

Objects tracking using a periodic array of volume: numerical evaluation

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Abstract. A method for tracking 3D objects using sensitivity phase of its Fourier transform to determine the position, displacement and orientation of a moving object is presented. The orientation and 3D position of the object from the 3D imaging system with subvoxel accuracy are calculated. A reference pattern should be fixed on the surface of the object and should be registered by the system of 3D images. This method is based on a prior knowledge of the association between the phase of the reference pattern and object motion, inherent property of the Fourier Transform. The reference pattern consists of a regular distribution of intensity that can be represented mathematically by three fringes of orthogonal patterns of volume whose 3D spectrum it allows to generate a 3D distribution of continued phase and absolute for each strip of the pattern orthogonal. A local maximum intensity value in the reference pattern represents a phase value $2\pi N$ where N corresponds to the fringe where is encountered the desired value. Reconstruction of the absolute phase associated with each set of fringes of volume in each direction orthogonal allows obtaining an exact point of the pattern, leading to the determination of the position of the object's location with subvoxel accuracy. The displacements from two consecutive positions are calculated. The accuracy of this method considering the parameters such as sampling, the number of holes, noise and contrast is evaluated numerically. Also its efficiency measuring the orientation is evaluated.

1. Introduction

Position and displacement measurement with high accuracy and resolution have been widely studied and applied in various fields of science and engineering. They have been proposed several devices that measure the position and rotation of an object with high precision through techniques optical, of radiofrequency, digital image processing for localization of marks between other [1].

A method in the field of the image processing which aims to locate a mark is computationally developed and simulated. The high sensitivity of the phase with respect to displacement allow measure the position and displacement of an object in a 3D scene, with high accuracy and resolution. This method is based on priori knowledge of the association between the phase of the reference pattern and the displacement of the object, inherent property of the Fourier transform [2]. The reference pattern or target consists of a regular intensity distribution which can be represented mathematically by three patterns of volume fringes orthogonal whose 3D spectrum can generate a three-dimensional distribution of continuous phase and absolute to each orthogonal fringes pattern allowing efficient analysis in the frequency space which is performed with the Fourier transform [3]. The spectral content of the reference pattern shows information of this periodic arrangement of volume in the frequency space. The reconstructions of the absolute phase associated at orthogonal fringes system



allows the precise location of the centre of the target that determine the positioning of the object with precision subvoxel. The displacements are calculated from two consecutive positions.

2. Measurement principle

A combination of strategies of Takeda and Sandoz is used [4,5]. In the setup, a cubic target is used as shown in the Figure 1, with $10 \times 10 \times 10$ holes dark of four pixels wide, this target is introduced into an empty matrix $256 \times 256 \times 256$ which is called workspace, distribution in the spatial frequencies is observed, which is obtained using Fourier Transform (TF),

$$F(u, v, w) = \int_{x,y,z} f(x, y, z) e^{2\pi i(xu+vy+zw)} dx dy dz \quad (1)$$

Where the spatial function $f(x, y, z)$ is the distribution of intensity of the image of volume acquired (target) and $F(u, v, w)$ is its Fourier Transform, which can be seen in Figure 2. Then, three of the first harmonics are filtered independently, one for each direction orthogonal of the fringes (see Figure 2) contained in the input information. The location of the corresponding harmonics can be found because of that the period of the target is known. In these positions the filters must be located. This can be seen in Figure 2. The filtering described mathematically as:

$$F_i(u, v, w) = F(u, v, w) \cdot W(u - u_i, v - v_i, w - w_i) \quad i = 1, 2, 3 \quad (2)$$

Where $W(u - u_i, v - v_i, w - w_i)$ is the filter type Hanning centred in (u_i, v_i, w_i) with a width of $2/3$ the cut-off frequency. Then, each harmonic are relocated in the origin of coordinates independently and three-dimensional inverse Fourier transform is applied to each one

$$f_i(x, y, z) = \int_{u,v,w} F_i(u, v, w) e^{-2\pi i(xu+vy+zw)} du dv dw \quad (3)$$

Once recovered the inverse Fourier transform, the modulus $M_i(x, y, z)$ and the phase $p_i(x, y, z)$ of each function is calculated:

$$\begin{aligned} M_i(x, y, z) &= f_i(x, y, z) \cdot f_i^*(x, y, z) \\ p_i(x, y, z) &= \arctan \left[\frac{\Im(f_i(x, y, z))}{\Re(f_i(x, y, z))} \right] \quad i = 1, 2, 3 \end{aligned} \quad (4)$$

Where $f_i^*(x, y, z)$ is the complex conjugate of the function $f_i(x, y, z)$. In the Figure 3, the module and the phase is obtained from the function $f_i(x, y, z)$ corresponding to lobe u_i, v_i, w_i . The module is used order to build a binary mask which when multiplied with the phase $p_i(x, y, z)$ allows to define the phase region to working.

In the Figure 3 is shown the phase of the work in each of the orthogonal directions. Similarly, the function $f_2(x, y, z)$ and $f_3(x, y, z)$ the same treatment are applied.

Once phase region is obtained and as the phase $p_i(x, y, z)$ recovered has jumps of phase of 2π and as they is wrapped between $-\pi$ and π , a three-dimensional algorithm the phase unwrapping is applied [6]. As observed in the Figure 4, the phase obtained is linear (hyperplane), the direction of growth for each of the harmonics filtered is orthogonal. From each phase obtained, the coefficients are calculated by polynomial approximation a_i, b_i, c_i and d_i [7,8], which help describe the hyperplane of phase $P_i(x, y, z)$ best fit of the calculated phase. The Figure 4 shows the hyperplanes of phase and the associated phase, which can be described by the following equation:

$$P_i(x,y,z) = a_i x + b_i y + c_i z + d_i, \quad i = 1, 2, 3 \quad y \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} P_1 - d_1 \\ P_2 - d_2 \\ P_3 - d_3 \end{bmatrix} \quad (5)$$

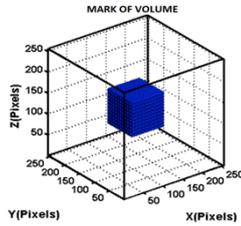


Figure 1. Target of volume inside of the workspace of 256x256x256 pixels.

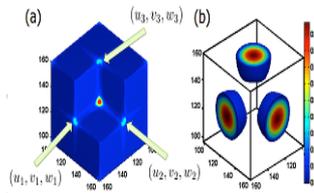


Figure 2. (a) Location of the harmonics at filter and (b) filter hanning for to extract the harmonics.

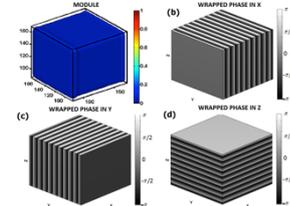


Figure 3. (a) Module of the inverse Fourier transform (b), (c) y (d) Phase wrapped in x, y, Z.

For to find the coordinates of the centre of the target or any other point on this, a phase value for each hyperplane is fixed, counting $2\pi N$ from the origin. The value of x, y, z is found solving the system of linear Equations (5), for the values of phase P_1, P_2, P_3 , corresponding to the desired location. The value of phase associated to the centre of the target is $P_1, P_2, P_3 = 0$, because of target symmetry. Similarly, for values of phase corresponding the faces and corners are calculated, Figure 8.

3. Results

For evaluating the positioning method are performed simulations in order to obtain the accuracy of the system with respect at position without displacement versus to sampling (pixels by each hole), the number of holes used in the target, the noise, the contrast and the measurement of rotation. The target is centred on a workspace comprised of 2563 voxels, therefore theoretically the centre coordinate should be (128.5, 128.5, 128.5). For each case a target with the desired characteristics is taken, are generated 100 targets with standard deviation of 2 grey levels and it proceeds to calculate the centre of each one with the positioning method implemented, then the parameter varies evaluation and the procedure is repeated. The results obtained are described in the Figure 5 and 6.

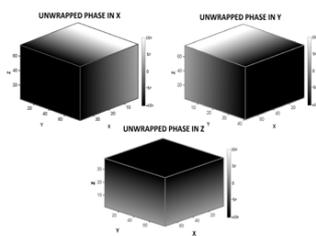


Figure 4. Hyperplanes of the unwrapped phase in each orthogonal direction.

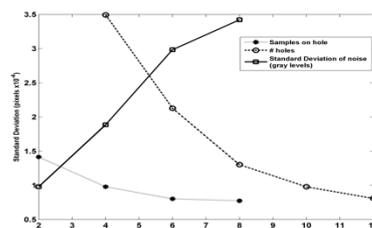


Figure 5. Analysis of the dependence of the accuracy in calculating the position with the sampling, number of holes and noise in the grey levels from 100 measurements.

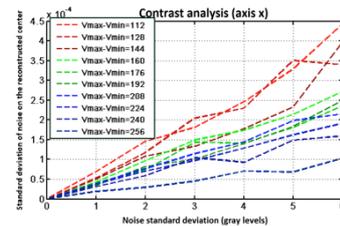


Figure 6. Standard deviation in the reconstruction of the centre vs. standard deviation of the greys levels for different contrasts of the target (date of axis x).

For measuring the rotation of the target, the reference points are calculated the centre (padding), a face (hole) and a corner (padding), Figure 7.

The Figure 9 shows the target of volume rotated 45° parallel to the axis z. This is located in the centre of the target of volume.

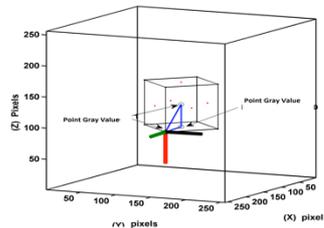


Figure 7. Location of points of reference in the target.

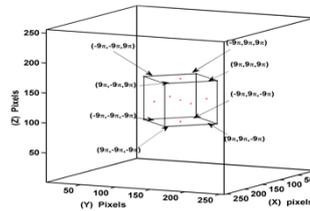


Figure 8. Phase values of the corners.

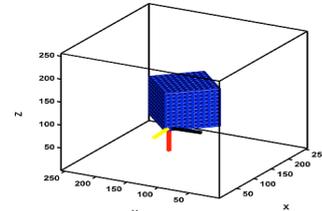


Figure 9. Target of volume rotated an angle of 45 degree.

4. Conclusions

In this paper, a method for measuring the positioning and rotation of an object are evaluated numerically using a periodic array of volume. This array is composed of periodic patterns of stripes in each orthogonal direction. The simulation shows that is possible to determine the position and angular displacement of an object, with subvoxel precision. The accuracy depends on the specific characteristics of the periodic array of volume used (sampling, number of hole, noise and contrast). Similarly, the effect of to add a defect on the volume target (padding a hole) allows for a rotation of 360° without degrading system accuracy in the positioning calculation of the target and without significant changes in the position calculation algorithm. Is observed that minimum accuracy of the system for when the target is of 10 holes, with a contrast of 240 grey levels and standard deviation of the noise of 2 grey levels, is the order of 4×10^{-4} voxels, that is, is possible to measure 2500 positions between voxels in any direction only when the target is sampled at 2 pixels per hole. When increasing the number of pixels per hole, increases accuracy, similarly the computational cost, because the area of phase interest is equivalent to the size of the target, this weighs in the calculation algorithm applied. When the target is sampled at 8 pixels per hole then the accuracy it is approximately doubled, that is, will could measure 5000 positions between voxels.

Acknowledgments

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