

# Ignition calculations using a reduced coupled-mode electron-ion energy exchange model\*

**W J Garbett and D A Chapman**

AWE plc, Aldermaston, Reading, Berkshire, RG7 4PR, UK

E-mail: warren.j.garbett @awe.co.uk

**Abstract.** Coupled-mode models for electron-ion energy exchange can predict large deviations from standard binary collision models in some regimes. A recently developed reduced coupled-mode model for electron-ion energy exchange, which accurately reproduces full numerical results over a wide range of density and temperature space, has been implemented in the Nym hydrocode and used to assess the impact on ICF capsule fuel assembly and performance. Simulations show a lack of sensitivity to the model, consistent with results from a range of simpler alternative models. Since the coupled-mode model is conceptually distinct to models based on binary collision theory, this result provides increased confidence that uncertainty in electron-ion energy exchange will not impact ignition attempts.

## 1. Introduction

Integrated calculations of ICF capsule implosions are generally performed using relatively simple, computationally inexpensive, plasma physics models for energy transport and exchange processes, such as those of Spitzer [1] or Brysk [2]. These models are derived using binary collision theory in ideal plasma and as such do not allow for energy transfer between electrons and ions through the interaction of collective excitations. Models based on quantum kinetic treatments which include the effects of coupled, collective modes can produce electron-ion energy exchange rates that differ significantly from Spitzer in some regimes [3,4], raising concerns about a potential lack of fidelity in ICF modelling.

During an ICF implosion, the capsule fuel accesses a range of conditions, including hot, dense, partially electron degenerate states, where coupled-mode effects may be pronounced. Although such effects are expected to be diminished as the fuel is heated to high temperatures, calculations predict electron-ion energy exchange to remain slower than the Spitzer rate even for temperatures in the kilo-electron-volt range [5]. However, it has been difficult to accurately assess the impact of coupled-mode models on capsule performance since full numerical evaluation is computationally expensive and unsuitable for direct implementation in radiation-hydrocodes.

A recently developed reduced coupled-mode model for electron-ion energy exchange [6], which accurately reproduces full numerical results over a wide range of density and temperature space, has been implemented in the Nym hydrocode [7]. The model has been applied to 1D calculations of a NIF ignition capsule to assess the impact on fuel assembly and performance.

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## 2. Energy exchange model

The coupled-mode model is derived for a two-component electron-ion plasma using a quantum statistical framework, which correctly describes the particle kinetics in dense, partially-degenerate systems [4]. In the weakly-coupled limit, the model is evaluated using the random phase approximation (RPA), which yields an expression for the energy transfer rate per unit volume as a double Fourier-space integral in terms of the plasma dielectric function  $\varepsilon$  and those for the individual components  $\varepsilon_{ee}$  and  $\varepsilon_{ii}$ .

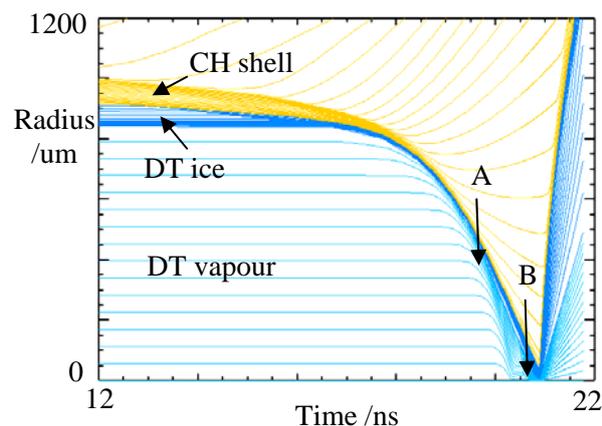
$$Z_{ei}^{CM}(t) = \frac{\hbar}{\pi^3} \int_0^\infty dk k^2 \int_0^\infty d\omega \omega [n_{Bi}(\omega; t) - n_{Be}(\omega; t)] \frac{\text{Im} \varepsilon_{ee}(k, \omega; t) \text{Im} \varepsilon_{ii}(k, \omega; t)}{|\varepsilon(k, \omega; t)|^2} \quad (1)$$

Here  $\omega$  and  $k$  are the frequency and wavenumber and  $n_B(\omega)$  is the Bose occupancy function for the plasma modes. This approach naturally captures coupling between the collective modes of each component. However, direct numerical evaluation of the expression is computationally expensive since sharp plasmon peaks occur in the mode spectrum near the zeroes of the dielectric function.

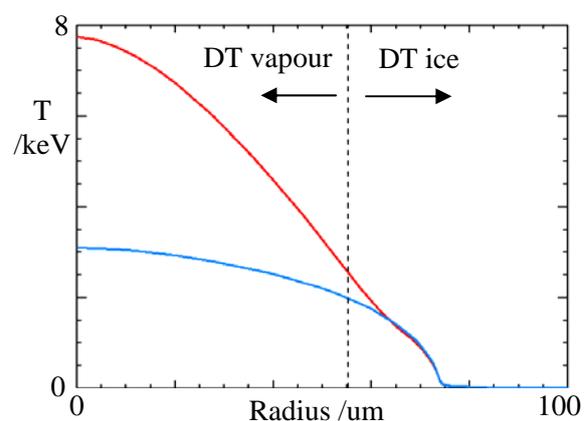
The reduced model uses the fact that the integrand is modulated by the spectral function of the ions, which decays over much smaller frequency scales than that of the electrons, to Taylor expand the electron term. Electron screening is also approximated to first order, including both the static real term and a non-vanishing imaginary term. This latter term, representing electron damping, was found to play a critical role, such that the coupled-mode effect cannot be reproduced by alternative reduced approaches using only static screening [6]. Although sharp plasmon peaks still occur, the inclusion of electronic damping means their amplitude increases more slowly than their width and the contribution to the integral decreases and remains calculable. Moreover, the position of the peak can be estimated *a priori*, allowing high numerical resolution of the integral to be focussed over a small interval. The reduced model shows excellent agreement with full numerical calculations over a wide range of temperature and density space.

## 3. Ignition calculations

Calculations of the NIF Rev 5 Ge-doped plastic ignition design, which was the initial target used during the ignition campaign [8] were performed in 1D using Nym (figure 1). Although details of the ignition design have evolved, the basic operation and fuel conditions are essentially unchanged.



**Figure 1.** Radius vs time for 1D Nym simulation of the Rev 5 Ge-doped plastic ignition capsule. Non-equilibrium occurs following shock breakout (A) and during self-heating (B).



**Figure 2.** Electron (blue) and ion temperatures (red) profiles through the hotspot during the self-heating phase, showing substantial separation.

Nominal calculations suggest two significant phases of operation where the electrons and ions are out of equilibrium. Firstly, as shocks break-out from the DT ice into the low density DT vapour they coalesce into a strong shock, producing significant temperature separation ( $T_i > T_e$ ). Thus there is potential for uncertainty in the energy exchange model to impact the calculated shock strength and subsequent energetics at stagnation. A slower exchange rate allows greater temperature separation and reduces radiative losses by maintaining a lower electron temperature. Following shock stagnation the fuel density continues to increase as the fuel shell converges, so that the temperatures tend to re-equilibrate.

The second period of separation occurs during the self-heating phase, when the ions in the DT fuel are again heated above the electrons (figure 2). This behaviour is perhaps counter-intuitive since at typical hotspot temperatures most of the alpha energy is deposited into the electrons [9]. However, despite lower energy input the ions also have much lower energy losses than the electrons, as the electrons and ions effectively become decoupled with increasing electron temperature. At these higher temperatures the hotspot is relatively weakly coupled and the electrons quickly become non-degenerate. Under these circumstances the uncertainty in the energy exchange rate may only be small.

Calculations were performed using both the Spitzer and coupled-mode energy exchange models. Integrated performance metrics, such as neutron yield, mean fuel temperatures or maximum areal density, show only small variation between the two models. Simulations show differences only during the self-heating and burn phases, and only in the central part of the fuel. The observed variation is much less than expected using a uniform multiplier on the exchange rate, but is consistent with the variation seen in a range of alternative models (Table 1). However, since the coupled-mode model is conceptually distinct from the various binary collision approaches, this result significantly strengthens confidence in those conclusions. Note also that in nearly all cases the alternative models reduce performance compared to Spitzer, albeit by only small amounts.

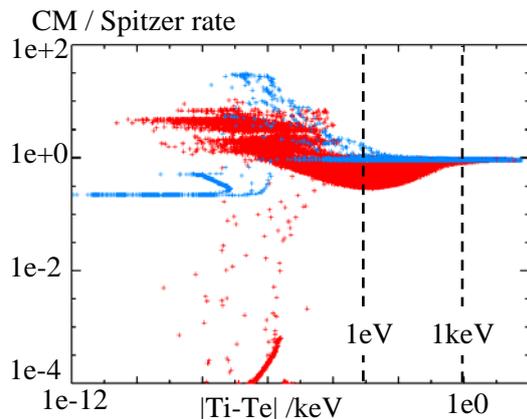
**Table 1.** Comparison of performance metrics for a range of models. The first line gives nominal values from simulations using the Spitzer model. Subsequent lines show deviation from nominal.

Model	Neutron yield	Ti (keV)	Te (keV)	$\rho R$ (g/cm <sup>2</sup> )
Spitzer	6.1e18	47.4	36.2	1.46
Coupled-mode	-2%	-3.5%	-2%	-1%
0.5 x Spitzer	-11%	-17%	-12%	-6%
Brysk	-3%	-5%	-3%	-2%
Chang [10]	-1%	-2%	-1.5%	-0.5%
Zha [11]	0	-0.5%	0	0
Gericke [12]	0	+0.5%	+0.5%	0

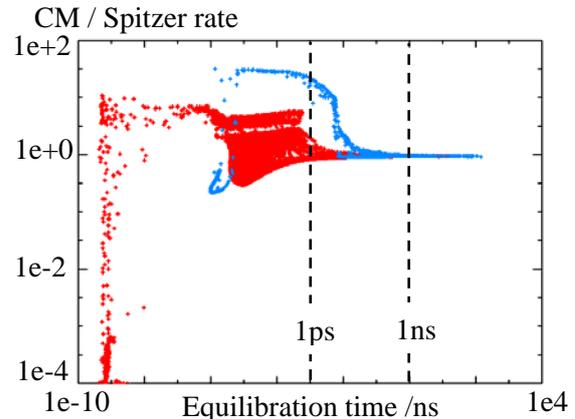
The lack of sensitivity to electron-ion energy exchange can be understood by detailed analysis of the simulations. The ratio of coupled-mode and Spitzer rates shows significant deviation from unity during the course of the implosion, but the largest deviations occur at early time when the fuel is cold and very strongly coupled, when neither model is strictly appropriate. The energy exchange rates have no impact at this time since the temperatures are low and the ions and electrons are in equilibrium.

When temperature separation is significant, the coupled-mode and Spitzer models tend to agree to better than 50% (figure 3). For the hotspot region the models show little variation in rate once temperature separation exceeds  $\sim 1$ eV. In the main fuel, the coupled-mode model can produce larger reductions relative to Spitzer (up to 70%) for temperatures separated by  $\sim 10$ eV, when the fuel is in the warm dense matter regime.

Furthermore, the largest differences between the coupled-mode and Spitzer rates occur not only when temperature separation is small, but also when the energy exchange rates are very high, typically for equilibration times much less than 1ps (figure 4). The coupled-mode model does show a large deviation from Spitzer for the hotspot when the equilibration time is a few picoseconds, but this corresponds to temperature differences of less than 1eV, so the impact is limited.



**Figure 3.** Ratio of coupled-mode to Spitzer energy exchange rate as a function of ion-electron temperature separation. Points represent values for the simulation cells in the hotspot (blue) and main fuel (red) at each time.



**Figure 4.** Ratio of coupled-mode to Spitzer ion-electron equilibration time, plotted for the hotspot (blue) and main fuel (red) cells at each time.

Examination of the exchange rate model ratio as a function of the (non-degenerate) plasma coupling parameter shows that the models agree to better than 25% within the weakly coupled regime ( $\Gamma < 0.1$ ). Although the coupled-mode model is derived in the RPA weak coupling limit, extension of the model to include strong coupling corrections is likely to have little impact on capsule performance, which appears insensitive to the already significant model variation in the strongly coupled regime. To have an impact on ignition capsule performance it appears that energy exchange models must deviate significantly from Spitzer in the high temperature, weakly coupled regime.

#### 4. Conclusions

Coupled-mode models for electron-ion energy exchange predict large deviations from conventional binary collisions models in some regimes. A recently developed reduced coupled-mode model has been implemented in the Nym hydrocode and applied to calculations of a NIF capsule. Simulations show little sensitivity to the coupled-mode model, consistent with results from a range of alternative models. This lack of sensitivity arises because the models are in reasonable agreement whenever electron and ion temperatures separate or equilibration times become significant. Although the coupled-mode model is evaluated in the weakly-coupled RPA limit, extending the model to include strong coupling corrections is not expected to impact this conclusion. These results provide some confidence that uncertainty in electron-ion energy exchange models will not impact ignition attempts.

#### 5. References

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