Control of atom-atom entanglement by cavity detuning

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Abstract. Using the atomic levels previously employed to demonstrate a two-photon maser, we show that the atom-atom entanglement produced by the successive passage of two three-level Rydberg atoms across a single-mode lossless cavity can be enhanced using the Stark shift. The atoms are assumed to be prepared in their excited states and to interact with the field during the same amount of time. Employing a physically motivated perturbation-theory approach, we obtain an effective two-level Hamiltonian. We show that, within the limits of validity of the approximation, atomic entanglement can be controlled by changing the frequency of the cavity field, and can be enhanced up to a maximum where the squared concurrence attains the value 16/27.

1. Introduction

Entanglement was initially discussed as a strange phenomenon which would demonstrate the lack of completeness of quantum mechanics [1]; today it is, without question, one of the most important concepts in the fields of quantum communication and quantum information [2]. Entanglement has been produced in several different systems, most of them involving matter-radiation interaction; in particular, in cQED (Cavity Quantum Electrodynamics) experimental setups [3]. In this context, one-photon processes have been extensively discussed [3]. Multiphoton processes, especially two-photon processes [4–7] are well known; however, the investigation of the production and control of entanglement employing these processes have received little attention. A remarkable exception is a recent report [8] in which, using ideas previously discussed in other systems [9, 10], two-photon precesses are proven useful to control the entanglement between two atoms which do not interact directly.

In this contribution, we reconsider the atomic states used in the reference [11] to demonstrate a two-photon maser, and show that the atomic entanglement of two atoms sent through a high-Q cavity can be controlled by an appropriate choice of detuning of the electromagnetic field with respect to the allowed atomic transitions. In section 2, we present the physical system and show why the analysis of [8] fails in this case. Using a physically motivated perturbation theory, we find an effective Hamiltonian; we show that the atomic concurrence can be controlled between zero and $4/3\sqrt{3}$ under a particular choice of interaction time (section 3).

2. Physical system

We consider a situation in which two Rubidium atoms sequentially traverse a high-Q microwave superconducting cavity. The relevant atomic states of the ^{85}Rb atoms are the ground state $39S_{1/2}$ ($|g\rangle$), the intermediate state $39P_{3/2}$ ($|i\rangle$) and the excited state $40S_{1/2}$ ($|e\rangle$) (see Figure 1). The transition frequency between the excited (intermediate) and ground state is $\omega_{eg}/2\pi=$ 136.83174GHz ($\omega_{ig}/2\pi=$ 68.37687GHz), and the frequency of the single-mode cavity field is ω. The atom-field coupling between the intermediate level and the other states is described by the one-photon Rabi angular frequencies $\Omega_{ei}\approx\Omega_{ig}\approx$0.7MHz$=\Omega$.

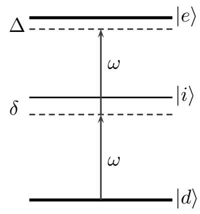

Figure 1. Relevant energy levels of the Rubidium atom.

The Hamiltonian of the system in the dipole- and rotating-wave approximations is

$$\widehat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \sum_{k=e,i,g}\hbar\omega_k\,|k\rangle\,\langle k| + \hbar\Omega\left(\hat{a}\hat{S}^+_{ig} + \hat{a}^\dagger\hat{S}^-_{ig} + \hat{a}\hat{S}^+_{ei} + \hat{a}^\dagger\hat{S}^-_{ei}\right), \tag{1}$$

Where the operators S^+_{kl} are the raising operators between levels $|k\rangle$ and $|l\rangle$, and the S^-_{kl} are the corresponding lowering operators. The transitions $|g\rangle \leftrightarrow |i\rangle$ and $|i\rangle \leftrightarrow |e\rangle$ are assumed to be almost resonant, i.e., the one-photon detunings are small in the sense that $\delta_g \equiv \omega_i - \omega_g - \omega \ll \omega_i - \omega_g$ and $\delta_e \equiv \omega_e - \omega_i - \omega \ll \omega_e - \omega_i$. We also define the two-photon detuning $\Delta = \delta_e + \delta_g = \omega_{ei} + \omega_{ig} - 2\omega$.

Employing a canonical transformation method the authors of [8] find the effective Hamiltonian

$$H_{\mathrm{GMN}} = [\Delta + (\beta_e + \beta_g)a^\dagger a]S_z + \frac{1}{2}(\beta_e - \beta_g)a^\dagger a + g(S^+_{eg}a^2 + S^-_{eg}a^{\dagger 2}), \tag{2}$$

Where $S_z = (|e\rangle\langle e| - |g\rangle\langle g|)/2$. Here, $g = \Omega^2(\delta_g - \delta_e)/(\delta_e\delta_g)$, and the Stark shifts associated with the levels e and g are, respectively, $\beta_e = \Omega^2/\delta_e$ and $\beta_g = \Omega^2/\delta_g$. The authors restrict themselves to the case $\beta_e = \beta = \beta_g$, and show interesting possibilities of control of atom-atom concurrence for the parameters $\beta = \Delta = 2g$ and $\beta = -\Delta = g$. We show that the particular choice used in Reference [8] breaks down in the case of a single intermediate level.

The equality of the Stark shifts of the levels $|e\rangle$, β_e, and $|g\rangle$, β_g, imply that the one-photon detunings are smaller than the two-photon detuning Δ, $\delta_g = \frac{1}{2}\Delta = \delta_e$. In this case it is inconsistent to ignore one-photon transitions, an implicit assumption in the derivation of Hamiltonian (2).

Taking as reference values those of zero Stark shift and exact two-photon resonance, striking differences on the values of atom-atom entanglement were obtained in [8] for $\beta/g = 1$ and $\Delta/g = -1$. We consider the slightly more general form $\beta/g = x^2$ and $\Delta/g = -y^2$, for x and y real parameters. If we multiply the dimensionless Stark parameter and the dimensionless two-photon detuning, we get $\beta\Delta = -(xyg)^2 < 0$, which is incompatible with the equality $\beta\Delta = \frac{\Omega^2}{\delta_g}\times 2\delta_g = 2\Omega^2 > 0$. Our results, to be presented in the following sections, complement those of Reference [8].

We find an effective two-level approximation by defining a physically motivated perturbation theory. Efficient two-photon processes between the ground and the excited state, and almost no population transfer to the intermediate state, are expected for large one-photon detunings ($|\delta_e|, |\delta_g| \gg \sqrt{n+1}$) and a small two-photon detuning ($|\Delta| \ll \sqrt{n+1}$). We define the one-photon detuning to be of zeroth order, the terms proportional to the dipole moments to be of first order, and the two-photon detuning to be of second order. The effective Hamiltonian, up to second order and ignoring the contribution of the intermediate level, is given by

$$H_{\text{eff}} = \hbar\Big\{\Delta - \frac{\Omega^2}{\delta_g}\left(\hat{a}^\dagger\hat{a}+1\right)\Big\}\left|e\right\rangle\left\langle e\right| - \frac{\hbar\Omega^2}{\delta_g}\left(\hat{a}^\dagger\hat{a}+2\right)\left|g\right\rangle\left\langle g\right| - \frac{\hbar\Omega^2}{\delta_g}\Big\{\hat{a}^2\widehat{S}_{eg}^{+} + \hat{a}^{\dagger 2}\widehat{S}_{eg}^{-}\Big\}. \tag{3}$$

3. Atomic entanglement

We consider a situation in which two atoms, initially prepared in their excited states, traverse the initially empty cavity. The atoms, which are never simultaneously present in the cavity, interact with the cavity field for the same amount of time. Though the atoms never interact directly, they become entangled. If the interaction time is set to $\tau{=}\frac{\pi}{\omega_2}$, the atomic concurrence is $C(s){=}2s\sqrt{1-s}$ where $s{=}\sin^2(\varphi)\sin^2\!\left(\pi\frac{\omega_0}{\omega_2}\right)$, $\tan\varphi{=}\frac{\Omega^2}{2\delta}\sqrt{2}/\left(\frac{\Delta}{2}+\frac{\Omega^2}{2\delta}\right)$, and

$$\omega_n \;=\; \sqrt{\left(\frac{\Delta}{2}+\frac{\Omega^2}{2\delta}\right)^2 + \left(\frac{\Omega^2}{2\delta}\right)^2 (n+1)(n+2)}, \quad n = 0, 2. \tag{4}$$

The interaction time, of the order of hundreds of microseconds, is reasonable in cQED experiments. When $s = 2/3$, concurrence attains its maximum value (under the condition of equal interaction times) $C_{\max}{=}\frac{4}{3\sqrt{3}}{\approx}\,0.77{<}1$. When the two-photon detuning is varied between -20kHz and 20kHz, atomic concurrence varies from almost zero to $C_{\max}$, then to a local minimum and again to $C_{\max}$, before decreasing to almost zero (as shown in Figure 2).

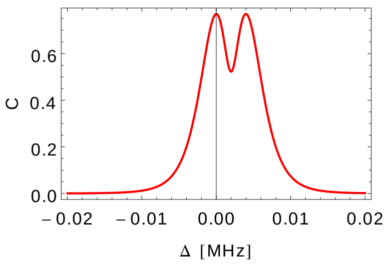

Figure 2. Atomic concurrence as a function of two-photon detuning.

4. Conclusions

We considered the sequential passage of two Rubidium atoms, prepared in specific Rydberg states, through a high-Q cavity, initially prepared in its vacuum state. We have shown that, choosing a particular interaction time, the two-photon detuning can be used to control the atomic entanglement. To assess the feasibility of the present scheme, it is necessary to include dissipation and small changes on the interaction time in the mathematical model.

Acknowledgments

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