

Dynamical systems for modeling the evolution of the magnetic field of stars and Earth

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Abstract.

The cycles of solar magnetic activity are connected with a solar dynamo that operates in the convective zone. Solar dynamo mechanism is based on the combined action of the differential rotation and the α -effect. Application of these concepts allows us to get an oscillating solution as a wave of the toroidal field propagating from middle latitudes to the equator. We investigated the dynamo model with the meridional circulation by the low-mode approach. This approach is based on an assumption that the solar magnetic field can be described by non-linear dynamical systems with a relatively small number of parameters. Such non-linear dynamical systems are based on the equations of dynamo models. With this method dynamical systems have been built for media which contains the meridional flow and thickness of the convection zone of the star. It was shown the possibility of coexistence of quasi-biennial and 22-year cycle. We obtained the different regimes (oscillations, vacillations, dynamo-bursts) depending on the value of the dynamo-number, the meridional circulation, and thickness of the convection zone. We discuss the features of these regimes and compare them with the observed features of evolution of the solar and geo magnetic fields. We built theoretical paleomagnetic time scale and butterfly-diagrams for the helicity and toroidal magnetic field for different regimes.

1. Introduction

It is generally accepted that the 11-year cycle of solar activity is connected with the action of the magnetic dynamo, the mechanism of which is based on the mutual work of the α -effect and the differential rotation [1]. It is assumed that Sun magnetic field has two components: the poloidal and the toroidal. The toroidal magnetic field originates from the poloidal one owing to the action of the differential rotation, which is situated inside the convective zone of the Sun. The reverse process of the transformation of the toroidal magnetic field into the poloidal one is realized as a result of the violation of the mirror symmetry of convection in a rotating body. The Coriolis force acting on the arising and diverging (descending and converging) vortices leads to the prevalence of right vortices in the northern hemisphere and left vortices in the southern one. The electromotive force, appearing as a result of the action of the Faraday's electromagnetic induction, after the averaging by velocity pulsations gain the component which is parallel to the mean magnetic field. It is this component that make close the self-excitation circuit in the Parker dynamo.

The 11-year cycle of solar activity is connected with its spot-producing activity. At the beginning of the solar activity cycle the spots appear at the mean latitudes and during



approximately 11 years approach to the equator. Such latitudinal-temporal sunspot distribution looks like butterflies and is called butterfly-diagram or Maunder butterflies.

According to conceptions of dynamo theory, the butterfly appearance is connected with the motion of the toroidal component of the magnetic field (dynamo-wave) from higher latitudes to the equator in each hemisphere. The latitudinal-temporal diagram of the level lines of the magnetic field toroidal component obtained even using simplest dynamo models, qualitatively reproduces observational data of the butterfly-diagram. The form of the butterfly-diagram obtained in models is in essential dependence from the controlling model parameters. In [2], [8], [10], [11] it was shown numerically and analytically that meridional circulation directed against the dynamo-wave propagation can essentially slow down its motion. Besides, intensive meridional circulation “blows away” the dynamo-wave to the poles. In [6] it was shown that the turbulent diffusivity coefficient also has ability to influence the duration of solar activity cycle. Also there was shown that the α -effect latitudinal profile has influence upon the form of the Maunder butterfly.

Thus, the characteristics of the theoretical latitudinal-temporal diagrams for the magnetic fields of the Sun and other stars significantly depend on the controlling parameters in the dynamo models. On the other hand, the attempt to reproduce the observational latitudinal-temporal distributions for the stars magnetic fields by the choice of the appropriate values and the controlling processes depending on the coordinates and time could give the information about the physics of the investigated process. However, it is should be remembered that the physics of the process will be limited by the bounds of the chosen model.

2. Dynamo Model

Here we use a simple dynamo model which is a straightforward generalization of the initial Parker [7] migratory dynamo. The governing dynamo equations are derived by averaging the magnetic field over the radius within the thin shell, where the dynamo mechanism operates, and neglecting terms describing the effects of curvature near the pole. In this case the dynamo equations have the form:

$$\frac{\partial A}{\partial t} = R_\alpha \alpha B + \beta \frac{\partial^2 A}{\partial \theta^2} - V \frac{\partial A}{\partial \theta} - \eta^2 \beta A, \quad (1)$$

$$\frac{\partial B}{\partial t} = R_\omega \sin \theta \frac{\partial A}{\partial \theta} + \beta \frac{\partial^2 B}{\partial \theta^2} - \frac{\partial(VB)}{\partial \theta} - \eta^2 \beta B. \quad (2)$$

Here B represents the toroidal magnetic field, A is the azimuthal component of the vector potential of the poloidal magnetic field, and θ is the latitude measured from the pole, η is characterizes the thickness of the convective zone, t is the time. The distances are measured in units of the star of planet radius R and the time is in units of the diffusion time R^2/β . Baliunas (2006) showed that the terms proportional to η^2 describe the radial magnetic field diffusion. Since the thickness of the convective zone in the Sun is 1/3 of the radius, $\eta \approx 3$ for the Sun. The factor $\sin \theta$ is the decrease in the differential rotation at higher latitudes [4]. The second equation neglects the small contribution of the α -effect, i.e. we use so-called α - Ω -approximation. Curvature effects are absent in the diffusion terms. In Eq. 1–2 the parameters R_α and R_ω describe intensity of the α -effect and the differential rotation, respectively, the parameter $D = R_\alpha R_\omega$ is the dimensionless dynamo-number (a measure of the intensity of dynamo action), and β is the turbulent diffusivity. We used a simple scheme for the stabilization of the magnetic field growth, namely, the algebraic quenching of the helicity. This scheme assumes that $\alpha = \alpha_0(\theta)/(1 + \xi^2 B^2) \approx \alpha_0(\theta)(1 - \xi^2 B^2)$, where $\alpha_0(\theta) = \sin \theta$ is the helicity in the unmagnetized medium and $B_0 = \xi^{-1}$ is the magnetic field for which the α -effect is considerably suppressed.

In this paper we restrict ourselves to the consideration of dipole symmetry of the magnetic field with boundary conditions at the poles: $A(0, t) = B(0, t) = A(\pi, t) = B(\pi, t) = 0$, because we find only the solutions with the dipole symmetry.

Following [8], we consider latitudinal profile of the meridional circulation: $V(\theta) = v \sin 2\theta$. Since in our model the latitude is measured from the pole, the value with a positive sign corresponds to the meridional circulation directed along the dynamo-wave propagation.

The basic idea of the low-mode approach is that the mean-field dynamo equations are projected onto a minimum set of several first eigenfunctions of the problem describing the decay of magnetic fields without any generation sources. In this case it is necessary to choose the minimum set of functions in such a way that the solution which is a set of several first time-dependent Fourier coefficients taking into account the generation sources will describe the general behavior of the magnetic field of a given object and will not describe this field with any lower set of functions. Substituting the chosen set of components of the magnetic field into the dynamo equations one can obtain a dynamical system of equations containing the selected modes.

According to [13], we describe the magnetic field as

$$\begin{aligned} B(\theta, t) &= b_1(t) \sin 2\theta + b_2(t) \sin 4\theta + b_3(t) \sin 6\theta, \\ A(\theta, t) &= a_1(t) \sin \theta + a_2(t) \sin 3\theta + a_3(t) \sin 5\theta. \end{aligned}$$

Substituting the chosen set of components of the magnetic field into the dynamo equations and collecting the coefficients of the sines of similar arguments, we can obtain a dynamical system of six equations containing the six modes [12], [13].

3. Results

As shown in [9], [13] dynamical system reproduces regimes of oscillations, vascillations (oscillations relative to a non-zero value of the magnetic field amplitude), dynamo-bursts and a special kind of oscillation, which is similar to the generation of quasi-biennial oscillation at the background of 22-year cycle. In [8], [12] [13] regimes of generation and their dependence on the control parameters v, D, β, η are discussed. However, in these papers the butterfly-diagrams of the helicity for these regimes and butterfly-diagrams of toroidal magnetic field for vascillations, dynamo-bursts and oscillations with two peaks which is similar to the dependence of Wolf number on time have not been built. Here we represent these butterfly-diagrams.

In Fig. 1a dependence of first component of the toroidal field on time is represented at $D = -2000$, $\beta = 1$, $v = 0$, and $\eta = 3$. Amplitude of the magnetic field and time are given in model units. Such dependence has oscillations with two peaks at maximum of the magnetic activity cycle which is similar to the dependence of Wolf number on time for solar cycles 22 and 23. For these dynamo parameters in Fig. 1b and Fig. 1c the butterfly-diagrams for the toroidal field and the helicity are represented. In Fig. 1d the butterfly-diagram for the helicity at $D = -2300$, $\beta = 1$, $v = 0$, and $\eta = 3$ is presented for combination of quasi-biennial and 11-years cycles. Theoretical butterfly-diagram for helicity represents Hale polarity rule similarly butterfly-diagram for observable current helicity in [5].

In Fig. 2a and Fig. 2b the butterfly-diagrams for the toroidal field and helicity are presented at $D = 90$, $\beta = 1$, $v = 0$, and $\eta = 0$ for the regime of vascillations. In Fig. 2c and Fig. 2d the butterfly-diagrams for the toroidal field and helicity are presented at $D = 128$, $\beta = 1$, $v = 0$, and $\eta = 0$ for the regime of dynamo-bursts. Fig.2a,b,c,d are built for fully convective stars.

It is suggested that generation of Earth magnetic field is carried out in outer core which is composed of iron and nickel. We use our dynamo model for modeling reversals of the geomagnetic poles. In [14] it was suggested that fluctuations in the α -effect are cause of geomagnetic reversals. It has been indicated that the reversal scale obtained in the scope of this model is rather close

to the observed scale in its properties. In our model we take into account meridional circulation and consider magnetic field as

$$B(\theta, t) = b_1(t) \sin 2\theta + b_2(t) \sin 4\theta,$$

$$A(\theta, t) = a_1(t) \sin \theta + a_2(t) \sin 3\theta,$$

$\beta = 1$ and $\eta = 0$.

We obtained regimes similar to the geomagnetic reversals at $D \sim 500 \div 600$ and $v \sim -3 \div 0$. In Fig. 2e the theoretical dependence of first component amplitude of the poloidal field on time (top plot), theoretical paleomagnetic time scale (middle plot) and observable paleomagnetic time scale (bottom plot) are represented at $D = 500$, $v = -1$. Here time and amplitude of the magnetic field are given in model units. In Fig.3e for theoretical and observable paleomagnetic time scales we can see various periods of reversals which have a chaotic sequence.

4. Conclusion

The simple non-linear dynamic systems for the dynamo equations reproduce main features of solar and geo magnetic fields. We obtained that theoretical butterfly-diagrams for helicity represent Hale polarity rule. Our dynamic system is able to reproduce combination of quasi-biennial and 11-years cycles, moreover, we found regime of oscillations with two peaks at maximum of the magnetic activity cycle which is similar to the dependence of Wolf number on time for solar cycles 22 and 23. We obtained the regime similar to the geomagnetic reversals at constant governing parameters of the dynamo model.

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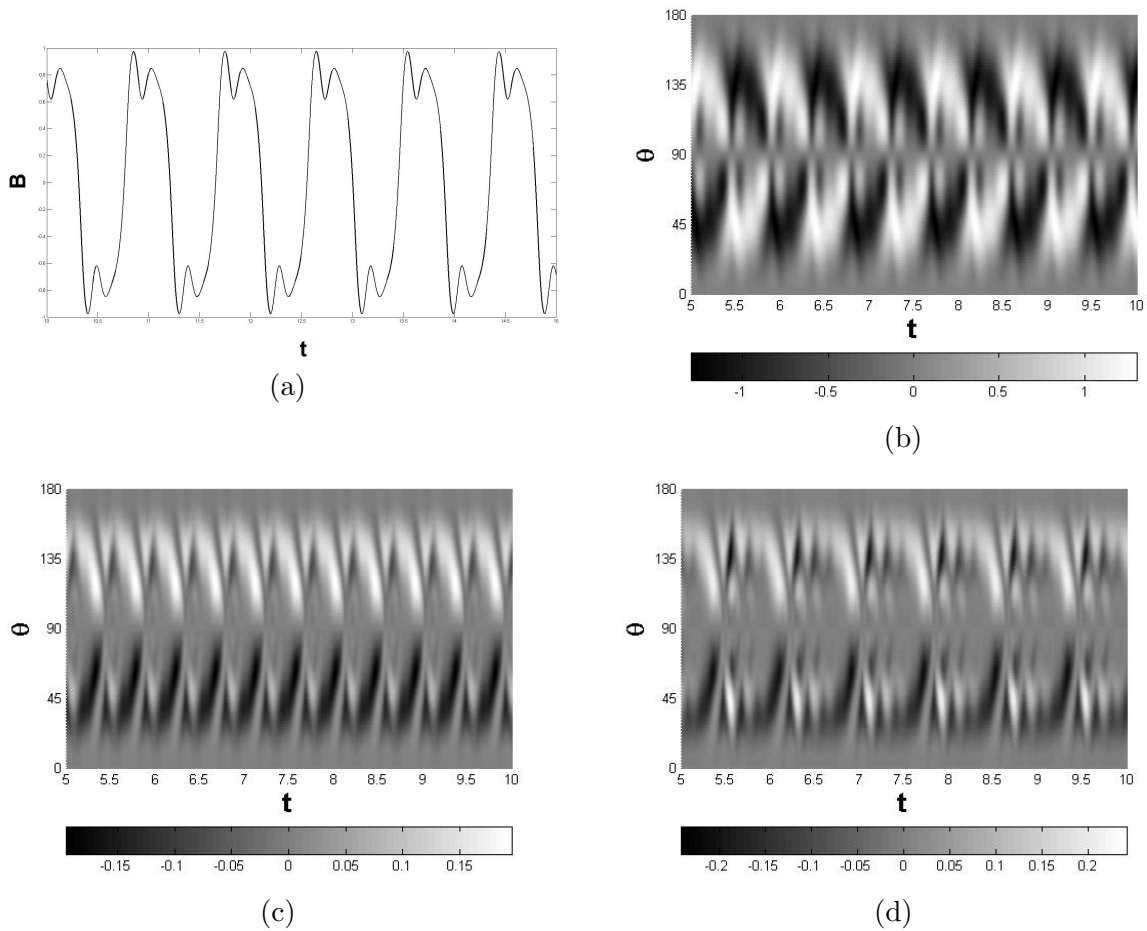


Figure 1. The dependence between the magnetic field amplitude and time (a), the butterfly-diagram for the toroidal field (b), the butterfly-diagram for the helicity (c) at $D = -2000$, $\beta = 1$, $v = 0$, and $\eta = 3$; the butterfly-diagram for the helicity at $D = -2300$, $\beta = 1$, $v = 0$, and $\eta = 3$

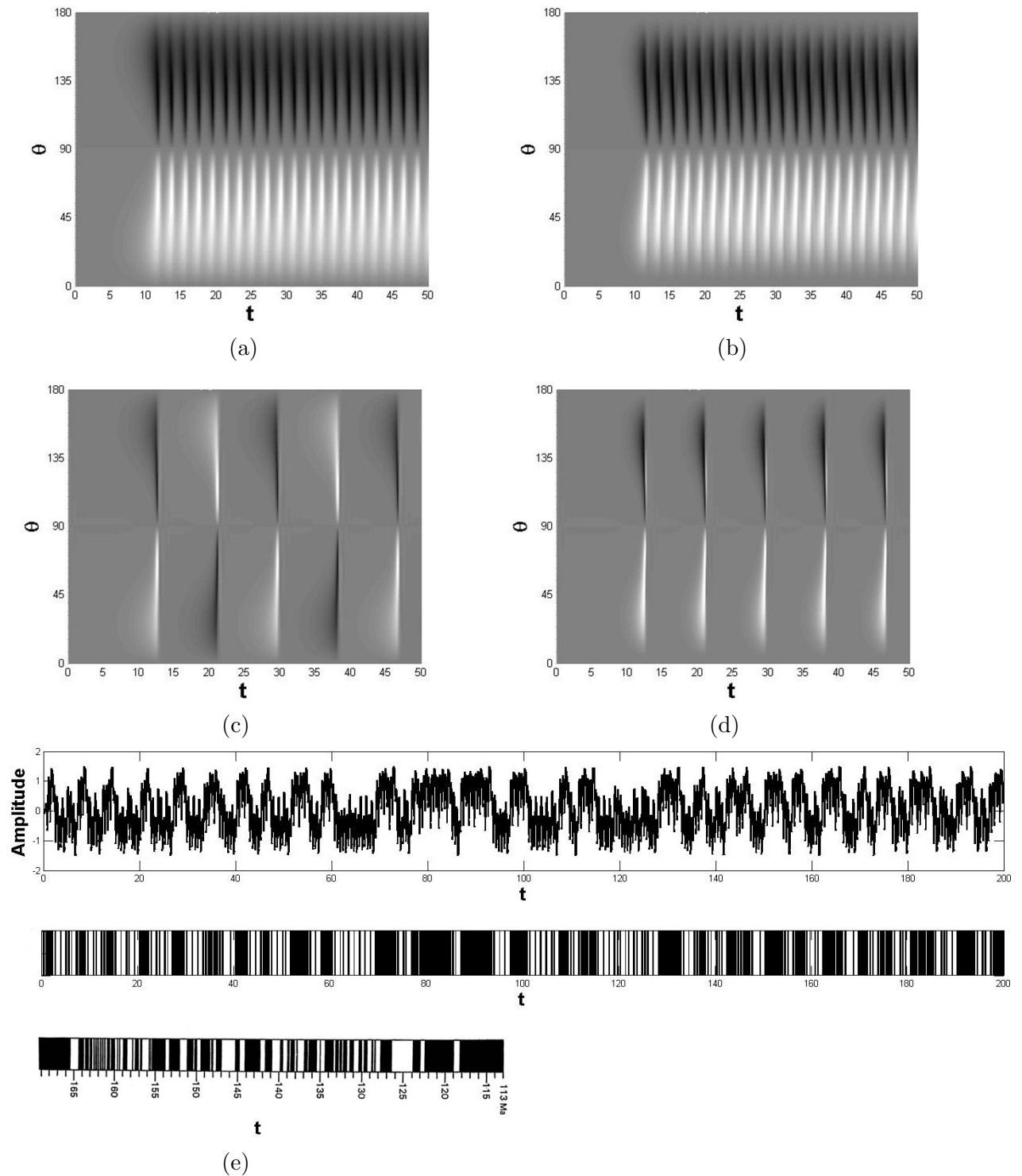


Figure 2. The butterfly-diagram for the toroidal field (a) and helicity (b) at $D = 90$, $\beta = 1$, $v = 0$, and $\eta = 0$; the butterfly-diagram for the toroidal field (c) and helicity (d) at $D = 128$, $\beta = 1$, $v = 0$, and $\eta = 0$; the theoretical dependence of first component amplitude of the poloidal field on time (top plot of (e)), theoretical paleomagnetic time scale (middle plot of (e)) and observable paleomagnetic time scale (bottom plot of (e)) at $D = 500$, $v = -1$, $\beta = 1$, and $\eta = 0$.