

Reconstruction of the local inflationary potential with different correlation levels

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Abstract. We review the puzzles of the standard Big Bang model and cosmic inflation as their possible solutions. The relation between inflation and the spectra of the cosmological perturbations is emphasized. In particular we focus on the local reconstruction of the shape of the inflationary potential from observations and the consequences of a direct detection of cosmological gravitational waves, exploring different correlation levels between the spectral index n_s and the tensor-to-scalar ratio r of the primordial perturbations.

1. Introduction

The standard Big Bang model explains important features of the universe: the Hubble expansion, the abundances of light elements (H, He, ...) and the presence and thermal nature of the Cosmic Microwave Background (CMB). Nevertheless, there are some observational evidences that remain unexplained inside this picture, the so-called puzzles of the standard cosmology [1–5]. They can be synthesized in the following questions:

- The flatness problem: why the universe looking back in time is so flat?
- The horizon problem: why the universe shows the same statistical properties also in non causally connected regions?
- The monopoles problem: where are the magnetic monopoles that were produced during the breaking of the Grand Unified Theory?

In addition, there is the problem of the origin of cosmic structures: in the standard scenario there is not a natural way to introduce the perturbations that lead to the formation of the large scale structures as well as the anisotropies in the CMB. The simplest solution to resolve these problems is to introduce an accelerated expansion of the early universe about 10^{-35} s after the initial singularity, on energy scales of the order of 10^{15} GeV [1–3], often called inflation. Mathematically, inflation is defined by $\ddot{a}(t) > 0$, where $a(t)$ is the scale factor of the universe, and it acts by producing:

- An early stretching of the constant-time hypersurfaces inducing the flatness.
- A causal horizon greater than the horizon in the standard hot big bang context.
- A dilution of the magnetic monopoles on the largest scales, reducing the possibility of their detection.



2. Inflationary cosmology and scalar fields

The simplest method to introduce an accelerated phase is to consider a neutral and homogeneous scalar field called inflaton ϕ that dominates the early times and that is minimally coupled to gravity with canonical kinetic term [1–3]. The inflaton field is also characterized by some potential $V(\phi)$ with an initial plateau or quasi plateau region, as shown in Figure 1. The cosmological action for early times becomes, in natural units $c = \hbar = 1$:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G} R - \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right\} \quad (1)$$

where R is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, g the determinant of the metric tensor and G the Newton constant. The Friedmann equations derived from this action are therefore:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho, \quad \frac{\ddot{a}}{a} = -\frac{8\pi G}{3} (\rho + 3p) \quad \text{and} \quad H(t) = \frac{\dot{a}}{a} \quad (2)$$

where H is the Hubble rate while the energy density and the pressure of the scalar field are:

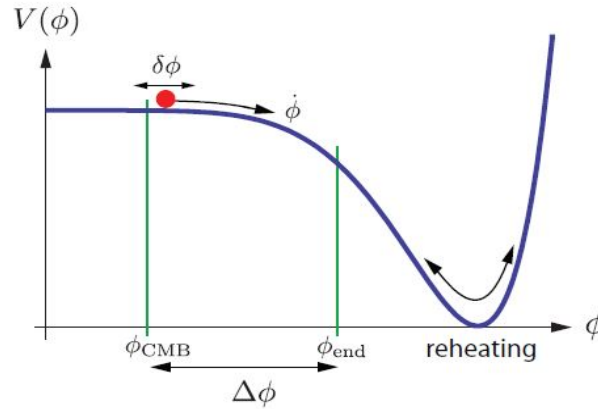


Figure 1. In the simplest scenario a scalar field is the energy source for the inflationary process.

$$\rho(\phi) = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p(\phi) = \frac{1}{2} \dot{\phi}^2 - V(\phi). \quad (3)$$

What happens is shown in Fig.1. When inflation starts the scalar field moves slowly through the quasi plateau region of the potential. Here the kinetic term is subdominant with respect to the potential one, i.e. $\dot{\phi}^2 \ll V(\phi)$, thus $\rho + 3p < 0$, realizing the acceleration $\ddot{a}(t) > 0$ in the second Friedmann equation. Later, the inflaton relaxes toward the global minimum, the expansion reduces and the final stage called reheating starts. The field oscillates and decays producing entropy and normal particles [6]. The entire mechanism can be codified by the so called slow roll parameters defined as:

$$\epsilon = -\frac{\dot{H}(t)}{H^2(t)}, \quad \eta = -\frac{\ddot{H}(t)}{H(t)\dot{\phi}(t)} \quad \text{or} \quad \epsilon_V = \frac{1}{2} M_p^2 \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \quad \eta_V = M_p^2 \left(\frac{V''(\phi)}{V(\phi)} \right). \quad (4)$$

In ϵ_V and η_V the $'$ represents the derivative with respect to the scalar field and in both cases M_p is the reduced Planck mass. In the slow roll limit $\dot{\phi}^2 \ll V(\phi)$, one has:

$$\epsilon \simeq \epsilon_V, \quad \eta = \eta_V - \epsilon_V. \quad (5)$$

Inflation takes place when $\epsilon < 1$ or $\epsilon_V \ll 1$.

3. Inflation and quantum fluctuations

In inflationary cosmology the problem of the origin of structures is also resolved [2, 3]. In the early universe the scalar field is dominant and from a quantum mechanics point of view it is characterized by quantum fluctuations $\delta\phi(x, t)$ with zero mean value in a macro time scale, by definition:

$$\phi(t) \rightarrow \phi(x) = \bar{\phi}(t) + \delta\phi(x, t).$$

Such fluctuations imply fluctuations on the stress-energy tensor $T_{\mu\nu}$ and by the Einstein equations, on the metric tensor $g_{\mu\nu}$. The accelerated expansion stretches these fluctuations on astronomical scales and here they freeze-out with a non-zero mean value in a macro time scale and become classical perturbations. These perturbations will induce the anisotropies of the CMB and will grow into the large scale structure of the universe. There are two useful quantities to describe a given cosmological perturbation field f :

- The power spectrum, which describes the presence of the perturbation on a given scale. It is defined as: $P(k) = \frac{k^3}{2\pi^2} |f(k)|^2$, where k is the wave number.
- The spectral index, which describes the variation of the perturbation with respect to the scale. It is defined as: $n(k) = \frac{dP(k)}{d \ln k}$.

Inflation excites scalar, vector and tensor modes [2, 3]. The power spectrum and the index of scalar and tensor sectors at first order in slow roll expansion are [5–7]:

$$P_s(k) = \frac{1}{8\pi^2 M_p^2} \frac{H^2}{\epsilon}, \quad n_s = 1 - 4\epsilon + 2\eta \quad \text{and} \quad P_t(k) = \frac{2}{\pi^2} \frac{H^2}{M_p^2}, \quad n_t = -2\epsilon \quad (6)$$

and it is possible to define the so called tensor-to-scalar ratio of the perturbation amplitudes:

$$r = 16\epsilon = -8n_t. \quad (7)$$

The vector sector is uninteresting because the related modes decay as the universe expands [2].

4. Reconstruction of the inflationary potential

The common way to study the inflationary phase consists in writing a lagrangian density with a given self-interaction potential, extracting the values for (n_s, r) using Eq. (4) and (5), and comparing them to cosmological data. This is particularly simple in the case of a canonical single field model. An alternative solution uses directly the data sets to constrain, at least locally, the shape of the inflationary potential [8]. This approach is justified by two pieces of evidence:

- The observable modes are stretched on astronomical scales (over the Hubble radius $R_H = c/H$, with c speed of light) about 60 e-folds before the end of inflation, when the inflaton field is on the plateau.
- The value of the inflaton field during this phase is approximately constant, ϕ_0 .

In this sense it is possible to perform a Taylor series expansion of the potential, around ϕ_0 and connect the coefficients of the expression with the inflationary observables:

$$V(\phi) = V(\phi_0) + V'(\phi_0)(\phi - \phi_0) + \frac{1}{2}V''(\phi_0)(\phi - \phi_0)^2 + \dots$$

To do this one needs an appropriate equation for the potential. Such expression is the so called Hamilton-Jacobi equation for the inflationary dynamics in which the independent variable is the inflaton field instead of cosmic time [7, 8]:

$$V(\phi) = 3M_p^2 H^2(\phi) - 2M_p^2 H'^2(\phi). \quad (8)$$

In the Hamilton formalism it is possible to define a complete hierarchy of Hubble slow roll parameters (HSRP) [7,8]:

$$\epsilon(\phi) = 2M_p^2 \left(\frac{H'(\phi)}{H(\phi)} \right)^2, \quad \beta_n(\phi) = 2M_p^2 \left(\frac{(H')^{n-1} H^{(n+1)}}{H^n} \right)^{1/n} \quad n \geq 1 \quad (9)$$

where the symbol $(n+1)$ indicates the derivative of order $n+1$ with respect to the field. These quantities codify order after order the dynamics of inflation. Using Eq. (8) and Eq. (9), and passing to the observables P_s , n_s and r by Eq. (6), at first order approximation in HSRP and up to quadratic order in $\Delta\phi$, one has¹:

$$V(\phi) = \frac{3}{2}\pi^2 M_p^4 P_s(k)r|_{\phi_0} + \frac{3}{4\sqrt{2}}\pi^2 M_p^3 P_s(k)r^{\frac{3}{2}}|_{\phi_0} \Delta\phi + \frac{\pi^2 M_p^2 P_s(k)r}{2} \left[\left(\frac{9r}{16} \right) - \frac{3}{2}(1 - n_s) \right] |_{\phi_0} \Delta\phi^2. \quad (10)$$

The derivation of this expression is given in [8].

5. Sampling the potential: results and conclusions

In this work we present the general results for the reconstruction of the potential both at first and second order in slow roll expansion, considering:

- Direct detection of inflationary gravitational waves, i.e. primordial tensor perturbations.
- Different possible correlation levels for the couple (n_s, r) .

In particular we assume that future observations will provide a Gaussian distribution both for n_s and r with a given correlation ρ . The assumed central value and standard deviation of the two distributions are $n_s = 0.968$, $\sigma_{n_s} = 0.006$ and $r = 0.05$, $\sigma_r = 0.01$ while $P_s = 20.08 \times 10^{-10}$ for $k = 0.05 \text{ Mpc}^{-1}$, compatible with the current Planck 2015 data [9]. In the first order framework we can normalize the expansion respect the 0-th order term and define:

$$V(\phi) = V(\phi_0) \left[1 + d_1 \Delta\phi + \frac{1}{2} d_2 \Delta\phi^2 \right], \quad d_1 = \frac{1}{2} \sqrt{\frac{r}{2}} \quad \text{and} \quad d_2 = \frac{1}{3} \left[\frac{9r}{16} - \frac{3}{2}(1 - n_s) \right].$$

Sampling N times the coefficients d_1 and d_2 using the previous statistics, and assuming $\rho = 0.1$ and $\rho = 0.5$ we get the results in Figures 2 and Figure 3. These plots show that in presence of higher correlation levels the distributions tend to compactify. This effect is reasonable because a more accentuate correlation constrains the ranges that the single variables can assume individually. For completeness we also report the results using the next order approximation in the slow roll parameters using expressions derived in [8]. In this case the square of the third-slow roll parameter $\xi^2(\phi)$ plays an important role and hence its physical counterpart, the running of the scalar spectral index defined as $\alpha_s = \frac{dn_s}{d \ln k}$. We assume a Gaussian distribution for it with central and standard deviation values given by $\alpha_s = -0.0084$, $\sigma_\alpha = 0.0082$, compatible with Planck data [9]. The results are showed in Figure 4 and Figure 5. In this case d_1 and d_2 are not normalized to the 0-th order term of the Taylor series expansion. The shapes of the distributions are quite different from the previous ones because of the different and more complex expressions of d_1 and d_2 . However, they show the same trend for different values of ρ . We can conclude this preliminary work summarizing some conclusions:

- The technique of reconstruction is an important tool to understand which order tends to dominate in the Taylor expansion.
- The technique is particularly sensitive to the available statistics: in the future a detection of r can improve the results (for an example, see [10]).
- The next step is to reproduce the previous analysis using an higher order Taylor expansion in the inflationary variables to get more information about the physics around ϕ_0 .

¹ In this work we assume that $\dot{\phi} < 0$ so $\sqrt{\epsilon} = +\sqrt{2}M_p H'/H$ and the first order term is positive.

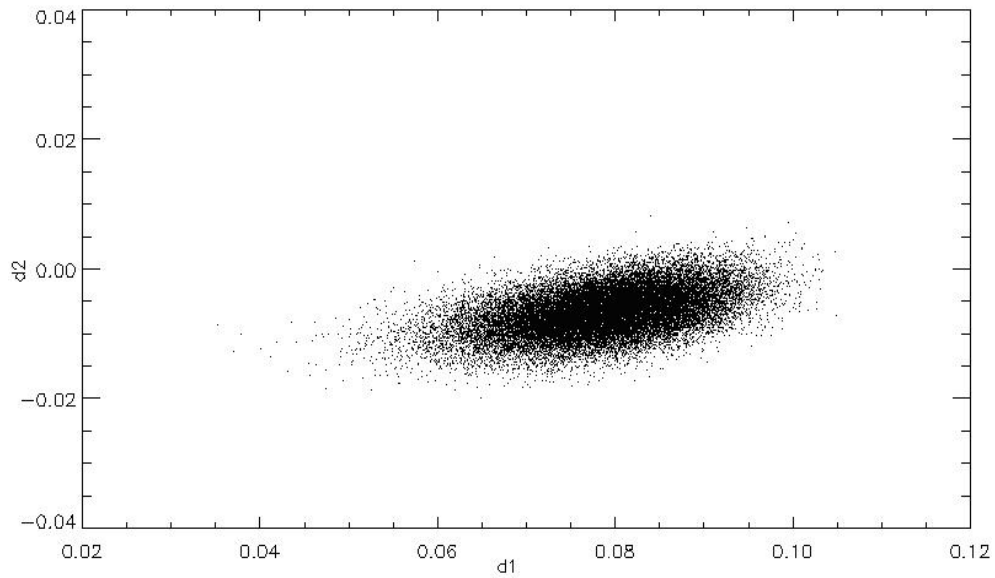


Figure 2. Distribution of d2 vs d1 at 1st order with $\rho = 0.1$ and $N=30000$.

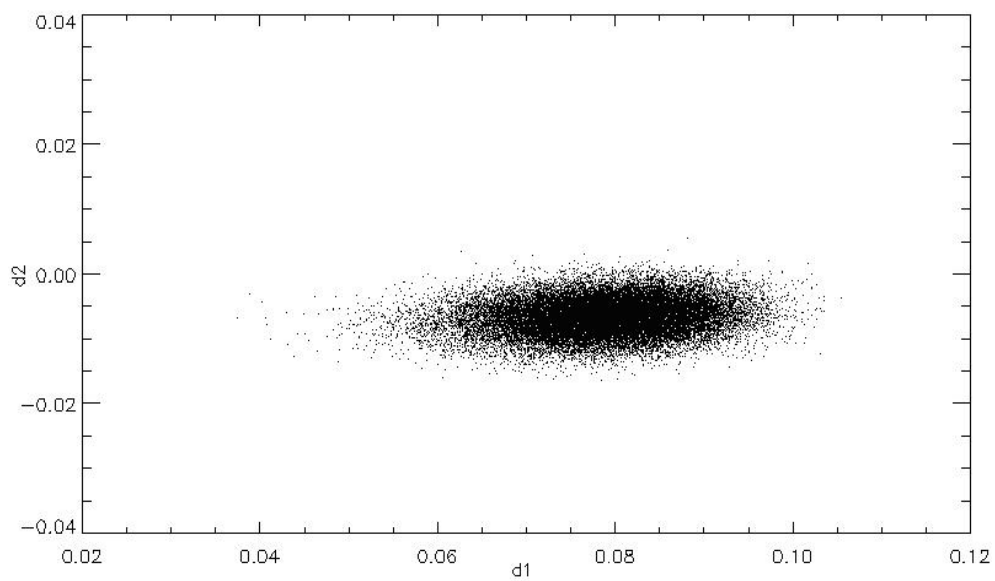


Figure 3. Distribution of d2 vs d1 at 1st order with $\rho = 0.5$: in this case the cloud of samples is denser with respect to the previous case.

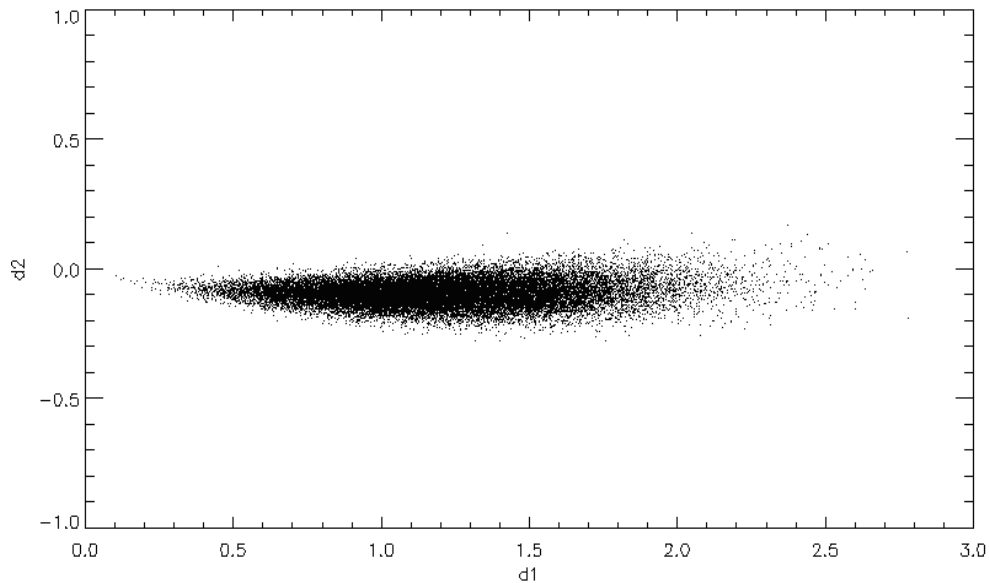


Figure 4. Distribution of d_2 vs d_1 at 2nd order with $\rho = 0.1$.

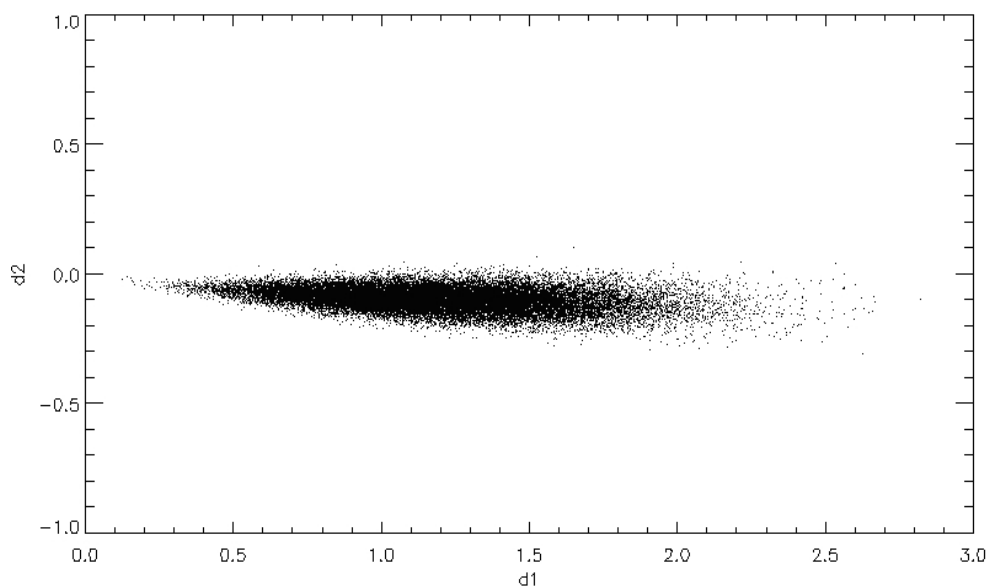


Figure 5. Distribution of d_2 vs d_1 at 2nd order with $\rho = 0.5$: the distribution becomes thinner.

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