

# Precision Measurement of Delbrück Scattering via Laser Compton Scattered $\gamma$ -rays

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**Abstract.** Precision measurements such as the muon anomalous magnetic moment have indicated deviations from the standard model and have in turn prompted higher precision theoretical calculations. Delbrück scattering is the scattering of photons off the Coulomb field of nuclei via virtual electron-positron pairs and has been measured using  $\gamma$ -rays from radioactivities and following neutron capture reactions. However, because low flux  $\gamma$ -rays from nuclear transitions have been used in the low photon energy regime fairly large uncertainty exists in the data. In addition, due to the complexity and time consuming nature of the theoretical calculation the scattering cross sections are obtained from tables with interpolation between the tabular values. In recent years high flux  $\gamma$ -ray sources via laser Compton scattering (LCS) using energy-recovery linacs have been proposed. These sources allow measuring the Delbrück scattering with high precision. We will present our own independent calculations for the scattering cross section and show what precision can be obtained using the new LCS  $\gamma$ -ray sources in the low photon energy regime.

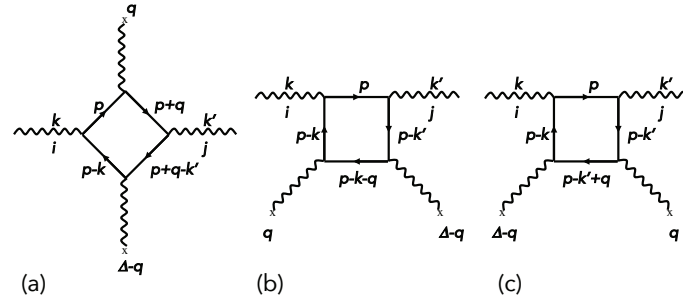
## 1. Introduction

In recent years deviations between quantum electrodynamics (QED) calculations and experimental measurements such as the muon anomalous magnetic moment have hinted at new physics beyond the standard model [1]. The verification of which has required having high precision theoretical calculations [2]. Delbrück scattering, which is the scattering of  $\gamma$ -rays off the Coulomb field of nuclei via virtual electron-positron pairs [3], has been experimentally measured and also shows deviations from the lowest order QED calculations for large  $Z$  nuclei [4]. In the low energy regime ( $< 20$  MeV) unpolarized sources were used and there was a fairly large uncertainty in the data [4]. Because of the difficulty of the theoretical calculation, the scattering cross sections can currently only be obtained from tables [5].

Soon high flux highly monoenergetic highly polarized  $\gamma$ -ray sources via laser Compton scattering (LCS) will become available [6, 7]. As a result, high precision measurements of the Delbrück scattering cross section will be possible and will also require high precision theoretical calculations. In this paper our own calculations for the scattering cross section are presented.

These calculations and comparison with experimental results could determine whether or not deviations from QED occur. In addition these calculations could be used to determine the cause of the expectedly large index of refraction of  $\gamma$ -rays measured for Silicon attributed to Delbrück scattering [8]. If confirmed by our calculations, this could lead to the development of  $\gamma$ -ray lenses which would have applications in nuclear physics [9] and in astrophysics [10].





**Figure 1.** Lowest order Feynman diagrams for Delbrück scattering. The incoming and outgoing photons are represented by their 4-vector momenta,  $k$  and  $k'$ , and polarization directions,  $i$  and  $j$ , respectively.  $\Delta = k' - k$  the momentum transfer.  $x$ 's are the Coulomb field where  $q$  is the momentum of the field.

## 2. Delbrück scattering

We use natural units where  $c = \hbar = 1$ . Figure 1 shows the lowest order Feynman diagrams for Delbrück scattering [3]. The differential scattering cross section calculated from these diagrams is given by [3]:

$$\frac{d\sigma}{d\Omega} = \frac{1}{(4\pi)^2} |\mathcal{M}_0^{(D)}|^2 \quad (1)$$

where

$$\mathcal{M}_0^{(D)} = ie^6 Z^2 \int \frac{d^3 q}{(2\pi)^3} \frac{M_0(k, k', q)}{(\vec{q}^2)(\vec{\Delta} - \vec{q})^2}, \quad (2)$$

$e$  is the electron charge,  $Z$  is the atomic number of the scattering nucleus,  $k = (\omega, \vec{k})$  and  $k' = (\omega', \vec{k}')$  are the incoming and outgoing photon 4-vectors, respectively,  $\vec{\Delta}$  is the momentum transfer taken as  $|\vec{\Delta}| = 2\omega \sin(\theta/2)$  and  $\theta$  being the scattering angle, and  $q$  is the momentum of the Coulomb field, with

$$M_0(k, k', q) = \frac{i\delta_{ij}}{24\pi^2} + \int \frac{d^4 p}{(2\pi)^4} \left\{ \frac{\text{Tr}[\gamma_i(\not{p} + m)\gamma_0(\not{p} + \not{q} + m)\gamma_j(\not{p} + \not{q} - \not{k}' + m)\gamma_0(\not{p} - \not{k} + m)]}{(p^2 - m^2)[(p + q)^2 - m^2][(p + q - k')^2 - m^2][(p - k)^2 - m^2]} \right. \\ + \frac{\text{Tr}[\gamma_i(\not{p} + m)\gamma_j(\not{p} - \not{k}' + m)\gamma_0(\not{p} - \not{k} - \not{q} + m)\gamma_0(\not{p} - \not{k} + m)]}{(p^2 - m^2)[(p - k')^2 - m^2][(p - k - q)^2 - m^2][(p - k)^2 - m^2]} \\ \left. + \frac{\text{Tr}[\gamma_i(\not{p} + m)\gamma_j(\not{p} - \not{k}' + m)\gamma_0(\not{p} - \not{k} - \not{q}' + m)\gamma_0(\not{p} - \not{k} + m)]}{(p^2 - m^2)[(p - k')^2 - m^2][(p - k - q')^2 - m^2][(p - k)^2 - m^2]} \right\} \quad (3)$$

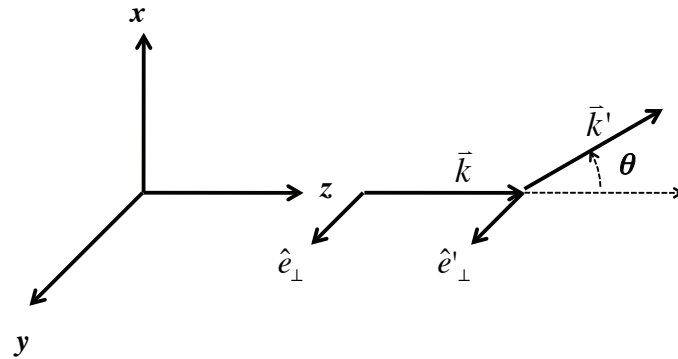
where the  $\gamma$ 's are the Dirac matrices,  $\not{p} \equiv \gamma_\mu p^\mu$ ,  $q' = \Delta - q$ ,  $m$  is the electron mass,  $(i, j)$  refer to the photon polarization directions, and "Tr" refers to the trace of the matrix expressions.

The explicit expressions including cancellations of similar terms are given elsewhere [3].

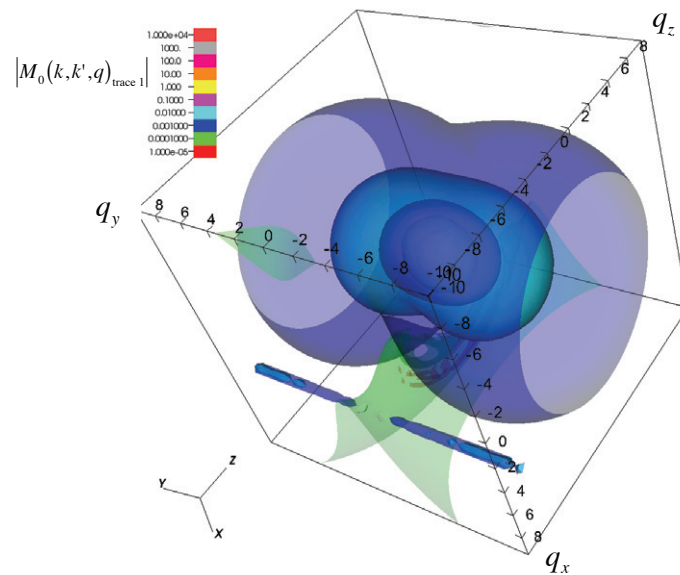
The integrals over  $p$  in Eqs. 3 can be calculated using the specialized package of routines, LoopTools [11]. The multi-dimensional integrals over  $q$  in Eq. 2 can be calculated via Monte Carlo methods in the package of routines, CUBA [12].

## 3. Preliminary Results

Here we present preliminary numerical results for scattering element  $M_0(k, k', q)$  for the first trace (trace 1) of Eq. 3. Figure 2 shows the geometry we choose for our calculations. For simplicity we calculate the particular component where the initial photon is polarized



**Figure 2.** Geometry for Delbrück scattering.  $\vec{k}$  and  $\vec{k}'$  are incoming and outgoing photon wavenumber vectors where the incoming photon is assumed to be propagating along the  $z$ -axis and both photon energies are equal to  $\omega$ .  $\theta$  is the scattering angle.  $\hat{e}_\perp$  and  $\hat{e}'_\perp$  are the initial and final photon polarization vectors, respectively, where  $\perp$  means that the polarizations are perpendicular to the scattering plane.



**Figure 3.** Preliminary values of  $|M_0(k, k', q)|_{\text{trace } 1}$  for a photon energy of 2.09 MeV with  $\theta = 24.6^\circ$  where the axes are in units of MeV.

perpendicular to the scattering plane with polarization vector,  $\hat{e}_\perp$ , and the final photon is also polarized perpendicular to the scattering plane with polarization vector,  $\hat{e}'_\perp$ . Figure 3 shows preliminary values of  $|M_0(k, k', q)|_{\text{trace } 1}$  for a photon energy of 2.09 MeV with  $\theta = 24.6^\circ$  where  $(q_x, q_y, q_z)$  are in units of MeV. We have calculated values for  $q$  up to  $\pm 10$  MeV using a  $50 \times 50 \times 50$  size grid in the  $q_x$ ,  $q_y$ , and  $q_z$  directions, respectively. As apparent from the figure near  $q_x \approx 4$ ,  $q_z \approx -8$ , and for all  $q_y$ , there are regions where the values are large at single grid points. For different values of scattering angle and photon energies these regions differ and can have very large values. These are most likely numerical instabilities, which typically occur in loop calculations, and are currently under investigation. One possible source of numerical instability is the occurrence of small Gram determinants, which could occur when obtaining the

coefficients used to represent the tensor integrals in Eq. 3 in terms of simpler scalar integrals [13]. We are investigating packages such as CutTools [14], which handle these and other types of numerical instabilities. In the next phase of the calculation we will work on eliminating these instabilities, calculate the values for  $|M_0(k, k', q)|$  of all the traces in Eq. 3, calculate the integral  $\mathcal{M}_0^{(D)}$  using the CUBA library[12], and then compare with published values [5].

#### 4. Discussion and Conclusions

Proposed future LCS  $\gamma$ -ray sources will have high total flux ( $10^{13}$  ph/s), almost 100% polarization, small energy spread (FWHM  $10^{-3}$ ), and be tunable in energy up to 13.2 MeV [6, 7]. These sources may allow measuring only the Delbrück scattering component among the various scattering mechanisms occurring at low photon energies with previously unavailable high precision scanning specific energies and scattering angles. Since accuracy goes as  $1/\sqrt{N}$  where  $N$  is the scattered photon numbers, obtaining high accuracy measurements of the cross sections should be possible with these new sources.

In this paper we have presented the calculation method for obtaining the Delbrück scattering cross section using packaged routines. We have presented some preliminary calculations of the scattering matrix for the first trace in Eq. 3, which show some numerical instability, for a simple configuration. In our next calculations we will examine the possibilities to suppress these instabilities, calculate the scattering matrix for all three traces, obtain the integrated cross section and examine the regimes where Delbrück scattering can be measured accurately for specific energies and scattering angles.

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