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Dressed Charge of Electron by Radiation Reaction

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Abstract. With the progress of ultraintense short pulse laser technologies, the maximum intensity of the lasers has reached the order of 10^{22}W/cm^2. Now, several institutes are aiming at higher intensities of over $10^{24\sim}$W/cm^2. It is expected that these lasers can investigate the regime of the ultra-intense field effects like electron-positron pair creation and annihilation, these high-intense laser fields enters into the non-linear QED regime. However, when an electron interacts with lasers with intensities over 10^{22}W/cm^2, it has been predicted that the radiation reaction effect becomes significant. The strong radiation field can induce the fluctuation of QED vacuum (vacuum polarization or photon-photon scatterings). Though the propagator for fields is deformed by it, some kind of a polarized charge exists as the dress. In this paper, we will discuss about this dress of an electron generated by radiation in the description of classical physics. It leads to the avoidance of the mathematical difficulty of the radiation reaction problems.

1. Introduction

Recently, the plans of the construction of ultra-high intense lasers are becoming more concrete and one can hear some interesting plans for investigating new physics [1]. This new regime includes the QED region, like electron-positron pair creation/annihilation [2]. These processes are regarded as being generated by the laser intensities of $10^{24\sim}$W/cm^2. However, there is an important physical process before these, radiation reaction [3] which is the main problem addressed in this paper. Radiation reaction is a very basic process. When an electron is accelerated, it emits the Liénard-Wiechert electromagnetic field. If this radiation energy loss becomes significant, the electron feels an additional force via radiation mechanism. This feedback from radiation is called radiation reaction [4~6]. This radiation reaction remains the mathematical dificulty, namely the "run-away" effect [6]. In this paper, we discuss this radiation reaction in the QED vacuum [7,8] with the stabilization of the run-away effect [9-10, 13]. Finally, the theory leads to the "dressed charge" via quantum vacuum fluctuations, the fermionic 1-loop.

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2. Radiation Reaction in Quantum Vacuum

What is reason why we need to consider QED vacuum fluctuations [7,8]? The answer is, the original theory of radiation reaction is equal to the model of an electron. H. A. Lorentz considered that an electron has the charge distribution of a spherical surface, each charge elements interact with others via the Liénard-Wiechert field [5, see Fig. 1]. When the elements are co-moving, the Liénard-Wiechert force has directivity. This field we call the "retarded field". P. A. M. Dirac suggested that the radiation field has not only the retarded field $F_{\mathrm{ret}}{}^{\mu\nu}$, but also an advanced field $F_{\mathrm{adv}}{}^{\mu\nu}$.

$$F_{\mathrm{LAD}}{}^{\mu\nu} = \left.\frac{F_{\mathrm{ret}}{}^{\mu\nu} - F_{\mathrm{adv}}{}^{\mu\nu}}{2}\right|_{x=x(\tau)} = -\frac{m_0\tau_0}{ec^2}\left(\frac{d^2w^\mu}{d\tau^2}w^\nu - \frac{d^2w^\nu}{d\tau^2}w^\mu\right) \tag{1}$$

This field on the electron's point $F_{\mathrm{LAD}}{}^{\mu\nu}$ is the field for radiation reaction (in this paper, we call it the LAD field). It should be regarded as the self interaction of the electron. This LAD field generates an additional acceleration and the equation of motion becomes,

$$m_0\frac{d}{d\tau}w^\mu = -eF_{\mathrm{ex}}{}^{\mu\nu}w_\nu - eF_{\mathrm{LAD}}{}^{\mu\nu}w_\nu\,. \tag{2}$$

This is the standard model of radiation reaction, named the Lorentz-Abraham-Dirac (LAD) equation [6]. But this equation has the mathematical difficulty of the exponential divergence $dw^\mu/d\tau = C^\mu(\tau)\times\exp(\tau/\tau_0)\to\infty$ during the small period, so called the "run-away" solution [6], the method for the stabilization has been required.

In QED, it allows the following processes. (1) An electron emits the photon field, (2) emitted photons enters the fermionic 1-loop (the sea of a "virtual" pair). (3) Then, this "virtual" pair production vanishes and goes back to the photons. The sea of the fermion 1-loop is regarded as a medium for light. This medium is called "QED vacuum" which is all around us according to QED. Since this effect leads the photon-photon scatterings, the propagator for fields should be modified and the permittivity of vacuum should be changed like $\varepsilon_0\mapsto\varepsilon$. Therefore, we have to consider the correspondence between $F=(\mathbf{E},\mathbf{B})$ acting on an electron and $F_{\mathrm{LAD}}=(\mathbf{D},\mathbf{H})$ which we observe via the vacuum polarization. Now, the field $F^{\mu\nu}$ makes the field of the QED vacuum fluctuation $M^{\mu\nu}$ (Fig. 2). For putting the symbol $\langle\bullet|\circ\rangle$ as $\langle A|B\rangle\equiv A_{\mu\nu}B^{\mu\nu}$, the vacuum fluctuation (the corrected behavior of propagating fields) is derived from the first order of the Heisenberg-Euler Lagrangian;

$$L = -\frac{1}{4\mu_0}\langle F|F\rangle + \frac{\alpha^2\hbar^3\varepsilon_0{}^2}{360m_0{}^4c}[4\langle F|F\rangle^2 + 7\langle F|{*F}\rangle^2] \tag{3}$$

$$M^{\mu\nu} = -\frac{\alpha^2\hbar^3\varepsilon_0{}^2}{45m_0{}^4c^2}[4\langle F|F\rangle F^{\mu\nu} + 7\langle F|{*F}\rangle\,{*F^{\mu\nu}}] \tag{4}$$

J. Schwinger demonstrated the Larmor formula from the LAD field $F_{\mathrm{LAD}}{}^{\mu\nu}$ [11] and showed,

$$\partial_\mu F_{\mathrm{LAD}}{}^{\mu\nu}(x) = 0\,. \tag{5}$$

From this Maxwell equation, $F_{\mathrm{LAD}}{}^{\mu\nu}$ is a source-less field which is observed at the outer position of an electron. Therefore, the LAD field is a superposition field of the bare field $F^{\mu\nu}$ and QED vacuum fluctuation $M^{\mu\nu}$,

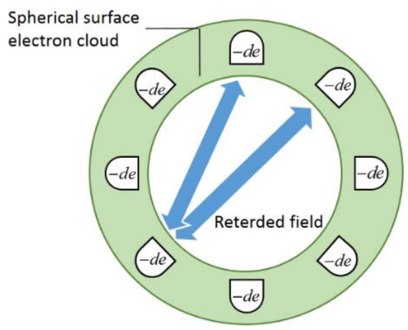

Fig.1 The electron model by Lorentz. The charge elements are distributed on a spherical surface. Each element interacts with others via the Liénard-Wiechert field (the retarded field).

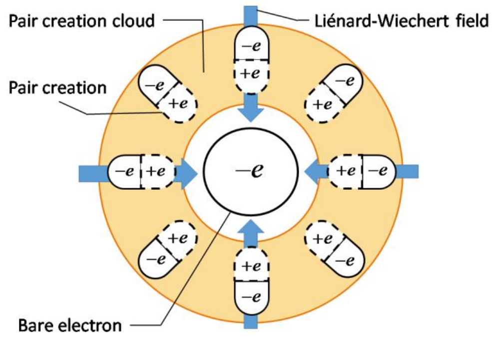

Fig.2 The new model of the electron. The bare electron dresses the QED vacuum fluctuation as a charge. This fluctuation comes from photon-photon scattering which is the diagram of the fermion 1-loop with two incoming photons and two outgoing photon. This dress has a total positive charge.

$$F^{\mu\nu} + \frac{1}{\varepsilon_0 c} M^{\mu\nu} = F_{\rm LAD}{}^{\mu\nu} . \tag{6}$$

Transforming Eq. (6) with the definition of (4) and using standard perturbation, the bare field becomes

$$F^{\mu\nu} = \frac{1}{1-\eta\langle F_{\rm LAD} | F_{\rm LAD}\rangle} F_{\rm LAD}{}^{\mu\nu} . \tag{7}$$

($\eta = 4\alpha^2\hbar^3\varepsilon_0 / 45 m_0{}^4 c^3$, $F_{\rm LAD\,\alpha\beta} F_{\rm LAD}{}^{\alpha\beta} \le 0$). If we take the limit of $\hbar \to 0 \Rightarrow \eta \to 0$, then $F^{\mu\nu} \to F_{\rm LAD}{}^{\mu\nu}$. From the above discussion, our new equation of motion becomes,

$$m_0 \frac{d}{d\tau} w^\mu = -\frac{e}{1-\eta\langle F_{\rm LAD} | F_{\rm LAD}\rangle} (F_{\rm ex}^{\mu\nu} + F_{\rm LAD}{}^{\mu\nu}) w_\nu . \tag{8}$$

We can demonstrate this equation is stable due to the factor of $1/[1-4\alpha^2\hbar^3\varepsilon_0 / 45 m_0{}^4 c^3\ (F_{\rm LAD\,\alpha\beta} F_{\rm LAD}{}^{\alpha\beta})]$, and Eq.(8) can avoid run-away (the detail of demonstration is in Ref. [9] or more strictly in Ref.[10]).

3. Dress of Charge via Radiation Reaction

In this section, we focus on the term of the radiation reaction force in Eq. (8).

$$f_{\rm reaction}{}^\mu = -\frac{e}{1-\eta\langle F_{\rm LAD} | F_{\rm LAD}\rangle} F_{\rm LAD}{}^{\mu\nu} w_\nu \tag{9}$$

This formula comes from two parts. The first part is the LAD radiation reaction force $f_{\rm LAD}{}^\mu = -e F_{\rm LAD}{}^{\mu\nu} w_\nu$ and the second is the additional term.

$$f_{\rm reaction}{}^\mu = f_{\rm LAD}{}^\mu - \frac{e\eta\langle F_{\rm LAD} | F_{\rm LAD}\rangle}{1-\eta\langle F_{\rm LAD} | F_{\rm LAD}\rangle} F_{\rm LAD}{}^{\mu\nu} w_\nu \tag{10}$$

Now, we regard the LAD force as the field which is generated by a bare electron. We consider the extra term in Eq. (10) as, coming from the fermionic 1-loop. Here, $-e = -e_{\rm bare}$ and $\delta e_{\rm dress}(\tau) = -e\eta\langle F_{\rm LAD} | F_{\rm LAD}\rangle / [1-\eta\langle F_{\rm LAD} | F_{\rm LAD}\rangle] \ge 0$, the radiation reaction force becomes,

$$f_{\rm reaction}{}^\mu = [-e_{\rm bare} + \delta e_{\rm dress}(\tau)] F_{\rm LAD}{}^{\mu\nu} w_\nu . \tag{11}$$

Sometime we use the word of the “coupling” for the charge since it couples the dynamics between a particle and fields. The new formula of radiation reaction arranges this coupling by the additional charge of $\delta e_{\rm dress}$ depending on the LAD field $F_{\rm LAD}$, we can consider it as the “dress of charge” by radiation. Actually, the Hisenberg-Euler Lagrangian renormalizes the propagation of radiation fields, we could incorporate a modern physics scheme into classical physics. From this fact, the dress is essentially equivalent to the stabilization of radiation reaction [9-10,13].

4. Summary

Radiation reaction is one of the important process in ultra-high intense laser - high energy electron interactions. The standard model for it, the LAD model was developed as an electron model which is a pure solution of the Maxwell equation. In this paper, we improved the electron model by using QED vacuum. The equation of motion becomes Eq. (8). This equation is stable and can avoid the run-away problem [9]. For these reasons, our equation (8) is a successor of Lorentz, Abraham and Dirac.

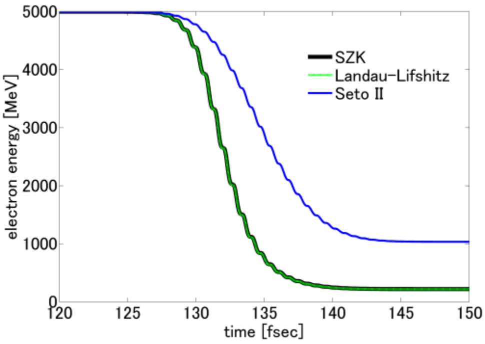

Fig.3 The time evolution of an electron's energy. It simulates the head on collision between a 5GeV electron and the intense laser ($5\text{PW} - 1\times10^{22}\ \text{W/cm}^2$ $\lambda = 0.82\mu\text{m}$, $\tau_{\text{pulse}} = 22\text{fs}$). We used the SZK Eq.(8) [9], the Landau-Lifshitz (LL) equation [12] and Seto II Eqs.(12-13) [13]. SZK converges to LL, it mean LL includes the main effects of QED vacuum fluctuation for the stability. The difference of the final energy derive the fact that Seto II equips QED radiation coupling. It will be carried out in ELI-NP [14].

The essence of our method comes from the dress of charge (the part of the coupling with the dependence of the LAD field F_{LAD}), δe_{dress} by QED vacuum fluctuations, the fermionic 1-loop. This method will become new reference for laser-plasma simulations. Recently, our group succeeded the next model with QED modified radiation from an electron. Defining $\mathfrak{F}_{\text{hom}} = F_{\text{ex}} + F_{\text{Mod-LAD}}$,

$$dm(x)\frac{dw^{\mu}}{d\tau} = -d\mathfrak{C}^{\mu\nu}{}_{\alpha\beta}(x)\mathfrak{F}_{\text{hom}}{}^{\alpha\beta}w_{\nu} \quad , \tag{12}$$

$$\frac{d\mathfrak{C}^{\mu\nu\alpha\beta}}{dm} = \frac{e}{m_0}\frac{(1-\eta\langle\mathfrak{F}_{\text{hom}}|\mathfrak{F}_{\text{hom}}\rangle)g^{\mu\alpha}g^{\nu\beta} + \eta\langle\mathfrak{F}_{\text{hom}}|{}^*\mathfrak{F}_{\text{hom}}\rangle\times\frac{1}{2!}\varepsilon^{\mu\nu\alpha\beta}}{(1-\eta\langle\mathfrak{F}_{\text{hom}}|\mathfrak{F}_{\text{hom}}\rangle)^2 + (\eta\langle\mathfrak{F}_{\text{hom}}|{}^*\mathfrak{F}_{\text{hom}}\rangle)^2} \quad . \tag{13}$$

By using this equation, we can find more active effect of an electron's anisotropic coupling $d\mathfrak{C}^{\mu\nu\alpha\beta}/dm$ by checking the time evolution of an electron's energy in Fig.3. The detail of this model is in Ref.[13].

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