

# From sequential to correlated tunneling of two bosons

R Avella<sup>1</sup>, R R Rey González<sup>1</sup> and K M Fonseca Romero<sup>1</sup>

<sup>1</sup> Universidad Nacional de Colombia, Bogotá, Colombia.

E-mail: kmfonsecar@unal.edu.co

**Abstract.** The sequential to correlated-tunneling transition of two identical bosons confined in a symmetrical double well potential is studied using a time-dependent variational formalism. Increasing the strength of the short-range boson-boson interaction, the probability of finding both bosons in the same well allow to identify three regimes. For small values of the boson-boson coupling constante, this probability display beatings. Then, the probability of finding both bosons in the same well becomes quasi-periodical in time, for almost all values of the coupling constant; however, there is a zero-measure set, which contains an infinity of points, for which this probability is time-periodic. Finally, for larger values of the coupling constant, this probability changes periodically. Based on the dynamics of the probability of finding one boson on each well and increasing the boson-boson interaction, we find that tunnelling changes from a regime where one boson tunnels after the other to a regime in which they tunnel together.

## 1. Introduction

Tunneling, one of the most characteristic quantum effects, has been studied in many systems. In the context of cold atoms, Josephson tunneling of Bose-Einsteins Condensates (BEC) in multi-well systems have received considerable attention [1–3]. Recent experimental techniques have allowed the exploration of the tunneling of a few bosons [4]. In fact, optical lattices permit a high control of both the geometry and the depth of confining potentials [5]. In this contribution, we study the tunneling of two bosons in a double-well potential using a time-dependent variational approach.

## 2. Physical system

We consider two <sup>87</sup>Rb atoms, with mass of  $144.42 \times 10^{-27}$  kg, to be confined in a double well potential (Figure 1, dashed line). The width of each well and of the  $h \times 3$  kHz potential barrier are  $10 \mu\text{m}$  and  $2 \mu\text{m}$  [6], respectively. Experimental reports of tunneling in these systems describe a sequential to correlated-tunneling transition [4, 7]. Previous theoretical treatments [8–11] show sequential tunneling for weak boson-boson interactions and correlated tunneling near the fermionization limit.

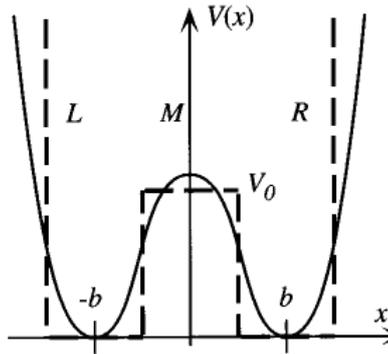
We use dimensionless quantities, distances in units of  $\xi = 1 \mu\text{m}$ , energies in units of  $\epsilon = 10^{-31}$  J, and times in units of  $\tau = \frac{\hbar}{\epsilon}$ . For example, the height of the potential barrier is  $v_0 \approx 19.88$  in these units. The single-particle Hamiltonian then becomes

$$h_0(x) = -\frac{\hbar^2}{2m\xi^2\epsilon} \frac{d^2}{dx^2} + v(x). \quad (1)$$



The (single-particle) eigenvectors can be given in analytic form, but in terms of the eigenvalues of  $h_0(x)$ , which are the solutions of a trascendental equation. The two smallest eigenenergies,  $E_0 \approx 0.03548$  and  $E_1 \approx 0.03552$  are well separated from the other eigenvalues,  $E_n > 0.14$  for  $n \geq 2$ .

We choose the first eigenfunction  $\psi_0(x) = \langle x|0 \rangle$  to be non-negative everywhere, and the second,  $\psi_1(x) = \langle x|1 \rangle$  to be positive for  $x \geq 0$ . Thus, the wavefunction  $\langle x|L \rangle = \psi_L(x) = \frac{1}{\sqrt{2}}(\psi_0(x) - \psi_1(x))$  ( $\psi_R(x) = \frac{1}{\sqrt{2}}(\psi_0(x) + \psi_1(x))$ ) describes a particle in the left (right) well. The tunneling time from one well to be other is  $7.5 \times 10^4$ .



**Figure 1.** Realistic (solid line) and square double well potentials (dashed line).

The two-particle Hamiltonian is  $H(x_1, x_2) = h_0(x_1) + h_0(x_2) + \lambda \delta(x_1 - x_2)$ , where  $x_1$  and  $x_2$  are the dimensionless positions of the two bosons, and  $\lambda$  is the interaction coupling constant.

### 3. Time-dependent variational principle

The time-dependent variational principle, which provides an approximation of the exact dynamics, corresponds to the minimization of the action functional  $S = \int_0^T dt (i \langle \Psi | \dot{\Psi} \rangle - \langle \Psi | H | \Psi \rangle) = \int_0^T dt L(t)$ . In order to investigate the tunneling dynamics, we use the trial wave function  $\Psi(x_1, x_2, t) = c_1(t)\psi_0(x_1)\psi_0(x_2) + \frac{c_0(t)}{\sqrt{2}}(\psi_1(x_1)\psi_0(x_2) + \psi_0(x_1)\psi_1(x_2)) + c_{-1}(t)\psi_1(x_1)\psi_1(x_2)$ .

The ‘‘Lagrangian’’  $L(t)$  depends on the value of the integrals  $I_{ij} = \int dx \phi_i^2(x)\phi_j^2(x)$ ,  $i, j = 1, 2$ . By setting  $E_0 = \frac{E_M - \delta}{2}$  and  $E_1 = \frac{E_M + \delta}{2}$ , and employing the approximation  $I_{ij} \approx I = 0.07235$ , with an error of about 3 parts per ten thousand, we obtain the Lagrangian,

$$L(t) = ic_{-1}^*(t)\dot{c}_{-1}(t) + ic_0^*(t)\dot{c}_0(t) + ic_1^*(t)\dot{c}_1(t) - \left\{ E_M + \delta(-|c_1(t)|^2 + |c_{-1}(t)|^2) + \lambda I \left[ 1 + |c_0(t)|^2 + c_{-1}(t)c_1^*(t) + c_{-1}^*(t)c_1(t) \right] \right\} \quad (2)$$

The tunneling process depends on the coefficients  $c_m^*(t)$ ,  $m=0, \pm 1$ , whose equations of motion are given by  $\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{c}_m^*(t)} \right) = \frac{\partial L}{\partial c_m^*(t)}$ .

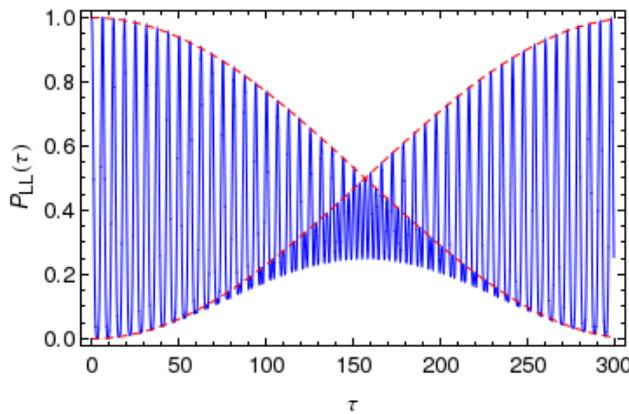
### 4. Return probability

In order to investigate the tunneling process, we assume that both bosons are initially on the left well, i.e., the initial state of the system is  $|LL\rangle$ . The probability that they remain on the left well is ( $g = \lambda I$ ),

$$P_{LL} = \frac{1}{2} \left( 1 + \cos(gt) \cos \left( t\sqrt{g^2 + \delta^2} \right) \right) + \frac{g \sin(gt) \sin \left( t\sqrt{g^2 + \delta^2} \right)}{2\sqrt{g^2 + \delta^2}} - \frac{\delta^2 \sin^2 \left( t\sqrt{g^2 + \delta^2} \right)}{4(g^2 + \delta^2)} \quad (3)$$

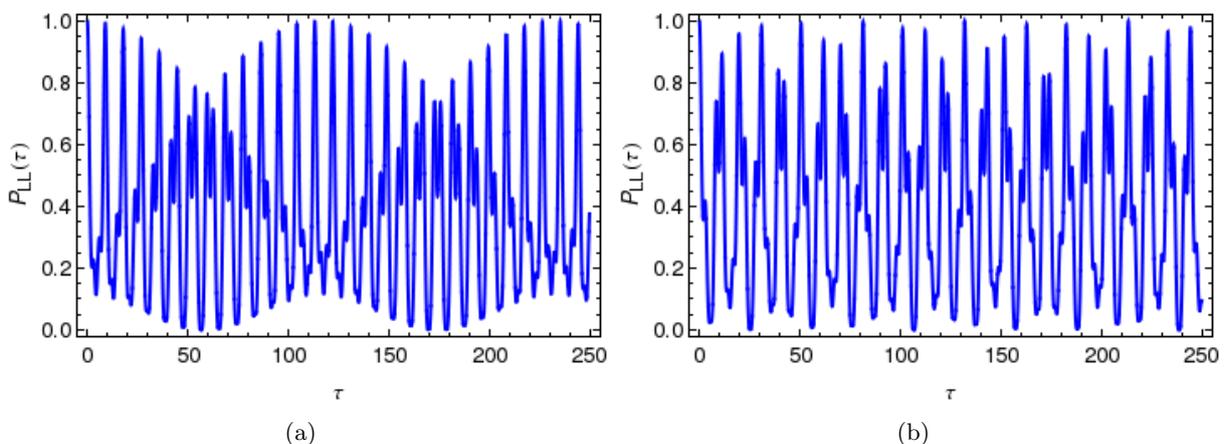
The probability of finding one boson on each well,  $P_{LR} = \frac{\delta^2}{2(g^2 + \delta^2)} \sin^2(\sqrt{g^2 + \delta^2}t)$ , reaches a maximum of  $\frac{1}{2}$  for vanishing boson-boson interaction and decreases to zero as the interaction is increased. Thus, the tunneling is sequential for small values of the interaction constant and correlated for large values.

The behaviour of the quantum return probability  $P_{LL}$  present three distinct regions. The first region, corresponds to very small values of the effective coupling constant  $g$ , when the return probability can be approximated as  $P_{LL} \approx \frac{1}{2} \left( 3 + 4 \cos(gt) \cos(\sqrt{g^2 + \delta^2}t) + \cos(2\sqrt{g^2 + \delta^2}t) \right)$ . In this region, the most important characteristic is the presence of beatings: a low-frequency envelope  $\cos(gt)$ , which modulates the high-frequency periodic oscillations  $\cos(\sqrt{g^2 + \delta^2}t)$ . The term  $-1 + \cos(2\sqrt{g^2 + \delta^2}t)$  becomes important when the amplitude of the beating,  $\cos(gt)$ , is small. These details are shown in Figure 2.

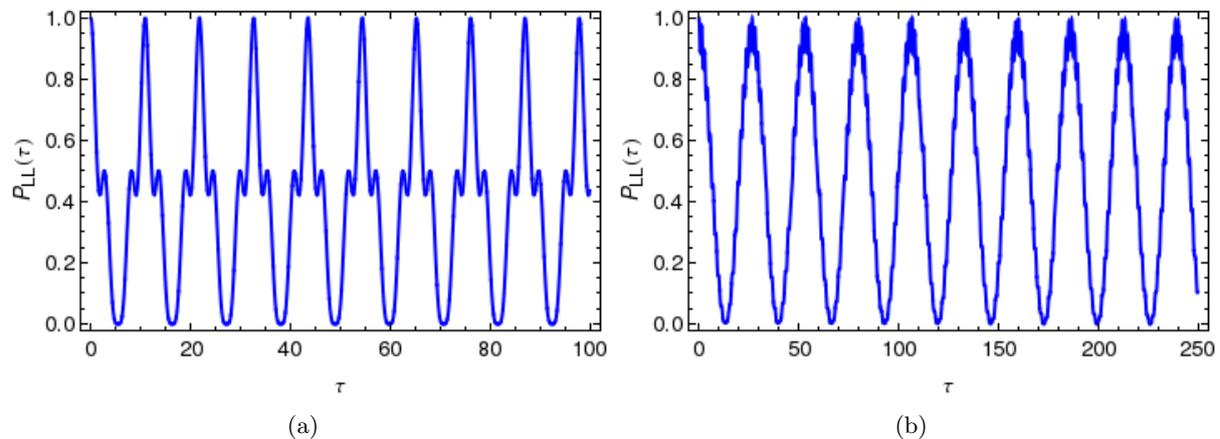


**Figure 2.** Population of the state  $|LL\rangle$  with time measured in units of  $1/\delta$ , for  $g = \frac{\delta}{100}$ .

The effective interaction coupling constant  $g$  and the energy  $\delta$  are of the same order in the second region, where the behaviour of the return probability is quasi-periodic (See Figure 3). In this region there are infinitely many values of the coupling constant where the return probability becomes periodic: when  $g = \frac{\alpha\delta}{\sqrt{1-\alpha^2}}$ , where  $\alpha$  is a rational in  $[0, 1)$  (see Figure 4(a)). In fact, in this case the return probability has contributions corresponding to the angular frequencies  $\omega, \omega(1 \pm \alpha)$ , with  $\omega = \sqrt{g^2 + \delta^2}$ .



**Figure 3.** Quase-periodic evolution of the population of the state  $|LL\rangle$  with time measured in units of  $1/\delta$ , for (a)  $g = \frac{\delta}{3}$  and (b)  $g = \frac{\delta}{2}$ .



**Figure 4.** Periodic evolution of the population of the state  $|LL\rangle$  with time measured in units of  $1/\delta$ , for (a)  $g = \frac{\delta}{\sqrt{3}}$  and (b)  $g = 2\delta$ .

The last region corresponds to an effective coupling constant much larger than the energy difference  $\delta$ , when the return probability becomes periodic  $P_{LL}(t) \approx \frac{1}{2} \left( 1 + \cos\left(\frac{\delta^2 t}{2g}\right) \right)$ , (see Figure 4(b)).

## 5. Conclusions

We have considered the problem of tunneling of two interacting cold bosons in a double well potential. Using a simple trial function, we have used the time-dependent variational principle to show that there is a transition from sequential to correlated tunneling as the coupling constant is increased. For small values of the coupling constant, the return probability displays frequency beating; for intermediate values, the return probability is quasi-periodic except for an infinity of values when it becomes periodic. For large values of the coupling constant, the return probability is periodic; the larger the coupling constant the larger the tunneling period.

## Acknowledgments

We acknowledge partial support from Universidad Nacional de Colombia under project 9366.

## References

- [1] Smerzi A, Fantoni S, Giovanazzi S and Shenoy S R 1997 *Phys Rev Lett* **79** 4950
- [2] Javanainen J 1986 *Phys Rev Lett* **57** 3164
- [3] Cataliotti F S, Burger S, Fort C, Maddaloni P, Minardi F, Trombettoni, Smerzi A and Inguscio M 2001 *Science* **293** 843
- [4] Fölling S, Trotzky S, Cheinet P, Feld M, Saers R, Widera A, Müller T and Bloch I 2007 *Nature* **448** 1029
- [5] Sebby J, Anderlini M, Jessen P and Porto J 2006 *Phys Rev A* **73** 033605
- [6] Yong-II S 2006 *Experiments with Bose-Einstein condensates in a double-Well potential* (USA: MIT)
- [7] K Winkler K, G Thalhammer G and Lang F 2006 *Nature Lett* **441** 853
- [8] Roy A and Reichl L 2008 *Phys Rev A* **77** 033418
- [9] Anlabha R and Reichl L 2009 *Time-of-flight signatures of two boson in a double well* arXiv:0903.5029v4
- [10] Zöllner S, Dieter H and Schmelcher P 2008 *Phys Rev A* **78** 013621
- [11] Zöllner S, Dieter H and Schmelcher P 2007 *Phys Rev Lett* **100** 040401