 10^{-3}.

5. Conclusions
We reviewed a time-dependent slave-bosons approach within a non-equilibrium Green's function formalism to study the quantum pumping of charge and spin in an interacting quantum dot in the limit of infinite Coulomb repulsion. The pump based mechanism is the time periodic modulation of two tunnel barriers whose transparencies are governed by the external top gates. In particular, we showed an equation of motion of the slave boson field $\mathcal{B}(t)$ which is equivalent to an infinite series of constraints, generalizing the adiabatic case. By using a finite set of constraints (single photon approximation) the expressions of the relevant Green's functions and

of the current have been derived. We showed that the pumped charge current contains, beyond a term related to the modulation of the out-of-phase external parameters, an additional term due to the internal dynamics of the Lagrange multiplier and the slave boson field (parasitic pumping current). While for a noninteracting quantum dot model the current depends only on the phase difference between the two external parameters, $\varphi_R - \varphi_L$, in the strongly interacting limit it depends separately on $\varphi_{L/R}$. The additional pumping contributions are originated by the phase difference of the external parameter phase (for instance φ_R) and the phase of an internal degrees of freedom (the slave boson field or the Lagrange multiplier). This mechanism is similar to that obtained in the presence of dynamical effects of a mechanical degree of freedom coupled to the QD[14].
Finally we showed how in the case of spin-dependent tunnel barriers with a finite polarization, the pump works as a spin current generator. Furthermore, by changing the energy level of the QD the pump can work as a pure spin current generator. This effect shows the possibility of all-electrical control of the spin current pumped through a QD and is relevant for spintronics.

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