

In-Service Teachers' Understanding on the Concept of Limits and Derivatives and the Way They Deliver the Concepts to Their High School Students

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Abstract. The aim of this study was to analyze the teachers' understanding on the concept of limits and derivative and the way they deliver the subjects to their students. The study was divided into two main phases during two years of research. This research was conducted in 7 high schools vary from general, Islamic and occasional schools. The participants of the study were 20 in-service mathematics teachers from 7 high schools. Questioners were given to find out how teachers' understanding on the concepts and how they organized their class. The teachers' level of complexity on the subject was analyzed by Structure of the Observed Learning Outcome (SOLO) Taxonomy, and teachers' class organizations was analyzed by assessing and classifying their responds written on the questioner sheets and discussion with the selected participants.. Based on the data, it can be figured out that the most teachers' position were on third level out of five level of SOLO Taxonomy. Data also told us that half of the teachers experienced the difficulties in teaching the concept due to their limitations on mastering subject. Data also showed that there is a relevance between teachers' level of understanding and teachers' ability in delivering subject to their students.

1. Introduction

Many issues arising from an activity which is the evaluation of the results may need to be discussed. One of the problems that arise from time to time when we talk about teaching and learning was the difficulty faced by students in learning. Learning about the concept of limit and derivative at first year university level, is essentially a continuation of subjects that were already introduced at the secondary school level. It the limit and derivative were introduced at high school in last year, then at university level, they were introduced in calculus course in the first year.

As lecturers for first year calculus course, researchers discovered quite a number of students who were experiencing difficulties so that they considered the calculus course was kind of a very difficult subject, particularly on limit and derivative. This might be due to their less mastered the initial subject which they head learned in school. Some previous researches also showed that students had lack understanding on some topics in calculus. Among others, [1] found that more than 45% of students faced problems in understanding subjects on Calculus I during their first year.

On the other side, based on the interviews with high school teachers, it was found that majority of them admitted that their students had a lack understanding on the limit and derivative concepts. The unfamiliarity if the students about the calculus in high school level would make their difficulties in



understanding the calculus at university level. In contrast, if learning about limit and derivative in mathematics school was going well, then students will be able to follow the calculus course at university level as well.

Deeper conversation and discussion with several teachers drove us to situations that teachers also admitted that even they themselves were not sure some concepts on limit and derivative. However, teachers' mastering and understanding the concepts and how they organized their classes related to the concepts would be a 'bridge' to the calculus course at the first year university level.

Chick in [6] suggested that discussion of teaching and learning are inseparable from the two aspects the teachers' capability, namely contents and pedagogy. Similar to this, [7] defined domains of 'mathematical content knowledge for teaching framework' in which contains two domains; 'Subject Matter Knowledge' and 'pedagogical content knowledge'. Subject Matter Knowledge (SMK) is related to level of understanding about the concept, while Pedagogical Content Knowledge (PCK) is related to the way subject is delivered to the students.

Related to the above problem, researchers were interested in conducting research on how in-service teachers' understanding on the concept of limit and derivative and the way they deliver the concepts to their high school students, especially in the city of Padang, the Capital of West Sumatera Province, Indonesia.

2. Teaching and Learning Calculus at High School

Calculus is one field of study in mathematics that have started to be introduced in high school. The main topics in high school calculus are basically include two things, differential and integral calculus. The emergence of these two concepts is based on the way we used to resolve problems that occur in everyday life. According to the Indonesian curriculum, differential calculus topics were introduced in the second semester of grade XI, while integral calculus topics were introduced in grade XII. The prerequisite for learning calculus, students must have a good understanding on real numbers and functions. Students will have difficulties in understanding a concepts if they had a lack of understanding on its prerequisite contents [2]. Students' understanding of derivative required the mastery of previous important concepts which are interlinked, such as functions, limit, slope, continuity, and rate of change [3]. By being lack of understanding those concept will affect students have difficulties in understanding the derivative concepts [4]. In fact, [5] figured out that difficulties and misconception students had in studying calculus were more likely in derivative, limit, and relationship with rate of change.

In Indonesian curriculum for high school level, the core competences for mathematics and natural sciences are to understand, implement, and analyze factual knowledge, conceptual and procedural. To these core competences, the goals of teaching and learning calculus in high school are: i) to understand and to analyze the concepts and properties of trigonometric functions, algebraic functions towards infinity as well as applying them in solving various problem; ii) understand the concepts of derivative of trigonometric functions and derive their properties and use them in solving problems; iii) understand the concepts and nature of derivatives of trigonometric and algebraic functions and to apply them to determine stationary points (points of maximum, minimum and turning point); iv) Analyze forms of mathematical models in the forms of equation of functions, as well as applying the concepts in nature of the derivative and the tangent line of the curve in estimating the value of the functions and values of the roots of algebraic equations.

As a learning processes becomes more complex, SOLO, which stands for the Structure of the Observed Learning Outcomes, is means of classifying outcomes in terms of their complexity, enabling us to assess students' works in terms of its quality, not of how many bits of this and of that they got right [8].

2.1. Subject Matter Knowledge

A teacher will not probably be able to teach what they do not know. Therefore, teachers' capability on mastering the content (Subject Matter Knowledge) is a must. Fennema and Franke in [9] stated that no

one questions the idea that that what a teacher knows in one of the most important influences on what is done in classroom and ultimately what students learn.

Based on mathematics curriculum for Indonesian high schools in differential calculus topics, a teacher was required to have a good understanding on material (subject content knowledge) that covered topics, especially of: i) real numbers; ii) function; iii) limit and continuity of functions; and, of course; iv) derivative and indeed all have linkages very closely.

For example, about the definition of the derivative of the function at a point a was defined as a value of

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \quad (1)$$

if h is set as small as possible towards zero, showing how function and limit function were required in discussing the concept of derivative. Understanding functions was the key to other concepts in mathematics, including calculus and its applications. Therefore, teachers were needed to have a good understanding and to master the subject properly. The concept of derivative itself can be represented in several ways: numerical representations, symbolic representations, and graph representation.

2.2. Pedagogical Content Knowledge

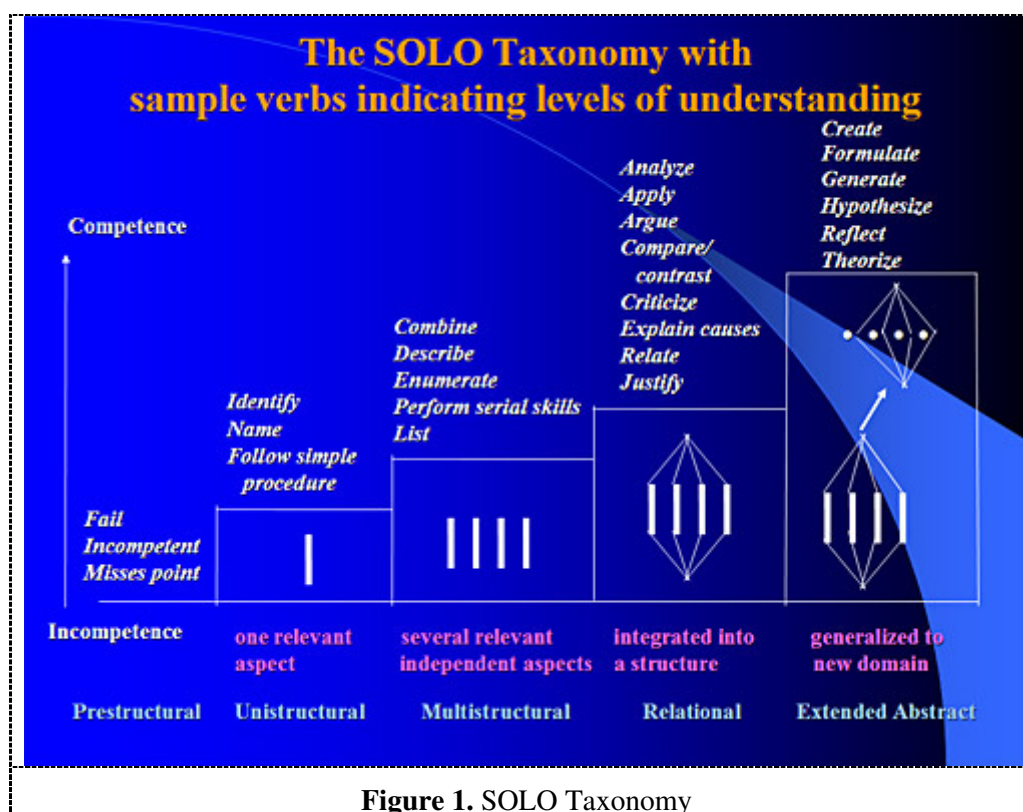
Pedagogical Content Knowledge (PCK) has become the focus of much research in mathematics and science education [10]. Mumfords in [11] recommended that in teaching calculus, there were at least three teaching methodologies to note. The first was to deliver the concepts so that it was remembered and understood; the second was using the variety of numerical and visual examples of everyday life which were experienced or felt by students; the last was that how problems to be presented which was based on theory of logic, and made identification of where concepts can apply.

Similar to this, [12] also noted that as educators, we constitute a profession that prides itself on professionalism, on ethos of quality performance and rigorous accountability since 'knowing something for oneself or for communication to an expert colleague is not the same as knowing it for explanation to a student'.

3. SOLO Taxonomy

SOLO Taxonomy which stands for Structure of the Observed Learning Outcomes developed by [8]. If Bloom separate knowledge from the intellectual abilities, or process that operate on this knowledge, so that Bloom Taxonomy is a systematic way of describing how a learner performance. SOLO Taxonomy provides a measure of cognitive learning outcomes or understanding of thinking. Surface or deep level of understanding can be planned for and assessed by coding a students' thinking performance.

SOLO Taxonomy is primarily based on the processes of understanding used by students or someone when answering the prompts. It means that knowledge, therefore, permeates across all levels of SOLO Taxonomy. The SOLO Taxonomy category is shown in the following figure.



Based on the SOLO Taxonomy above, it was acceptable to measuring teachers' complexity on knowledge of a concept by using SOLO Taxonomy. There are five levels or five categories of thinking performance in SOLO Taxonomy as follow:

Table 1. Level of SOLO Taxonomy

Level	Classification
1	Prestructural
2	Unistructural
3	Multistructural
4	Relational
5	Extended Abstract

4. Research Methods

This study was a descriptive research that was conducted in high school in Padang City, Capital of West Sumatera Province, Indonesia. This study could be divided into two terms or two phases. This first year of research was aimed at collecting information on how in-service teachers' understanding about the concepts of limit and derivative and the way they delivered the concepts to their students, based on their own information through a sheet of questionnaire given. For the second year, which is next year, the research will be conducted to figure out more comprehensively on how in-service teachers master the concept of limit and derivative and what steps do they follow in introducing and

teaching concepts of limit and derivative, including classroom observation. From the questionnaire, the teachers' responds on contents or subject were analyzed to figure out in what level of SOLO Taxonomy the teachers' complexity of understanding on limit and derivative topics. This was done by relating teachers' explanations with the level of verbs indicating level of understanding that was described by SOLO Taxonomy Table. For the questions on the way they delivered the subject and how their students in the class, data from teachers' responds will be collected to get more comprehensive pictures on what happened during the teaching and learning process. The relevance between the level of SOLO Taxonomy of teachers and how they teach related subject was also analyzed by looking at linear correlation between them.

The participants of this study were mathematics teachers from several general high schools, vocational schools and Islamic high schools around Padang city. From those schools, by using the purposive sampling, the total of 20 mathematics teachers were selected as research participants. They were asked to fill out the questionnaire. The distributions of those teachers regarding to kind of schools are as follows:

Table 2. Distribution of teachers from various schools

Schools	Numbers of teachers selected
Public General High School	7
Private General High School	8
Public Vocational School	2
Private Vocational School	1
Islamic High School	2

In this study, in order to collect relevant data, questionnaire was designed to be filled out by those selected teachers. The questionnaire were given to teachers to sought their understanding on the concepts of limit and derivative, specifically on their Subject Content Knowledge, as well as their Pedagogical Content Knowledge, including their opinion and their feeling regarding to what they originally did in their classroom . For the second year, or next year, there will be more data to be collecting, including the direct classroom observation as well as further questionnaire and interviews.

In every given question in the questionnaire, research participants were asked not only to write the answer, but they were also asked to explain their reasons. Questions about their understanding on the concepts of limit and derivative were analyzed by assessing their complexity of their answer. Overall, data from questionnaire about Subject Content Knowledge, or data from the description of the Subject Matter Knowledge and the teachers' answers or their responds regarding to the concepts on limit and derivative were analyzed by using the Structure of the Observed Learning Outcomes (SOLO) Taxonomy. Data acquired from teachers' responds regarding to their Pedagogical Content Knowledge, were analyzed by describing and listing the way they delivered subject matter in their classrooms.

5. Results and Discussions

The results were organized into two major sections. The first focused on the subject matter knowledge owned by teachers and were classified according to SOLO Taxonomy. The second examines classrooms' atmosphere towards the way teachers were teaching in their classrooms. Based on data from questionnaire, it can be noted some findings as follows:

5.1. Description of Teachers Understanding on the Concept of Limit and Derivative

Two Questions in questionnaire (question one and question two) required 20 in-service teachers to explain what the limit and derivative are respectively. Based on SOLO Taxonomy with sample verbs indicating level of understanding in Figure 1, teachers were asked to explain what limit and derivative is. There were told to explain as best as they could. Teachers' answers were classified and put into how deep and how comprehensive their responds. If teachers gave a wrong answer or let their sheet blank, or no respond for the question, then it was classified into prestructural. If teachers are just able to explain limit or derivative as text book definition, then by sample verb in figure 1, it was classified into unistructural. If teachers are able to explain in more detail, or gave appropriate example then it was classified into *multistructural*. Some teachers were also able to explain limit and derivative, and made some connections with others aspect such as simple applications in daily life, then their responds were classified into relational. Teachers who were able to explain, to give examples, explaining formula related to limit and derivative, and made connections between applications, showed their deep understanding on limit and derivative, then they were categorized as they reaches an extended abstract of SOLO Taxonomy. Table 3 provides the number of teachers awarded each of the five level (see Table 1) used to categorise the range of teachers' responds.

Table 3. Number of teachers in each of level of SOLO Taxonomy.

Level	Limit	Derivative
Prestructural	1	2
Unistructural	7	10
Multistructural	8	5
Relational	4	3
Extended Abstract	0	0

Of 20 in-service teachers, none of them were able to provide comprehensive explanation of limit nor derivative. One teacher left the question one remained blank and two teachers did the same for question eight. Majority of them just rewrite definitions as they were provided in text books. Several of them were able to explain and gave related examples.

5.2. Major Findings on Teachers' Answer related to Teaching and Learning Process

Questionnaire also consisted of some questions to figure out what teachers' approaches when delivering the limit and derivative, and what they did during their teaching.

Question three and question four required in-service teachers described what approached they used in introducing the concept of limit and derivative, and their responds were provided in Table 4 below.

Table 4. Some approaches used by teachers when introducing concept of limit and derivative

Introducing Limit	Introducing derivative
By definition of limit	Starting back with limit points
By definition of limit and some examples	Introduce the rate of change
Ask students to measure an object until smaller size	Using the graph
Give students a number then approached from left to right	Explain the definition of derivative
By using sentences with a word 'almost,	Giving examples in daily life

near, close to

Using line number

Motivate students with the goal and
advantage of learning derivative

Using tangent line

Table 4 told us that teachers, in general, used various approaches in introducing limit and derivative concepts. From teachers' responds, it was noted that teachers use various approaches as they also admitted that those ideas they picked up from what text books provided. Most teachers then noted "*I introduce limit to students by letting them know what the definition of limit is*".

Question five was set to find out what teachers' thought about topics on limit that high school students were usually have trouble with. Their responds were quite similar. Some of them gave the same answers. For example, there were 7 teachers pointed out that students have trouble in checking limit by separated them into left and right limit. Table 5 and Table 6 indicated what teacher's though about their students related to some difficult topics in limit. Based on the teachers' view, part of limit and derivative contents that were hard by students.

Table 5. Part of limit contents that were hard by students according to teachers' point of view

Topics	Number of teachers who answer (out of 20)
left and right limit	7
Existence of a limit at a point	7
Relationship between limit and continuity	5
Limit of trigonometric function	10
Determine the value of limit at a point	1
Basic definition of limit	8

Ten out of 20 teachers acquired that limit of trigonometric functions was commonly a hard for students. Some teachers wrote "*I have to explain limit of trigonometric function several times to make them understand*", while other said "*It seems that students hardly understand when they have to determine the limit on trigonometric functions*". An interesting fact was that to determine the value of limit at a point was not very difficult for students, and most of teachers argued that majority of students usually calculate a limit of function at a point by simply inserting that number to the function. It followed the teachers' thought that students' understanding on limit basically was not very well.

Question six was given to collect information from teachers about what efforts did they do in helping students who had trouble in understanding limit or derivative. The teachers' responds showed us that teachers used various efforts that could be seen in the following table:

Table 6. Some efforts used by teachers when helping students who had trouble in understanding concept of limit or derivative

Efforts	Number of teachers who did the effort (out of 20)
Repeat explanation	7
Give more examples	5
Add more exercise with different types	4
Ask students to remember and understand the formula	3
Ask students to study the prerequisite subject matter	1
Ask student to read more books	1
Use additional media	2
Provide simpler definition	1

From Table 6, it can be noted that most teachers tended to repeated explanation or gave more examples whenever students did not understand what they taught, just one teacher tried to motivate their students to read more books or asked them to learn the prerequisite subject. One teacher also responded in more detail “I also used media to explain derivative. I show them different tangent lines of a graph of a given function”.

Question seven was aimed at finding information on what application the teachers gave as examples in teaching limit and derivative. Table 7 indicated applications in derivative that were usually given by teachers

Table 7. Application of derivative that were usually given to students

Topics	Number of teachers who answer (out of 20)
Distance, velocity, acceleration	20
Maximum height of throwing ball	3
Calculate the marginal point	3
Calculate the growth rate	7
Specify gradient and tangent line	4
Time limitation and deadline for students in finishing task	1
Volume of sphere	1

The important finding in this study was that there were some teachers who were not sure the concept of limit and derivative so they did not answer the question. In fact, according the data, 50% of teachers admit that who did not have difficulties in understanding the concepts. Most teachers usually just repeat explanation, rather than trying different efforts in helping students who were in trouble in understanding limit or derivative. Most teachers tend to but not limited to use distance, velocity, and acceleration as application of derivative. Another interesting fact was that one teacher said that he used volume of sphere as kind of application of derivative that he explained to his students, but he failed to give more explanation when researcher asked what kind of problem that was used.

6. Conclusion

Based on the questionnaire filled out by research participants, it could be noted that majority of teachers are in level unistructural and multistructural. None of them are in the extended abstract level. It meant that none of them have complexity understanding on the concepts. Even though they used various approach in delivering the concept, their lack of mastering the concepts affected their capability in delivering concepts. If their students did not understanding what were being taught and asked, teachers tended to just repeat explanations, rather than explain more detail in many ways. Questionnaire also gave us info that teachers' examples in application of derivative were quite monotonous, most of them have no idea but limited to discuss about distance, velocity and acceleration. Questionnaire also informed us that limit of trigonometric function were one of the topics that seemed to be difficult for students.

Based on information about the level of teachers' understanding on limit and derivative, and the way they delivered the subject to their students based on the open questioner they filled out, it gave us some pictures about what teaching and learning calculus topic, particularly on limit and derivative at high school level. Data also told us that the better teachers' understanding on limit and derivative, or the higher level of SOLO Taxonomy they were, then they deliver the subject to their students better. To get more information, university lecturers and high school teachers might sit together and discuss about what can be done to help high school students as well as university students learn calculus.

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References

- [1] Desfitri, R 2008 Students' understanding on prerequisite topics of Calculus I (in Indonesian language) *Preprint* gr-qc/0401010
- [2] Diekei R, and Isleten T 2004 Investigation of relation and function of some variables on the challenges *J. Kastamonu Education* **11** (2) 105-116
- [3] Bingolbali E 2008 Suggestions for learning difficulties and conceptual understanding of the concepts of derivative *J Mathematical Misconception and Solution* 223 - 253
- [4] Kultur M 2011 Identifying the learning difficulties of freshmen in mathematics teachers training department in Function, Graph, and Derivative *J Humanities and Social Science* **1** (7) 119 - 124
- [5] Balci 2008 *General Mathematics* (Ankara: Balci Press)
- [6] Chick H 1998 Cognition in the Formal Modes: research Mathematics and SOLO Taxonomy *Mathematic Education Journal* **10** (2) 4-26
- [7] Ball D, Thames M and Phelps G 2008 Content Knowledge for Teaching: What makes it Special *Journal of Teacher Education* **59** 389-407
- [8] Biggs J B and Collis K F 1982 *Evaluating the quality of learning: the SOLO Taxonomy* (New York: Academic Press)
- [9] Fennema E and Franke M L 1992 Teachers' knowledge and its impact *Handbook of research on mathematics teaching and learning* (New York: MacMillan)
- [10] Shulman L S 1987 Knowledge and teaching: Foundation of the new reform *Harvard Educational Review* **57**(1) 1-22
- [11] Mumford D 1997 Calculus Reform – For the millions *Notices of the AMS* **44** (5) 559-563
- [12] Bass H 1997 Mathematicians as educators *Notices of the AMS* **44** (1) 18-21