

A fuzzy goal programming model for biodiesel production

D S Lutero¹, E M U Pangue¹, J M Tubay¹ and S P Lubag^{1,2}

¹ Mathematics Division, Institute of Mathematical Sciences and Physics, University of the Philippines Los Baños, Laguna, Philippines 4031

E-mail: dslutero@up.edu.ph

Abstract. A fuzzy goal programming (FGP) model for biodiesel production in the Philippines was formulated with Coconut (*Cocos nucifera*) and Jatropha (*Jatropha curcas*) as sources of biodiesel. Objectives were maximization of feedstock production and overall revenue and, minimization of energy used in production and working capital for farming subject to biodiesel and non-biodiesel requirements, and availability of land, labor, water and machine time. All these objectives and constraints were assumed to be fuzzy. Model was tested for different sets of weights. Results for all sets of weights showed the same optimal allocation. Coconut alone can satisfy the biodiesel requirement of 2% per volume.

1. INTRODUCTION AND LITERATURE REVIEW

Biodiesel production has been in place since 1991 to address the pressing need for a renewable source of diesel and diminished carbon emission. To sustain biodiesel production however requires external incentives such as tax exemptions or subsidies provided by the government [1]. Moreover, biodiesel are mostly harvested from food crop feedstocks [2]. Consequently, it has been seen as a cause of increase in food prices between 2003 and 2008. Extraction of biodiesel from non-food feedstock such as jatropha and algae is seen as a solution to the food problem [3].

Republic Act 9367 (Biofuels Act of 2006) mandates the 2% blend of biodiesel per volume of diesel in the country. The current feedstock source of biodiesel in the country is coconut (*Cocos nucifera*). Jatropha (*jatropha curcas*) was also identified as a non-food feedstock source for biodiesel. Although Philippines is one of the top coconut oil producers in the world, RA 9367 mandates an increase in biodiesel blend of up to 10% per volume by 2020. The price and supply of coconut oil in the local and international market may be affected by the increase in blend.

Biodiesel production requires intensive production planning that involves steady supply of raw materials and use of efficient and cost effective production processes. Moreover, feedstocks have food or non-food uses other than for biodiesel. Optimal allocation of feedstock is essential to maintain a desirable trade-off among its uses.

Several authors have used multiple objective decision making (MODM) approaches in production planning. Goal programming is often used for problems that involve multiple objectives such as that in production planning. Originally introduced by Charnes and Cooper

² Present address: de La Salle University Dasmariñas, Cavite, Philippines 4115



(1955), GP can arrive at an optimal solution to multiple objectives [4]. The goal programming approach transforms an originally multiple objective problem into a single goal [4]. Moreover, most real world problems function under an uncertain environment that involves imprecise and inexact data. Fuzzy set theory of Zadeh enables the inclusion of vagueness, imprecision and inexactness in models through the use of membership functions [5]. It was in 1980 that Narasimhan first integrated fuzzy set theory in goal programming [6]. Since then, fuzzy goal programming has been used in production planning and renewable energy management [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

In this paper, a fuzzy goal programming model was formulated to determine an optimal allocation of feedstocks used in biodiesel production given food and non-food requirements. Coconut and jatropha were the biodiesel feedstock sources. Objectives and constraints were assumed to be fuzzy. Models were tested using different sets of weights.

2. FUZZY GOAL PROGRAMMING

2.1. Fuzzy Set Theory

Let U be a universal set and a set $A \subseteq U$. Unlike an ordinary mathematical set, a fuzzy set is defined by a membership function, often denoted by μ , that takes on function values ranging within the interval $[0, 1]$. If A is considered a fuzzy set, then the degree of membership of x to A , denoted by $\mu_A(x)$ can be any value between 0 and 1 inclusive. A higher degree of membership of x in A implies a closer value of $\mu_A(x)$ to 1. With the definition of a membership function, fuzzy sets allow partial membership of elements in a (fuzzy) set.

2.2. Fuzzy Set Theory in Goal Programming

Fuzzy set theory was integrated in goal programming formulations by using membership functions that allow inexactness, ambiguity and imprecision of parameters in the model.

The conventional goal programming approach converts a multiple (linear) objective problem to a conventional (linear) single objective problem by assigning goals to each of the objectives and combining all these goals in a single objective function [4]. Both objectives and constraints can be treated as fuzzy and will take the form of one of the fuzzy goal types.

2.3. Fuzzy Goal Types

Fuzzy goals can either be " \lesssim " (at most), " \gtrsim " (at least) or " \approx " (almost equal to). In the discussion, we will only consider the first two types: $AX \lesssim b$ or $AX \gtrsim b$.

If t_l is the allowed tolerance for the l^{th} fuzzy goal of the form $AX_l \lesssim b_l$, then the membership function is

$$\mu(AX_l) = \begin{cases} 1 & \text{if } AX_l \leq b_l \\ 1 - \frac{AX_l - b_l}{t_l} & \text{if } b_l \leq AX_l \leq b_l + t_l \\ 0 & \text{if } AX_l > b_l + t_l \end{cases}$$

Similarly, if t_g is the tolerance for g^{th} fuzzy goal of the form $AX_g \gtrsim b_g$, the membership function is

$$\mu(AX_g) = \begin{cases} 1 & \text{if } AX_g \geq b_g \\ 1 - \frac{b_g - AX_g}{t_g} & \text{if } b_g - t_g \leq AX_g \leq b_g \\ 0 & \text{if } AX_g < b_g - t_g \end{cases}$$

The general fuzzy goal programming formulation can be defined as

Maximize

$$\sum_{l=1}^L \mu((AX)_l) + \sum_{g=1}^G \mu((AX)_g) \quad (1)$$

subject to

$$\begin{aligned} \mu((AX)_l) &\leq 1 - \frac{(AX)_l - b_l}{t_l}, \forall l \in \{1, \dots, L\} \\ \mu((AX)_g) &\geq 1 - \frac{b_g - AX}{t_g}, \forall g \in \{1, \dots, G\} \\ X &\geq 0, \mu((AX)_l) \geq 0, \mu((AX)_g) \geq 0 \end{aligned}$$

The objective function can be modified such that weights are assigned to each of the fuzzy goals. Then Equation 1 becomes

$$\sum_{l=1}^L \omega_l \mu((AX)_l) + \sum_{g=1}^G \omega_g \mu((AX)_g) \quad (2)$$

where ω_l and ω_g is the weight assigned to l^{th} and g^{th} fuzzy goal, respectively.

3. MODEL FORMULATION

What makes every general model powerful is its flexibility to be modified and tailor-fitted according to the assumptions and data of a particular problem. As many objectives and constraints can be integrated in the model provided the availability of data. More importantly, FGP model can accommodate assumptions on imprecision and uncertainty.

We chose feedstock allocation for the 2% blend requirement of biodiesel in the Philippines as the problem where choice of objectives, constraints and their targets and tolerances were based.

The general modelling process using fuzzy goal programming can be summarized in these steps:

1. Define each objective and constraint
2. Assign target value(goal) and tolerance to each fuzzy objectives and constraint
3. Formulate objective function

3.1. Identify and define objectives and constraints

3.1.1. Identification and definition of variables The variables in the general model are defined as follows

f	–	index of feedstock $f \in \{1, 2, \dots, F\}$
P_f	–	amount of feedstock f produced (ton)
TP_f	–	total production target for feedstock f (ton)
D_f	–	amount of diesel from feedstock f (liters/ton)
DY_f	–	amount of yield (ton) of feedstock f allocated to biodiesel production
NDY_f	–	amount of yield (ton) of feedstock f allocated to non-biodiesel production
EP	–	expected revenue (PHP) for all feedstock f
I_f	–	average investment (PHP/ton) for feedstock f
WC	–	working capital for farming (PHP)
E_f	–	energy requirement (KW-hr/ton) to produce a ton of feedstock f
TE	–	total energy available (KW-hr)
DR	–	annual biodiesel requirement
NDR	–	annual non-biodiesel requirement
L_f	–	area of land (ha) used to yield a ton of feedstock f
TL_f	–	expected total area of land (ha) available for feedstock f
LR_f	–	labor requirement (man-days/ton) required to produce a ton of feedstock f
TLR	–	expected total available labor (man-days)
RD_f	–	revenue (PHP/liter) from diesel produced from feedstock f
RND_f	–	revenue (PHP/liter) from non-biodiesel use produced from feedstock f
MT_f	–	required machine time to produce a ton of feedstock (hours/ton) f
TMT	–	expected annual machine hours available (hours)
W_f	–	volume of water (cubic meters) required for a ton of feedstock f
TW	–	expected total volume of water (cubic meters) available for irrigation

3.1.2. Fuzzy objectives description and formulation These were the objectives considered in the model for biodiesel production. All objectives are considered fuzzy.

1. Feedstock production
Maximize production output of biodiesel feedstock sources such that output should be greater than or equal to the target production output.
2. Overall revenue
Maximize overall revenue for every feedstock such with its allocation being between biodiesel and non-biodiesel uses.
3. Energy requirement
Minimize energy used in processing feedstocks to biodiesel to avoid waste of energy resources.
4. Working capital for farming
Minimize working capital for farming that includes purchasing of land and machinery, establishment of planting area among others.

3.1.3. Fuzzy constraints description and formulation All these constraints have fuzzy right hand side values.

1. Biodiesel Requirement
According to RA 9367, a 2% blend is required for all diesel sold in the Philippines before 2015.
2. Non-biodiesel requirement
The non-biodiesel requirements such as food or non-food demands for every feedstock should be met.

3. Land Availability

Every feedstock uses land during a given period of time. The land occupied by the feedstock should not exceed the total available land provided for the feedstock.

4. Labor Availability

The manpower who will handle the feedstock farming should not exceed total available labor.

5. Machine Time Availability

Total machine time for farming and processing the feedstock should not exceed the machine time available.

6. Water Availability

Every feedstock should be provided the minimum amount of water for it to grow but should not exceed the total amount of water available.

3.1.4. Non-fuzzy Constraint Description and Formulation The total yield per feedstock should be greater than or equal to the amount of feedstock allocated to biodiesel and non-biodiesel requirements.

$$P_f \geq DY_f + NDY_f \forall f \quad (3)$$

3.2. Assign target value(goal) and tolerance to each fuzzy objectives and constraint

The fuzzy objectives and constraints are reformulated in terms of their corresponding membership function. Every fuzzy objective and constraint is assigned with a target value and tolerance.

If the o^{th} fuzzy objective is of the " \gtrsim " form, let

$$\alpha_o \leq 1 - \frac{b_o - AX_o}{to_o} \Rightarrow AX_o + to_o(1 - \alpha_o) \leq b_o$$

So the fuzzy objectives of this type are transformed as

$$P_f + to_1(1 - \alpha_{1f}) \geq TP_f \forall f \in \{1, 2, \dots, F\} \quad (4)$$

$$\sum_{f=1}^F DY_f RD_f + \sum_{f=1}^F NDY_f RND_f + to_2(1 - \alpha_2) \geq EP \quad (5)$$

Similarly, if the o^{th} fuzzy objective is of the " \lesssim " form, let

$$\alpha_o \leq 1 - \frac{AX_o - b_o}{to_o} \Rightarrow AX_o - to_o(1 - \alpha_o) \leq b_o$$

So the fuzzy objectives of this type are transformed as

$$\sum_{f=1}^F E_f DY_f - to_3(1 - \alpha_3) \leq TE \quad (6)$$

$$\sum_{f=1}^F I_f P_f - to_4(1 - \alpha_4) \leq TI \quad (7)$$

Following the same transformation for the fuzzy constraints, the c^{th} fuzzy constraint with membership indicator β_c and tolerance tc_c is defined as

$$\sum_{f=1}^F D_f DY_f + tc_1 (1 - \beta_1) \geq DR \quad (8)$$

$$NDY_f + tc_2 (1 - \beta_2) \geq NDR_f \forall f \quad (9)$$

$$L_f P_f - tc_3 (1 - \beta_3) \leq TLR_f \forall f \quad (10)$$

$$\sum_{f=1}^F LR_f P_f - tc_4 (1 - \beta_4) \leq TLR \quad (11)$$

$$\sum_{f=1}^F MT_f P_f - tc_5 (1 - \beta_5) \leq TMT \quad (12)$$

$$\sum_{f=1}^F W_f P_f - tc_6 (1 - \beta_6) \leq TW \quad (13)$$

3.3. Formulate objective function

The objective of fuzzy goal programming is to find a solution that maximizes membership of all goals and constraints.

Based on the formulation, the weighted sum should be maximized to obtain the maximum membership grade. The objective function is formulated as

$$Z = w_1 \left(\sum_{f=1}^F a_f \alpha_{1f} \right) + \sum_{j=2}^4 w_j \alpha_j + w_5 \left(\omega_1 \beta_1 + \omega_2 \beta_2 + \omega_3 \sum_{f=1}^F \rho_f \beta_{3f} + \sum_{j=4}^6 \omega_j \beta_j \right) \quad (14)$$

where $w_j, j \in 1, 2, 3, 4$ correspond to weights associated with the objectives while w_5 is the total weight for all constraints. Note that $\sum_{j=1}^5 w_j = 1$. a_f is the sub-weight for production of feedstock f . $\omega_j, j \in \{1, 2, 3, 4, 5, 6\}$ are the sub-weights for each of the six constraints, and ρ_f is the sub-weight for the land availability constraint of feedstock f .

The weights denote the contribution of the goals and constraints to the value of the objective function. In decision-making, unless all weights are equal, the goals or constraints with greater weight are those that are more important to the decision-maker.

3.4. Final FGP model

The general FGP model is

Maximize Equation 14
subject to
Equations 4 to 7
Equations 8 to 13
Equation 3
nonnegativity constraints

4. ILLUSTRATIVE EXAMPLE

4.1. Data for model

Data were collected from different government units in the Philippines, offices from University of the Philippines Los Baños and Jatropha Project of the same university.

Table 1. Crisp numerical coefficients

Feedstock	RD_f	RND_f	E_f	I_f	D_f	L_f	LR_f	MT_f	W_f
Coconut ($f = 1$)	23,940	1,000	253,4679	2,600	630	0.2	17.65	5.05	0.49
Jatropha ($f = 2$)	20,000	0	253.4679	6,562	250	0.17	9.22	2	0.49

These are the crisp numerical coefficients of the fuzzy objectives and constraints.

Table 2. Target and Tolerance of Fuzzy Objectives

Fuzzy Objective	Target	Tolerance
Production (ton)		
Coconut	12,877,605	128,776.05
Jatropha	2,160	21.6
Overall revenue (PHP)	169,095,045,000	1,690,950,450
Energy Requirement (Kw-hr)	17,031,000,000	170,310,000
Working capital for farming (PHP)	35,796,527,900	357,965,279

The target values for each objective shown in the table were taken from various government agencies namely, Department of Energy, Bureau of Agricultural Statistics and Philippine Coconut Authority. Tolerance for every fuzzy objective was estimated to be 1% of the target.

Table 3. Target and Tolerance of Fuzzy Constraints

Fuzzy Constraint	Target	Tolerance
Biodiesel Requirement	173,000,000	1,730,000
Non-biodiesel requirement		
Coconut	1,113,912.83	11,139.1283
Jatropha	0	0
Land availability		
Coconut	3,379,740.90	337,974.09
Jatropha	1,100,000	11,000
Labor availability	962,480,000	9,624,800
Machine-time availability	1,320,000,000	13,200,000
Water availability	431,000,000	4,310,000

The target values for the constraints were taken from the same sources of Table 2. Tolerance for every fuzzy constraint was also estimated to be 1% of the target.

4.2. Results and sensitivity analysis

The model was ran using GAMS with CPLEX 12.1.0 as solver. Results were obtained in less than 0.03 seconds for all sets of weights.

Table 4. Resulting values for variables

Variable	Value
P_1	12,878,000
P_2	2,160
DY_1	274,600
DY_2	0
NDY_1	12,603,000
NDY_2	0

The model was tested for several sets of weights (equal and unequal). The results were the same for all sets of weights used. All the goals and constraints were satisfied for both equal and unequal weights. This means that 2% biodiesel blend in all diesel sold in the country is feasible. Coconut alone can satisfy the biodiesel requirement.

Table 5. Ranging Analysis for the Right Hand Side Values

Equation	Lower Limit	Current RHS Value	Upper Limit
Coconut Production	8,401,000	12,880,000	13,760,000
Jatropha Production	0	2,160	352,800
Overall Revenue	$-\infty$	169,100,000,000	258,600,000,000
Energy Requirement	69,600,000	17,030,000,000	$+\infty$
Working Capital for Farming	33,500,000,000	35,800,000,000	$+\infty$
Biodiesel Requirement	0	173,000,000	7,411,000,000
Non-biodiesel requirement	$-\infty$	1,114,000	12,600,000
Land for coconut	2,576,000	3,380,000	$+\infty$
Land for jatropha	367.2	1,100,000	$+\infty$
Labor availability	227,300,000	962,500,000	$+\infty$
Machine-time availability	65,040,000	1,320,000,000	$+\infty$
Water availability	6,311,000	431,000,000	$+\infty$

The right-hand side values for each constraint shown above can be adjusted within the specified range one at a time without making the solution infeasible. For example, the current value for the coconut production is 12,880,000 tons of coconut. We can decrease the production to as low as 8,401,000 tons and increase it to as high as 13,760,000 tons without making the solution infeasible.

5. CONCLUSION AND DISCUSSION

This paper proposed a fuzzy goal programming model for biodiesel production in the Philippines. Results showed that all objectives and constraints of the study were satisfied for both equal and unequal weights. This means that the 2% biodiesel blend requirement is possible. Coconut alone can provide for the 2% blend requirement. Jatropha can act as buffer if coconut declines in supply and fails to meet biodiesel requirement or if coconut feedstock will be allocated to non-biodiesel requirements to achieve other benefits, say, higher profit. All membership grades were equal to one which means that the solution can be improved by changing the right hand side values within the specified ranges shown in the sensitivity analysis one at a time.

6. RECOMMENDATIONS

Other feedstocks can be included in the FGP model such as algae. Increased blend requirement can also be considered to assess its feasibility. More biodiesel and non-biodiesel requirements can be incorporated in the model. Land use planning should accommodate the possible changes in land allocation for feedstock when the blend is increased. With this general formulation, FGP approach is envisioned to provide a different view in terms of decision making in the stream of biodiesel production. Math models in biodiesel production can incorporate more ambiguity and inexactness if models are considered to assume fuzziness. Moreover, decision makers have more flexibility in modifying the goals and constraints in a FGP model.

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