Plasma physics effects on thermonuclear burn rate in the presence of hydrodynamic mix

Xian-Zhu Tang, Zehua Guo, Grigory Kagan, Christopher McDevitt, and Bhuvana Srinivasan

Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545

E-mail: `xtang@lanl.gov`

Abstract. Hydrodynamic mix can significantly degrade thermonuclear burn rate in an inertial confinement fusion (ICF) target. Successful mitigation requires a detailed understanding of the physical mechanisms by which mix affects burn. Here we summarize the roles of three distinct plasma physics effects on burn rate. The first is the well-known effect of enhanced thermal energy loss from the hot spot and the mitigating role of self-generated or externally-applied magnetic field. The second is the fuel ion separation via inter-species ion diffusion driven by the powerful thermodynamic forces exacerbated by mix during the implosion process. The third is the fusion reactivity modification by fast ion transport in a mix-dominated ICF target, where hot plasma is intermingled with cold fuel.

1. Introduction

Fusion yield in an inertial confinement fusion experiment primarily comes from the reaction between deuterium (D) and tritium (T) for their significantly greater fusion cross section in the multi-keV energy range. The thermonuclear burn rate, or the fusion power density, is [1]

$$p_f \equiv n_D n_T \langle \sigma v \rangle E_f, \tag{1}$$

with $n_{D,T}$ the number densities of D and T, $\langle \sigma v \rangle \equiv (n_D n_T)^{-1} \int \sigma |\vec{v}_D - \vec{v}_T| f_D f_T d\vec{v}_D d\vec{v}_T$ the fusion reactivity integral, and E_f the energy release per fusion reaction. With the cross section σ and energy release E_f determined by nuclear physics, the DT fusion yield rate is affected by plasma physics through the product of n_D and n_T, and the normalized ion velocity distribution functions $f_{D,T}/n_{D,T}$ in the fusion reactivity integral. In a dense ICF plasma, f is close to be a Maxwellian f^M, so one can write $f = f^M + \delta f$ with $\delta f = f - f^M$ small in magnitude compared with f^M. The fusion reactivity integral can then be written in two parts,

$$\langle \sigma v \rangle = \underbrace{\frac{1}{n_D n_T} \int \sigma |\vec{v}_D - \vec{v}_T| f_D^M f_T^M d\vec{v}_D d\vec{v}_T}_{\widehat{\langle \sigma v \rangle}} + \underbrace{\frac{1}{n_D n_T} \int \sigma |\vec{v}_D - \vec{v}_T| (\delta f_D f_T + f_D \delta f_T) d\vec{v}_D d\vec{v}_T}_{\widetilde{\langle \sigma v \rangle}}, \tag{2}$$

where $\widehat{\langle \sigma v \rangle}(T_i)$ is determined by ion temperature T_i alone, and $\widetilde{\langle \sigma v \rangle}$ is a function of the deviation of the distribution function from a Maxwellian ($\delta f_{D,T}$). Distinctly different plasma transport physics enter these three yield factors: $n_D n_T$, $\widehat{\langle \sigma v \rangle}$, and $\widetilde{\langle \sigma v \rangle}$.

2. Effect of mix on yield via hot spot thermal energy loss – $\widehat{\langle\sigma v\rangle}$

Hydrodynamic mix can significantly degrade the fusion yield rate in ICF experiments. The most-recognized effect is the cooling of the hot spot due to mix-enhanced thermal loss by electron thermal conduction. Since the fusion reactivity $\widehat{\langle\sigma v\rangle}$ is a strong function of T_i, temperature drop in the multi-keV range can lead to a drastic reduction in $\widehat{\langle\sigma v\rangle}$. The way that mix enhances thermal energy loss from the hot spot is through a much enlarged hot/cold interface Σ, which enters the energy exchanged between hot (gas) and cold (ice) plasma as, $Q = \int_\Sigma \kappa_{\perp e}\nabla T_e \cdot d\mathbf{S}$. The rapid (exponential in early stage) growth in Σ can have a large impact on Q. The enhanced hot spot thermal energy loss produces (1) a lower T_e and hence T_i via electron-ion collisional equilibration in the hot spot, and (2) additional ice mass that is heated up and becomes part of a more massive but cooler hot spot. The trade-off between additional hot spot mass and lowered hot spot temperature can be illustrated with a simplified model that assumes a spatially uniform hot spot, where the volume integrated fusion yield rate is

$$\mathcal{P}_f = \int p_f d\mathbf{x} = \frac{4\pi R_{hs}^3}{3}\frac{p_D p_T}{T^2}\langle\sigma v\rangle E_f \approx \frac{4\pi R_{hs}^3}{3}\frac{p_D p_T}{T^2}\widehat{\langle\sigma v\rangle} E_f. \tag{3}$$

In the usual ICF range of $T_i = 1 - 8$ keV, $\widehat{\langle\sigma v\rangle} \propto T^4$, so the total yield rate of a hot spot with given radius (R_{hs}) and pressure (p_D and p_T) scales as T^2. Interestingly enough, if one is able to drive a hot spot to $T_i > 8$ keV but < 25 keV, the reactivity tapers off as $\widehat{\langle\sigma v\rangle} \propto T^2$, so the temperature dependence of $\mathcal{P}_f$ cancels out in leading order.

Mitigation of mix-enhanced hot spot thermal energy loss, i.e. reducing Q, can be realized by decreasing the electron thermal conductivity $\kappa_{\perp e}$ across the mix interface and/or limiting the dynamical growth in Σ. Magnetization of electrons is an effective way of reducing $\kappa_{\perp e}$ across the magnetic field. For this to work in an ICF target, the magnetic field needs not only to be strong enough to magnetize the thermal electron, but also to align with the mix interface. It turns out that both self-generated and externally-applied magnetic fields can satisfy that, but by different physical mechanisms. By self-generation, we mean that there is no seed magnetic field, so the Biermann's battery effect [2], which is attributed to a cross gradient of electron density and temperature, is responsible for a sufficiently large magnetic field that wraps around the bubble and spike of the mix interface [3, 4]. Although it is the electrons that drive the Biermann term, the magnetic field self-generation is closely linked to the ion fluid vorticity [5, 6]. To be more specific, the ion vorticity associated with the interpenetration of hot and cold plasmas under hydrodynamic mix is where the Biermann-battery-induced magnetic flux bundles concentrate. Since the vorticity roll-up is at the mix interface where the flow shear is the largest, the induced magnetic field is aligned with the mix interface. An externally applied seed magnetic field offers valuable control for the field magnitude and hence the level of electron magnetization. Hydrodynamic mix can efficiently amplify the field strength by the conventional magnetohydrodynamic dynamo mechanism, also known as the stretch-and-fold dynamo action. Even more importantly, the stretching action is most active where the flow shear is large, namely at the mix interface, and hence it rapidly aligns the amplified field with the mix interface [6], which is typical of a chaotic dynamo [7]. The fact that a mix-interface-aligned magnetic field not only reduces thermal loss but also stabilizes the short wavelength Rayleigh-Taylor modes [6, 8] suggests an intriguing strategy to control hydrodynamic mix by external magnetic field, which can be effective even though the magnetic pressure remains orders of magnitude smaller than the kinetic energy density in the implosion flow $\rho v^2/2$ and the plasma thermal pressure [6].

3. Effect of mix on yield via fuel ion separation – $n_D n_T$

For an assembled target with a specific $n_e = n_D + n_T$, the burn rate is maximized when $n_D = n_T$ everywhere in the burn region. This optimal fuel arrangement is usually satisfied initially by design in both the gas fill and the ice fuel layer, but the implosion dynamics can drive D and T separation [9, 10] even given the initial condition that n_D/n_T is a global constant. The underlying physics is inter-species ion diffusion driven by thermodynamic cross terms, namely the ion pressure gradient (baro-diffusion), the ambipolar electric field (electro-diffusion), and the ion and electron temperature gradients (thermo-diffusion). This effect can be quantified in a two-ion component (DT) plasma by following the time evolution of the D relative mass concentration, $c \equiv m_D n_D/\rho$ with $\rho = m_D n_D + m_T n_T$, $\rho\frac{\partial c}{\partial t} + \rho\vec{u}\cdot\nabla c + \nabla\cdot\vec{i} = 0$, where $\vec{u}$ is the plasma flow velocity and the inter-species ion diffusive flux is given by [11, 12, 13]

$$\vec{i} = -\rho D\left[\nabla c + \overbrace{k_p\nabla\log p_i}^{\text{barodiffusion}} + \underbrace{\left(ek_E/k_BT_i\right)\nabla\Phi}_{\text{electrodiffusion}} + \overbrace{k_T^{(i)}\nabla\log T_i}^{\text{ion thermo-difusion}} + \underbrace{k_T^{(e)}\nabla\log T_e}_{\text{electron thermo-difusion}}\right]. \quad (4)$$

The severity of fuel ion separation is determined by two factors: (1) the magnitude and sign of various relative diffusion coefficients ($k_p, k_E, k_T^{(i)}, k_T^{(e)}$), and (2) the gradient length scales of ion pressure p_i, plasma ambipolar potential Φ, ion and electron temperature. For given two species of ions, $k_p, k_E, k_T^{(i)}$, and $k_T^{(e)}$ are known functions of c only [11, 13, 14]. In a DT mixture, k_p and k_E are positive and of similar value in the range of (0,0.11). The ion thermo-difusion coefficient $k_T^{(i)}$ is also positive and larger than both k_p and k_E. The electron thermo-diffusion coefficient $k_T^{(e)}$ is negative and of a magnitude slightly smaller than k_p and k_E. All of the four relative diffusion coefficients achieve maximum value at $c = 0.5 - 0.6$ [14].

In a spherically symmetric implosion, the gradient length scales are on the order of the hot spot radius R_{hs}. When hydrodynamic mix is present, hot plasma and cold fuel interpenetrate, forming bubbles and spikes that later develop into a turbulent cascade toward smaller structures. The gradient length scales for p_i, Φ, T_i, T_e are then set by the width of the hot plasma filament l_{mix}, which can be far smaller than the nominal hot spot radius R_{hs}. This rapid scale reduction by hydrodynamic mix aggravates the inter-species ion diffusion and hence DT separation in the hot plasma region. It should be noted that DT fuel ion separation is mostly a problem in the hot plasma where the ion temperature is high. This can be understood from the temperature scaling of the classical DT inter-ion diffusivity, $D \propto T^{5/2}$. The ice fuel layer suffers little species separation from DT inter-diffusion, until it is heated up into the hot plasma.

4. Effect of mix on yield via fusion reactivity changes by fast ion transport – $\widetilde{\langle\sigma v\rangle}$

The third and last factor in the fusion burn rate p_f is $\widetilde{\langle\sigma v\rangle}$ which accounts for the deviation of the ion distribution from a Maxwellian. In a collisional plasma anticipated for a cryogenic ICF target, $\delta f_{D,T}$ and hence $\widetilde{\langle\sigma v\rangle}$ should be negligibly small unless the plasma is close to a hot/cold interface. The exception near the interface is due to the well-known Knudsen layer effect [15, 16] which comes about because the mean-free-path of a tail ion scales as E^2, so the fast ions from the hot plasma side tend to suffer free-streaming loss into the cold but dense fuel layer. Since fast ions in the Gamow peak (at $E_g \sim 3 - 4T_i$ for $T_i = 10$ keV, and even larger E_g/T_i for lower T_i) contribute the most to $\langle\sigma v\rangle$, the tail ion loss can significantly degrade the fusion reactivity in the hot spot Knudsen layer next to the hot/cold interface. In other words, $\widetilde{\langle\sigma v\rangle}$ can be negative and has a magnitude that is a substantial fraction of $\widehat{\langle\sigma v\rangle}$ in a Knudsen layer of width $L_{Kn} = (E_g/T_i)^2\lambda_i$, where λ_i is the mean-free-path of a thermal ion [17].

Similar to the fuel ion separation issue, the Knudsen reactivity reduction is less of a concern if the implosion is spherically symmetry for an ignition target in which $R_{hs} \gg L_{Kn} > \lambda_i$, since the volume affected $\sim 4\pi R_{hs}^2 L_{Kn}$ is small compared with the hot spot volume $(4/3)\pi R_{hs}^3$. Hydrodynamic mix can fundamentally alter this picture, as the interface area Σ can grow much larger than $4\pi R_{hs}^2$ as mix progresses. The degree of mix can be characterized by a mix zone width $L_{\rm mix}$ and the typical width $l_{\rm mix}$ of the small scale hot plasma filament in the mix zone. For $L_{\rm mix} \sim R_{hs}$ and $l_{\rm mix} \sim L_{Kn}$, the Knudsen layer effect on fusion reactivity would be present throughout the hot spot. A subtlety arises for cryogenic targets where the escaping fast ions from hot plasma can fuse with the cold DT fuel layer, so while $\widetilde{\langle\sigma v\rangle}$ is negative on the hot plasma side of the mix interface, it can be positive on the other side [17]. This so-called inverse Knudsen layer effect on fusion reactivity in the cold region can substantially recover the lost reactivity in the hot region if the hot plasma has $T_i > 8 - 10$ keV and the cold fuel reaches $T_i = 1.5$ keV. If the hot plasma is of sub-ignition temperature, say $T_i = 1 - 2$ keV, the Knudsen layer reactivity reduction is the dominant mechanism. This is quantified by the solution of Fokker-Planck equation for a hot plasma surrounded by cold fuel, taking into account pitch angle scattering, slowing down, and energy scattering against background ions and electrons, and the ambipolar electric field [18, 19].

5. Summary

Hydrodynamic mix is intrinsically an interfacial phenomenon which can impact fusion yield rate through (1) enhanced thermal loss, (2) fuel ion separation, and (3) modification of fusion reactivity. These bring a number of plasma physics issues to the forefront in order to understand the impact of mix on burn. Among them, we have investigated (1) the subtle role of magnetic field in mitigating thermal loss and controlling mix itself; (2) the fundamental inter-species diffusive transport theory that underlies ion species separation by thermodynamic forces; and (3) the competition between Knudsen and inverse Knudsen effect on fusion reactivity in a mix-dominated cryogenic ICF target.

Acknowledgments

We wish to thank numerous useful discussions with Steve Batha, John Benage, Bob Chrien, Jim Cooley, Hans Herrmann, Nelson Hoffman, Scott Hsu, Ray Leeper, Kim Molvig, Jas Mercer-Smith, and many colleagues at MIT, Univ. of Rochester, and LLNL. This work was supported by U.S. Department of Energy via the LANL LRDR program.

References

[1] Atzeni S and Meyer-Ter-Vehn J 2004 *The physics of inertial fusion* (Oxford University Press, Inc.).
[2] Mima K, Tajima T, and Leboeuf J N 1978 Phys. Rev. Lett. **41** 1715-1719.
[3] Srinivasan B, Dimonte G, and Tang X Z 2012 Phys. Rev. Lett. **108** 165002.
[4] Srinivasan B and Tang X Z 2012 Phys. Plasmas **19** 082703.
[5] Kulsrud R M, Cen R, Ostriker J P, and Ryu D 1997 Ap. J. **480** 481-491.
[6] Srinivasan B and Tang X Z 2013 Phys. Plasmas **20** 056307.
[7] Tang X Z and Boozer A H 2000 Phys. Plasmas **7** 1113.
[8] Chandrasekhar S 1961 *Hydrodynamic and hydromagnetic stability* (Oxford University Press, Inc.).
[9] Amendt P, Landen O L, Robey H F, Li C K, and Petrasso R D 2010 Phys. Rev. Lett. **105** 115005.
[10] Casey D T, Frenje J A, Johnson M G, Manuel M J E, Rinderknecht H G, Sinenian N, Séguin G H, Li C K, Petrasso R D 2012 Phys. Rev. Lett. **108** 075002.
[11] Kagan G and Tang X Z 2012 Phys. Plasmas **19** 082709.
[12] Amendt P, Bellei C, and Wilks S C 2012 Phys. Rev. Lett. **109** 075002.
[13] Kagan G and Tang X Z 2012 Phys. Rev. Lett. **109** 269501.
[14] Kagan G and Tang X Z 2013 Thermo-diffusion in inertially confined plasmas *submitted for publication.*
[15] Henderson D B 1974 Phys. Rev. Lett. **33** 1142-1144.
[16] Molvig K, Hoffman N M, Albright B J, Nelson E M, and Webster B 2012 Phys. Rev. Lett. **109** 095001.
[17] Tang X Z, McDevitt C J, Guo Z, and Berk H 2013 Fusion yield recovery by escaping hot-spot fast ions in the neighboring fuel layer *submitted for publication.*
[18] Tang X Z, Berk H, Guo Z, and McDevitt C J 2013 Reduced Fokker-Planck models for fast particle distribution across a transition layer of disparate plasma temperatures *preprint* LA-UR-13-27676.
[19] McDevitt C J, Tang X Z, Guo Z, and Berk H 2013 A comparative study of tail ion distribution with reduced Fokker-Planck models *preprint* LA-UR-13-27598.