

Dynamics of a liquid drop in porous medium saturated by another liquid under gravity

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Abstract. The work deals with numerical simulations of settling or ascension process of a liquid drop in porous media saturated by another liquid. The calculations were carried out using the Darcy model by Level set method with adaptive mesh refinement algorithm that dynamically refines computational mesh near interface. It is shown that the drop is unstable and the finger instability develops at the forefront of moving drop for any ratio of the viscosities of liquids. Under modulated pressure gradient small-scale perturbations of interface are suppressed and in the case of modulation with large enough intensity drop becomes stable.

1. Introduction

In the framework of the conventional Darcy model a planar displacement front in a porous medium is known to be unstable if the displacing fluid has lower viscosity than the displaced one [1]. The instability is associated with development of penetrating fingers of the less viscous fluid. They grow due to lower friction with porous matrix. The process is virtually unimpeded by viscous momentum transfer, which is negligible in porous media. This mechanism explains the short-wave nature of the instability.

In practice the displaced fluid often forms compact inclusions surrounded by the displacing fluid. In this case one should consider displacement fronts on both sides of the inclusion. In the framework of the linear theory of stability the dynamics of the steady settling of spherical inclusion of one liquid in a porous medium filled with another liquid is studied in [2]. It is shown that monotonically increasing perturbations are localized at the forefront of moving droplets, in agreement with the Rayleigh-Taylor mechanism. On the other hand, a two-parameter family of eigenmodes was found, with the spectrum of eigenvalues filling the entire complex plane.

In the present paper the numerical simulation of a liquid drop behaviour in a porous medium under gravity is performed and the effect of time-modulation of external pressure gradient is analyzed. The problem is studied under assumption of thin front between the phases, interpenetration of phases typical for Buckley-Leverett model [3] is not considered. Calculations are performed using the Level set method. The method was developed in [4–6] where interface between fluids of different properties is represented as transition layer of finite thickness so fluids are considered as one medium with sharp change of properties in the transition region. In [7] a new enhanced treatment of the Level set method is introduced that allows to perform simulation for fluids having large density and viscosity ratios.



The paper is organized as follows. In Sec. II we formulate the problem in terms of nondimensional governing equations and interface boundary conditions and introduce the nondimensional parameters. The method of computation of drop dynamics is described in Sec. III. In Sec. IV and Sec. V, we present the results of calculations. Section VI contains the discussion and concluding remarks.

2. Problem statement

We consider a uniform isotropic porous medium filled by the fluid wherein single drop of another fluid with different density and viscosity moves within the first fluid in gravity field, Fig. 1. The drop settles or ascends depending on density ratio in a cylindrical channel with rigid boundary. It was shown in [2] that this problem can be transformed into the problem of a drop driven by an imposed pressure gradient.

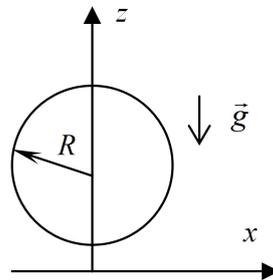


Figure 1. Problem geometry

The drop is initially spherical and the interface thickness between liquids is supposed to be small, much less than the drop radius. We neglect the effect of capillary forces. Let us introduce the coordinate system such that z -axis is directed vertically upward (see fig. 1).

The dynamics of system under consideration is described by Darcy model and the continuity equation in each liquid:

$$\nabla p_j + \frac{\varepsilon \eta_j}{k} \mathbf{v}_j = \rho_j g \boldsymbol{\gamma} \quad (1)$$

$$\text{div } \mathbf{v}_j = 0 \quad (2)$$

where p_j is pressure, \mathbf{v}_j is hydrodynamic velocity of liquid (not the filtration rate), k and ε are permeability and porosity coefficients, η_j is dynamic viscosity, ρ_j is liquid density, g is acceleration of gravity, $\boldsymbol{\gamma}$ is unit vector directed downwards, j denotes inner 1 or outer 2 fluids.

At the interface the kinematic condition and the condition of continuity of the normal component of velocity are held:

$$\frac{\partial F}{\partial t} + (\bar{\mathbf{v}} \nabla) F = 0, \quad [v_n] = 0. \quad (3)$$

where brackets $[f]$ denotes the jump of function f across the interface, so $[f] = f_1 - f_2$; $F(x, y, z, t) = 0$ is the equation determining the shape of the interface. At rigid boundary the condition of impermeability is imposed.

3. Numerical method

Numerical modeling of the liquid droplet motion is performed using the Level set method [7]. The basic idea of the method is the introduction of the distance function - an auxiliary function, which has different sign in different phases, and zero value of this function corresponds to the interface location. At the same time the interface is treated as the transition layer with sharply varying parameters. As a result, a two-phase system can be described as a single medium with parameters (density, viscosity) depending from the distance function. The calculations were performed for a cylindrical cavity with height H_0 and radius R_0 bounded by solid boundaries. We considered axisymmetric dynamics of the drop.

In accordance with the idea of the method, the equations (1) should be supplemented with the transport equation for the distance function and equations that determine the distribution of the density and dynamic viscosity:

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \mathbf{v}\nabla\phi = 0 \quad (4a)$$

$$\rho(\phi) = \rho_1 + (\rho_2 - \rho_1) H(\phi) \quad (4b)$$

$$\eta(\phi) = \eta_1 + (\eta_2 - \eta_1) H(\phi) \quad (4c)$$

where $H(\phi)$ is Heaviside function defined by the expression

$$H(\phi) = \begin{cases} 0 & \text{if } \phi < -\varepsilon \\ \frac{1}{2} \left[1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi} \sin(\pi\phi/\varepsilon) \right] & \\ 1 & \text{if } \phi > \varepsilon \end{cases} \quad (5)$$

where ε is half-thickness of the transition layer. We assumed that $\varepsilon = n\Delta x$, Δx is the distance between the nodes of computational mesh, $n = 3$ is the number of nodes located in the half-thickness of the transition layer.

As the typical scales of the problem, we chose R_0 for length, $kg(\rho_2 - \rho_1)/\varepsilon(\eta_2 + 0.5\eta_1)$ for velocity, $\rho_1 g R_0$ for pressure, ρ_2 for density and η_2 for viscosity. Note, that selected dimension of velocity corresponds to known analytical formula for the velocity of steady motion of spherical droplet in porous medium under vibrations [2]. Conducting the standard nondimensionalization of the problem (1-3) and equations (4) we obtain the following nondimensional equations:

$$\eta\mathbf{v} = A(\rho\boldsymbol{\gamma} - \nabla p) \quad (6a)$$

$$\text{div } \mathbf{v}_j = 0 \quad (6b)$$

$$\frac{\partial\phi}{\partial t} + \mathbf{v}\nabla\phi = 0 \quad (6c)$$

$$\rho(\phi) = \lambda + (1 - \lambda) H \quad (6d)$$

$$\eta(\phi) = \mu + (1 - \mu) H \quad (6e)$$

The problem contains the following dimensionless parameters: $\lambda = \rho_1/\rho_2$ is ratio of the liquids densities, $\mu = \eta_1/\eta_2$ is the ratio of viscosities, $A = \mu + 0.5/(\lambda - 1)$ is the dimensionless viscosity, $\varsigma = H_0/R_0$ is the height of the cavity, and $r_c = r_0/R_0$ is the drop radius.

The level set method requires high accuracy treatment near the interface of the drop where parameters of medium sharply changes. Therefore we implement adaptive mesh refinement (AMR) algorithm that allows maintaining fine enough mesh near the interface during all time of calculations. The AMR algorithm is realized using Paramesh libraries [8] that supports parallel computations based on Message-Passing Interface and dynamic load balancing procedure for

distributing the workloads across multiple computing resources. Thus, the mesh is automatically changed to minimize diffusion of the grid near the interface. In addition, at each time step, the corrections of the position of the interface is performed such that the thickness of the transition layer is kept constant [7]. As result, variation of the drop mass during calculations did not exceeded 1%.

We introduce the flow function to describe incompressible flow with axial symmetry. Poisson equation for the stream function is solved implicitly by Generalized minimal residual method [9]. To implement this method the library Aztec is used [10], which can effectively solve the system of linear equations with a sparse matrix distributed among computational nodes. The time derivative is approximated by second-order scheme. An approximation of the spatial derivatives is carried out by finite volume method based on the integral form of the conservation equations.

4. Droplet dynamics in gravity field

Calculations were performed for cavity with $\zeta = 4$ and the droplet radius $r_c = 0.4$. The initial coordinate of the drop mass center is $z_0 = 3\zeta/4$. At top and bottom of the cavity the condition of periodicity is imposed.

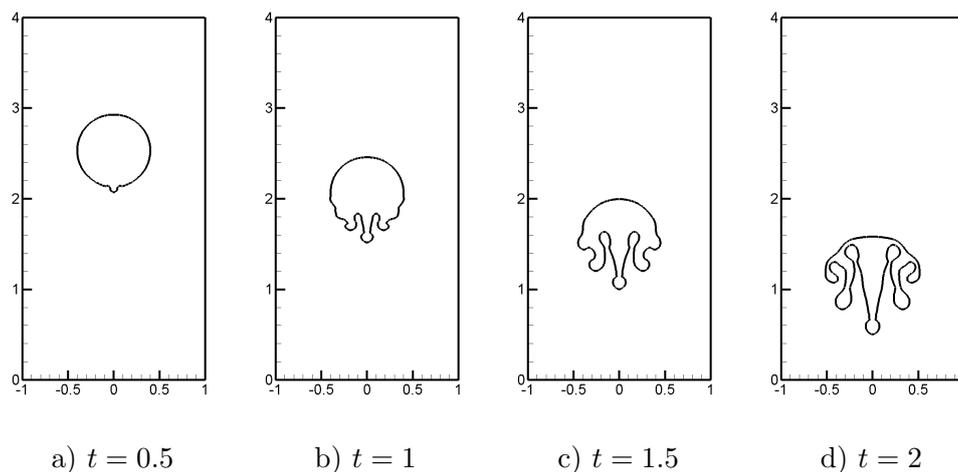


Figure 2. Drop settling for $A = 12.5$, $\lambda = 1.2$, $\mu = 2$

Numerical modeling of the droplet dynamics subjected to gravity showed that the droplet is unstable and instability develops at the forefront of moving droplets regardless of the ratio of fluids viscosities (see Figs. 2, 3). The results are in good agreement with the conclusions made in the paper [2], where in the frame work of linear stability analysis it is shown that the spectrum decrements fills the entire complex plane and monotonically increasing perturbations are localized at the forefront of moving droplets.

Due to the fact that the drop is unstable with respect to perturbations of an arbitrarily large wavenumber, the result of numerical simulation depends on parameters of the computational mesh. With the increase of the mesh resolution the perturbations of smaller scale become capable to be resolved. The rate of their growth is larger then growth rate of other perturbation so the evolution of the drop shape changes significantly. The effect is illustrated in Fig. 3.

5. Drop stability under modulated pressure gradient

Consider the drop stability in the gravity under the influence of time-modulated vertical pressure gradient. Effect of linearly polarized vibration on the stability of plane displacement front in

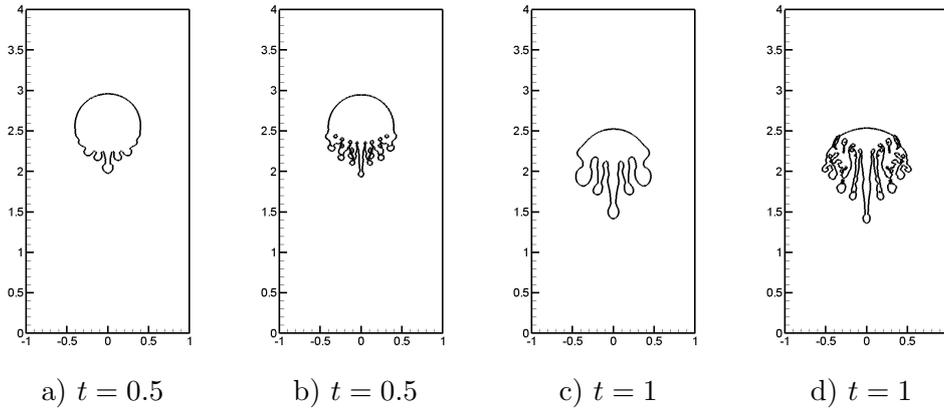


Figure 3. Evolution of the drop shape for $A = 5$, $\lambda = 1.2$, $\mu = 0.5$. Number of mesh nodes per unit of length near the interface equals 128 for (a), (c) and 256 for (b), (c)

a porous medium is investigated in [11]. It was shown that vibrations suspended short-wave perturbations of the displacement front. One could expect that the modulated pressure gradient is able to stabilize the drop.

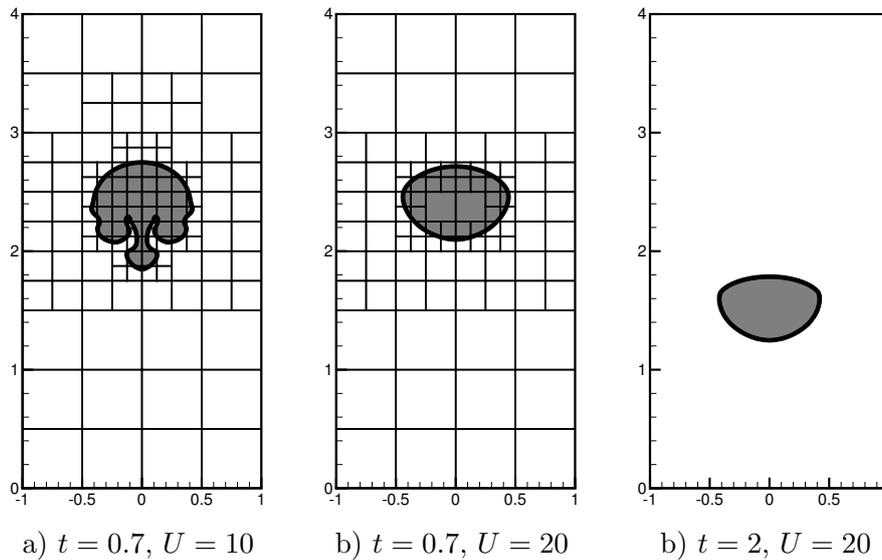


Figure 4. Drop shape under modulated pressure gradient for $W = 1000$, $A = 5$, $\lambda = 1.2$, $\mu = 0.5$

The influence of pressure gradient was taken into account by changing the boundary condition for the velocity field. According to (1) the modulated pressure gradient leads to vertical oscillating flow of fluid far from the drop. Therefore at the upper and lower boundaries of the computational domain vertical velocity component should be

$$v_z = U \cos(Wt) \tag{7}$$

where U is the dimensionless amplitude of the velocity and W is the dimensionless frequency of

modulation.

Calculations show that small-scale perturbations of interface are suppressed by modulated pressure gradient and only large-scale perturbations are developed (see Fig. 3 and Fig. 4a). Increasing of modulation amplitude and frequency leads to intensification of stabilizing effect. For large enough intensity of modulation drop becomes stable (Figs.4b, 4c). It is worth noting that for the case of stable drop results of calculation do not depend on calculation mesh.

Fig. 4 also illustrates the structure of calculation mesh. As it is mentioned above we implement AMR algorithm. Computational mesh consists of a number of blocks of equal size, near the interface additional blocks are created to maintain fine enough mesh. The size of each block shown in Fig. 4 is 16x16.

6. Conclusion

Numerical modeling of the liquid drop dynamics in porous medium saturated with another liquid under gravity is carried out. Simulations showed that the droplet is unstable and regardless of the ratio of liquid viscosities, the finger instability develops at the forefront of moving drop.

The results of modeling performed in the framework of Darcy filtration model depends on computational mesh parameters due to the fact that the drop is unstable to perturbations of an arbitrarily large wave number. If the density of mesh nodes is increased, then perturbations of smaller scale can be realized.

Under modulated pressure gradient small-scale perturbations of interface are suppressed and in the case of modulation with large enough intensity drop is stable.

Acknowledgments

The work was carried out under financial support of Russian Science Foundation (grant No. 14-01-00090).

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