

# Radiation Trapping in a Cold and Dense Atomic Ensemble in a Magnetic Field

**Igor Sokolov**

Peter the Great St.Petersburg Polytechnic University, Polytechnicheskaya st. 29,  
Saint-Petersburg, 194251, Russia

E-mail: [ims@is12093.spb.edu](mailto:ims@is12093.spb.edu)

**Abstract.** We report a theoretical investigation of cooperative spontaneous decay of atomic excitation in cold ensemble under condition that average interatomic separation is comparable with resonant light wavelength. We show that magnetostatic field causes some acceleration of decay at shot time interval upon turn off the excitation pulse and essential decreasing of decay rate of long-lived sub-radiant states. The dependence of this effect on duration of excitation pulse as well as on its carrier frequency is considered.

## 1. Introduction

Analysis of time-dependent fluorescence of cold atomic ensembles excited by pulse radiation is one of the most effective methods of studying of properties of such ensembles [1]-[7]. This analysis attracts particular interest recently in connection with cooperative effects in dense atomic clouds where both mean free path of resonant photons and averaged interatomic separation are comparable with wavelength [5, 6]. In dense and cold ensembles resonant interatomic dipole-dipole interaction can cause a number of fascinating quantum optical processes, including Anderson localization of light and atom-based random lasing. Atomic fluorescence can be an indicator of these processes.

The main goal of the present work is to study theoretically the influence of static magnetic field on cooperative spontaneous decay of excited atomic states in dense and disorder cold atomic ensembles. Magnetostatic field changes essentially properties of separate atomic scatterers in the ensemble. It leads to significant modification of multiatomic effects which take place under multiple light scattering. Such modification was studied for highly excited Rydberg states in [8]. In paper [9] it was shown that magnetic field can cause transition from extended to localized quasimodes for light in a gas of immobile two-level atoms in a magnetic field. This transition takes place either upon increasing the number density of atoms in a strong field or upon increasing the field at a high enough density. In the absence of the field the transition is impossible [10].

The conclusions about the role of static magnetic field made in the paper [9] were based on numerical analysis of collective multiatomic states. It is important to verify the predictions of the work [9] experimentally. One of the possible ways to do this is to measure dynamics of afterglow of atomic clouds located in magnetic field. In the present work we analyze such possibility. We calculate dynamics of total population of all excited atomic states of all atoms in the ensemble which determines the rate of fluorescence summarized over all angles. Comparison



of results obtained for the cases with and without magnetic field gives us opportunity to reveal the role of the latter. In the present paper we restrict ourselves by the case when collective effect and particularly effects of recurrent light scattering inside considered media are important but the density is not high enough to expect localization transition. Such intermediate densities can be rather easily achieved in dipole trap.

## 2. Basic assumptions and approaches

At present there are several approaches to description of light interaction with atomic clouds which allow taking into consideration effects of recurrent scattering. These approaches give us opportunity to describe influence of resonant dipole-dipole interaction on optical properties of dense and cold gases [11]-[19]. In this work we use the consistent quantum-posed theoretical approach developed previously in [18]. In the framework of this approach we solve the nonstationary Schrodinger equation for the wave function of the joint system consisting of  $N$  atoms and a weak electromagnetic field. We restrict the total number of states taken into account to those with no more than one photon in the field. It allows us to obtain a finite set of equations for amplitudes of states with one excited atom in the ensemble. This set of equations is solved numerically and amplitudes of the other states are calculated through found ones. The procedure allows us to find approximately the wave function of the system and consequently to analyze both the properties of atomic system and the light. In papers [18, 20] general approach is justified and laid out in detail and we will not reproduce it here. The generalization of this approach on the case of pulse excitation was performed in the paper [5]. Influence of external magnetic field was considered in [9]. The reader is referred to those papers for the theoretical development and justifications.

For the purpose of this paper we will consider a spherically-shaped atomic cloud with a radius  $R$ . The random distribution of atoms inside the cloud is assumed to be uniform, on average. Total number of atoms is  $N$ . All atoms are identical with a ground state  $J = 0$  and excited state  $J = 1$ . The latter is split into three Zeeman sublevels ( $m_e = -1, 0, 1$ ). The natural linewidths of the three Zeeman sublevels of this state are the same and are equal to  $\gamma$ . The Zeeman splitting  $\Delta$  of the three sublevels is determined by external magnetic field.

Atoms are assumed to be motionless. The influence of the residual motion typical for atomic traps is taken partially into account by means of examining the statistical ensemble of clouds with the random distribution of atoms. The main results presented below are obtained by averaging over random spatial distribution using the Monte Carlo method. The intensity of the radiation is assumed to be sufficiently small that all nonlinear effects are considered negligible. This approach, as well as the other approximations depicted above, allows us to use explicit expressions for time dependent population of total atomic excitation obtained in [5, 18, 20]. In the next section, we will use these expressions (which we do not reproduce here) to calculate the time dependent population of all excited states in dense atomic ensemble and analyze, on this foundation, the process of spontaneous decay in such ensemble.

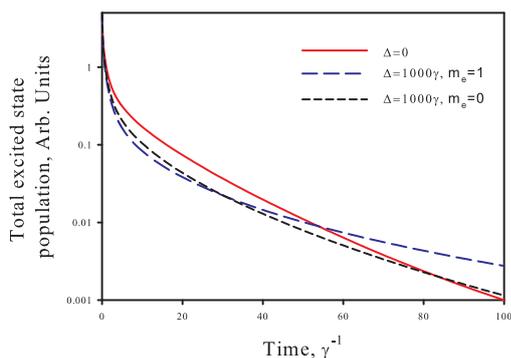
## 3. Results

All calculation in this work are performed for spherical cloud with radius  $R = 10$ . The average atomic density is  $n = 0.1$ . Hereafter in this paper we use the inverse wave number of the resonance probe radiation in vacuum  $k_0^{-1} = c/\omega$  as a unit of length. As it was shown in [18, 20, 21] considered density is high enough to guarantee manifestation of cooperative effect caused by recurrent scattering. At the same time it is not sufficient for localization transition even for strong magnetic field (see [9]).

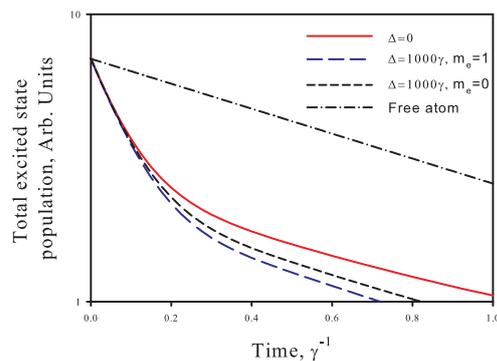
We choose the value of magnetic field strength such that Zeeman splitting is much more than collective level shifts caused by dipole-dipole interaction. Specific calculations are done for  $\Delta = 1000\gamma$ .

In the figure 1 we show the result of our calculation of dime dependent afterglow of considered ensemble excited by short pulse. The pulse duration is  $0.025\gamma^{-1}$ . This pulse has a relatively big spectral width and excites a big number of collective atomic states. The carrier frequency of the light coincides with transition frequency of the free atoms. The detuning of the carrier frequency is zero  $\delta = 0$ . The solid line in the figure corresponds to zero magnetic field. It demonstrates typical behavior. Initially time dependence is determined by superradiant collective states and the decay rate exceed that for free atom (for comparison see a figure 2). The rate gradually decreases and as time passes we see practically one-exponential decay.

The other two curves in the figure 1 describe afterglow in strong magnetic field. These two curves differ from each other in conditions of excitation. Long-dashed curve corresponds to excitation of level  $m_e = 1$  (exactly the same dependence take place for  $m_e = -1$ ), whereas the short-dashed one describes the case of  $m_e = 0$ . It is clearly seen that in due course both these curves decrease noticeably slower that solid line. Thus we see that magnetic field increases the lifetime of subradiant, long-lived states. And the influence of the field is more essential in the case of  $m_e = \pm 1$ . This difference is manifestation of optical anisotropy of the ensembles induced by magnetic field and this result confirms prediction of the paper [9].



**Figure 1.** Time dependence of the total excited state population of a uniform spherical sample of atoms. Excitation is with a temporally short,  $0.025\gamma^{-1}$  and resonant,  $\delta = 0$  pulse. Curves with  $\Delta = 1000\gamma$  describe the case of strong magnetic field, in the case when  $\Delta = 0$  magnetic field is absent.



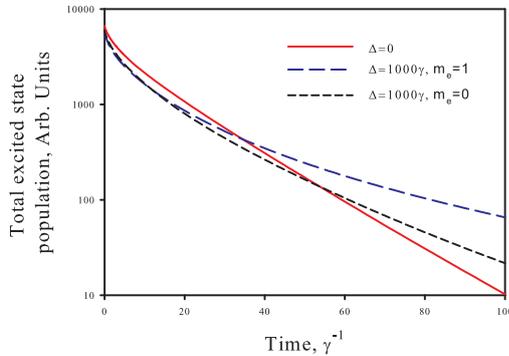
**Figure 2.** The same as in the figure 1, but for small time interval. Dash-dotted curve describes time dependence of free atom. Curves  $m_e = 0$  and  $m_e = 1$  correspond to different Zeeman sublevel excitation.

Note that decreasing of decay rate means that trapping time increases. This result seems unexpected if we analyze only far field effects. Really, magnetic field causes Zeeman splitting and thus increases resonant photon mean free path. So observed result tells us that magnetic field changes near field effects as well, i.e. dipole-dipole interatomic interaction. Moreover, this change is more important that changes in far field.

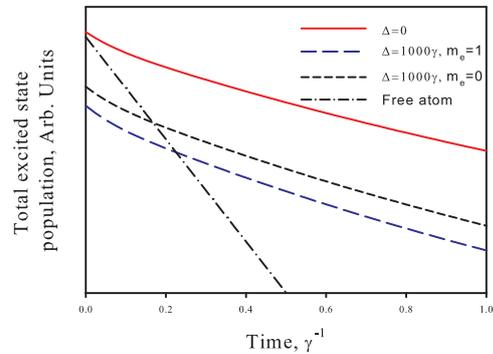
Magnetic field influences also on short-lived, superradiant states. From the figure 2 we see that magnetic field accelerates excitation decay at initial stage. And again this influence differs depending on the magnetic quantum number of the sublevels which take part in formation of corresponding collective state.

In the figures 1 and 2 we show the case of resonant excitation. For short pulse the picture varies only slightly with carrier frequency of the radiation because considered pulse covers very wide spectral region with  $\Delta\omega \simeq 40\gamma$ . For the density  $n = 0.1$  this region contains practically

all excited collective states (see for example [5, 10]). The situation changes for long pulse with narrow spectrum. Corresponding dependencies are shown in figure 3-6. These figure are drawn for long,  $1000\gamma^{-1}$  pulses.



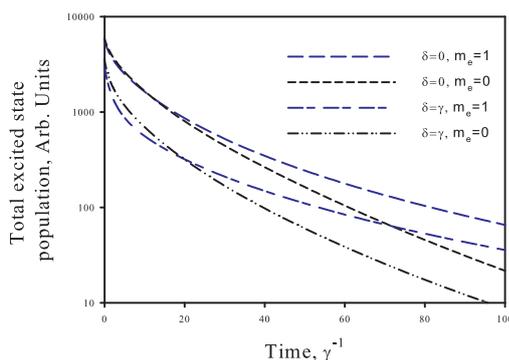
**Figure 3.** The same as in the figure 1, but for excitation with a temporally long,  $1000\gamma^{-1}$  resonant pulse,  $\delta = 0$ .



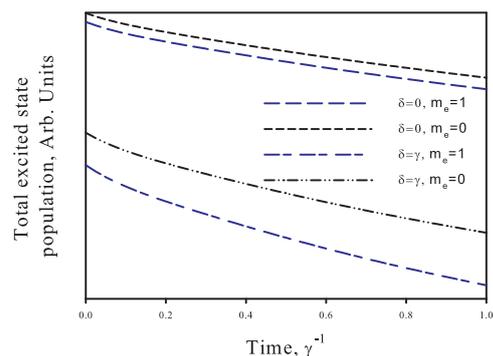
**Figure 4.** The same as in the figure 3, but for small time interval.

Figures 3 and 4 describe excitation with resonant light. The detuning of the carrier frequency of the radiation from free atom transition frequency is zero,  $\delta = 0$ .

Narrowing of spectral width of the pulse leads to variation of excitation probability for both sub- and superradiant multiatomic states. Nearby resonance frequency subradiant states are excited more efficiently. It is well seen if one compares figures 3 and 1. For big time interval increasing of pulse duration causes decreasing of decay rate. For short-lived states the situation is opposite. The excitation efficiency decreases with pulse duration (see figure 4). Moreover, for considered conditions the rate of decay is less than for free atom at the beginning of decay process. It means that subradiant states do not become apparent here.



**Figure 5.** Comparison of resonant,  $\delta = 0$ , and nonresonant,  $\delta = \gamma$ , excitation of dense atomic clouds.



**Figure 6.** The same as in the figure 5, but for small time interval.

As our analysis shows (see [9, 10]) the density of long-lived states differs for different part of the spectrum. The results shown in figures 5 and 6 confirm that. In these figures we compare dynamics of spontaneous decay in ensembles excited by long pulses with different carrier frequencies.

For radiation detuned from resonance at  $\delta = \gamma$  the efficiency of excitation decreases but the influence of magnetic field becomes far more essential. Decay rate decreases for level  $m_e = 0$  and even more for levels  $m_e = \pm 1$ . It means that in spectral region nearby  $\delta = \gamma$  the density of superradiant states is higher than nearby  $\delta = 0$ .

#### 4. Conclusion

In conclusion, we have found that the magnetic field has an influence on a collective spontaneous decay of cold and dense atomic ensembles. This influence manifests itself mainly by means of modification of dipole-dipole interatomic interaction. In our opinion experimental verification of predicted effects is of great interest. Measurement of frequency dependence of resonant fluorescence as well as its dependence on exciting pulse duration will allow one to get detailed information about peculiarities of collective effects in dense atomic clouds placed in magnetic field.

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