

Analysis of Relationship between Wavelength Selectivity and Angular Selectivity of Rugate coating

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Abstract. Based on Bragg law, Airy's formulae, and second-order Taylor series expansion, the relationships between wavelength selectivity and angular selectivity of the ordinary and phase-shifted Rugate coatings are investigated, respectively. And their expressions of the wavelength selectivity bandwidth and the angular selectivity bandwidth of these two types of Rugate gratings are put forward. The results show that when the incidence angle is far away from 0 rad, the bandwidth of the wavelength selectivity is proportional to that of the angular selectivity. And when the incidence angle approaches or even equals 0 rad (frequently-used cases), the bandwidth of the wavelength selectivity is squarely proportional to that of the angular selectivity. The results are instructive for the design and application of Rugate coatings.

1. Introduction

Rugate coating is a kind of thin film with the refractive index changing continuously along its thickness^[1]. Compared with the traditional graded coatings, Rugate coating has a lot of merits, such as no or few harmonious reflective bands, low stress between different layers, high laser damage threshold, and so on^[2-4]. With the development of the modern fabrication methods, the characteristics of Rugate coatings are analyzed extensively^[5-8]. Nowadays, the most widely-used spatial filter is pinhole filter. Due to its focusing characteristics, pinhole filter cannot be adjusted easily to the application for the high power laser beam. So the non-focusing method has attracted attention. Due to the fine wave vector selectivity of the grating, the non-spatial filtering for laser beam based on gratings has been put forward since 2000s^[9-12]. The fine wave vector selectivity denotes the fine wavelength selectivity at some angular domain and the fine angular selectivity at some wavelength domain. So Rugate coatings may be a potential candidate for the pinhole filter, especially in the high power laser field, with its fine angular selectivity^[13]. However, there is little attention paid to the wavelength selectivity for the



performance degradation of Rugate coatings' application upon the spatial filter of laser beam, especially upon the laser pulse. In order to make the Rugate coating suitable for the spatial filter of the short pulse laser, its relationship between the angular selectivity and wavelength selectivity is analyzed in order to search for structure of Rugate coatings with the fine angular selectivity and weak wavelength selectivity. From the refractive index's distribution, Rugate coating can be categorized into four types, such as ordinary, chirped, apodized, and phase-shifted types. In this paper, we just analyze the ordinary Rugate coating and phase-shifted Rugate coating. The other two types can be investigated with the similar method.

2. The definition of angular and wavelength selectivity bandwidths

For each Rugate grating, there is a wave vector at which the transmittance or reflectance efficiency is the highest. So we define the wave vector as the central wave vector k_0 . And there must be the incidence angle θ_0 and wavelength λ_0 in corresponding to the central wave vector. The angular selectivity bandwidth $\Delta\theta$ is defined as the two times incidence angle when the transmittance or reflectance efficiency η descends to one half of its maximum η_{max} . And $\Delta\theta$ can be expressed as

$$\begin{cases} \eta(\lambda_0, \theta_0) = \eta_{max} \\ \eta(\lambda_0, \theta_0 + \Delta\theta/2) = \eta_{max}/2 \end{cases} \quad (1)$$

And with the similar definition, the wavelength selectivity bandwidth $\Delta\lambda$ can be expressed as

$$\begin{cases} \eta(\lambda_0, \theta_0) = \eta_{max} \\ \eta(\lambda_0 + \Delta\lambda/2, \theta_0) = \eta_{max}/2 \end{cases} \quad (2)$$

where the parameters has the same definition as those in equation (1).

3. The relationship between angular and wavelength selectivity of ordinary Rugate coating

The reflection of laser beam by ordinary Rugate coating is shown in figure 1.

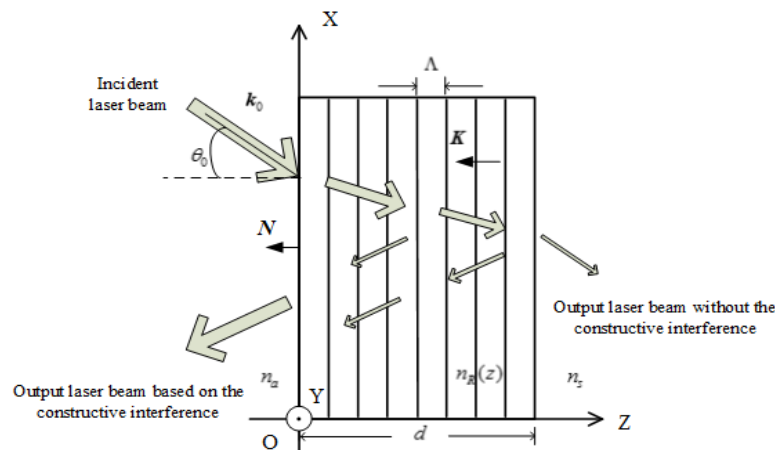


Fig1. Reflection of laser beam by ordinary Rugate coating

In figure 1, N denotes the normal to the front surface. n_a and n_s denote the refractive indexes of the ambient media and substrate, respectively. d is the thickness. K denotes the coating's vector, and is equal to $2\pi/\Lambda$, where Λ is the period of the coating. k_0 denotes the

central wave vector of the incident laser. And θ_0 is the incidence angle. The refractive index of ordinary Rugate coating is sinusoidally modulated. It can be expressed as

$$n_R(z) = n_0 + n_1 \sin(Kz) \quad (3)$$

where n_0 and n_1 denote the average and modulated refractive index, respectively. From equation (3), we can conclude that the structure of the ordinary Rugate coating is the same as that of reflecting volume phase grating^[14].

Due to the fact that the structure of the ordinary Rugate coating is the same as that of the reflecting volume phase grating, the reflecting characteristics of ordinary Rugate coating can be analyzed by Bragg law. From Snell's theorem and Bragg law, the central wave vector must satisfy the following equations, which can be expressed as:

$$n_a \sin \theta_0 = n_0 \sin \theta'_0 \quad (4)$$

$$2n_0 \Lambda \cos \theta'_0 = \lambda_0 \quad (5)$$

where θ'_0 denotes the angle of the incidence angle θ_0 in the Rugate coating.

We make the second order Taylor series expansion for λ with the variable θ at the point (λ_0, θ'_0) . And then λ can be expressed as:

$$\Delta \lambda = -2n_0 \Lambda \sin \theta'_0 \Delta \theta' - n_0 \Lambda \cos \theta'_0 \Delta \theta'^2 + o[\Delta \theta'^2] \quad (6)$$

where $\Delta \theta$ denotes $\theta' - \theta'_0$, while $\Delta \lambda$ denotes $\lambda - \lambda_0$. $o[\Delta \theta'^2]$ denotes the higher-order infinitesimal of $\Delta \theta'^2$.

According to the angle θ_0 , there are three situations classified for the relationship between wavelength and angular selectivity.

3.1. $\theta_0 = 0$

When θ_0 is equal to 0, the equation (6) can be expressed as:

$$\Delta \lambda = -n_0 \Lambda \Delta \theta'^2 \quad (7)$$

where the higher-order infinitesimal of $\Delta \theta'^2$ is omitted.

We make the derivative for the equation (4), which is expressed as:

$$\Delta \theta' = [n_a \cos \theta_0 / (n_0 \cos \theta'_0)] \Delta \theta \quad (8)$$

Due to $\theta_0 = 0$, $\Delta \theta' = (n_a / n_0) \Delta \theta$.

So the relationship between wavelength selectivity bandwidth and angular selectivity bandwidth can be expressed as:

$$\Delta \lambda = -(n_a^2 \Lambda / n_0) \Delta \theta^2 \quad (9)$$

The result shows that the wavelength selectivity bandwidth is squarely proportional to the angular one, when the central vector is normally incident into the ordinary Rugate coating.

3.2. $\theta_0 \in (0, 0.1 \text{ rad})$

According to equations (4) and (8), the absolute quotient of the first two terms of the right side of equation (6) can be expressed as:

$$|2n_0 \Lambda \sin \theta'_0 \Delta \theta' / (n_0 \Lambda \cos \theta'_0 \Delta \theta'^2)| = |2 \tan \theta'_0 / \Delta \theta'| = |2 \tan \theta_0 / \Delta \theta| \quad (10)$$

Usually, $\Delta \theta$ is less than 0.1 rad . So when $\theta_0 \in (0, 0.1 \text{ rad})$, the quantity of (10) is less than 1. The second term of the right side of (6) cannot be omitted. And with the omission of the higher order infinitesimal of $\Delta \theta'^2$ and equation (4), (5), and (8), $\Delta \lambda$ can be expressed as:

$$\Delta \lambda = \{(4n_0^2 \Lambda^2 - \lambda_0^2)[4(n_a^2 - n_0^2) \Lambda^2 + \lambda_0^2]\}^{\frac{1}{2}} \frac{1}{\lambda_0} \Delta \theta + [4(n_a^2 - n_0^2) \Lambda^2 + \lambda_0^2] \frac{1}{2\lambda_0} \Delta \theta^2 \quad \left(\frac{\lambda_0}{2n_0} < \Lambda < \frac{\lambda_0}{2(n_0^2 - n_a^2)^{\frac{1}{2}}} \right) \quad (11)$$

3.3. $\theta_0 \in [0.1, \pi/2 \text{rad})$

When $\theta_0 \in [0.1, \pi/2 \text{rad})$, the quantity of (10) is much larger than 1. So the second term of the right side of (10) and the higher order infinitesimal of $\Delta\theta'^2$ can be omitted, $\Delta\lambda$ can be expressed as:

$$\Delta\lambda = \{(4n_0^2\Lambda^2 - \lambda_0^2)[4(n_a^2 - n_0^2)\Lambda^2 + \lambda_0^2]\}^{\frac{1}{2}} \frac{1}{\lambda_0} \Delta\theta \quad \left(\frac{\lambda_0}{2n_0} < \Lambda < \frac{\lambda_0}{2(n_0^2 - n_a^2)^{\frac{1}{2}}}\right) \quad (12)$$

The result shows that the wavelength selectivity bandwidth is proportional to the angular one at this situation.

4. The relationship between angular and wavelength selectivity of phase-shifted Rugate coating

The transmission of a laser beam by phase-shifted Rugate coating is shown in figure 2.

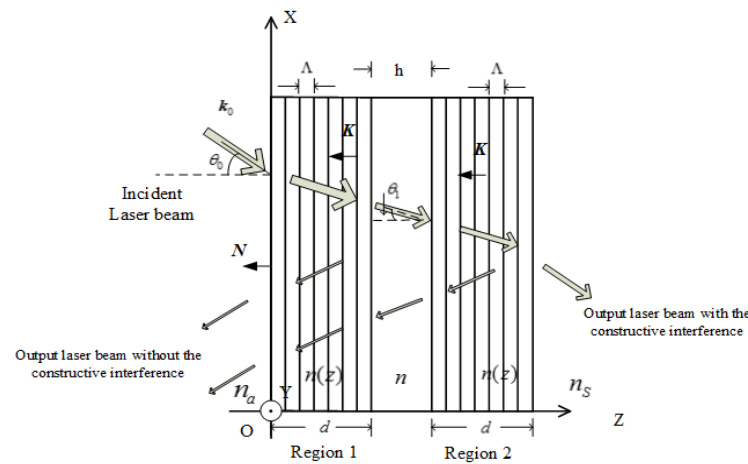


Fig2. Transmission of a laser beam by phase-shifted Rugate coating

The parameters in figure 2 are defined similarly as that in figure 1. Region 1 and region 2 are identical to each other. And their average refractive index is the same as n .

From the point that the two highly reflective coatings are placed parallel, the structure of the phase-shifted Rugate coating is similar to the Fabry-Perot interferometer. And their optical characteristics are similar, except that the Fabry-Perot interferometer has transmission harmonics, while Rugate coating has not. We just analyze the relationship between the wavelength bandwidth and angular bandwidth of the phased-shifted Rugate coating. So by Airy's formulae and without loss, the intensity of the transmitted laser beam can be expressed as^[15]

$$I^T = (1 + F \sin^2 \frac{\delta}{2})^{-1} \quad (13)$$

where

$$F = 4R/(1 - R)^2 \quad (14)$$

$$\delta = 4\pi n h \cos \theta_1 / \lambda \quad (15)$$

R denotes the reflectivity of region 1.

When I^T reduces to the half of its maximum, δ changes to δ' , which can be expressed as

$$\delta' = 2 \arcsin(F^{-0.5}) \quad (16)$$

From equation (15), the relationship between λ and θ_1 is expressed as:

$$2\pi n h \cos \theta_1 / \arcsin(F^{-0.5}) = \lambda \quad (17)$$

Comparing equation (17) and (5), the relationships between λ and θ_0 of these two types of Rugate coatings are similar to each other. So through the similar method, we can deduce the relationship between the wavelength selectivity bandwidth and angular selectivity bandwidth of phase-shifted Rugate coating. The results are as follows.

When $\theta_0 = 0$, their relationship can be expressed as

$$\Delta\lambda = -\{2n_a^2\pi h/[n_0\arcsin(F^{-0.5})]\}\Delta\theta^2 \quad (18)$$

When $\theta_0 \in (0, 0.1\text{rad})$, their relationship can be expressed as

$$\begin{aligned} \Delta\lambda = & \left\{ \left(4n_0^2 \left[\frac{\pi h}{\arcsin(F^{-0.5})} \right]^2 - \lambda_0^2 \right) \left[4(n_a^2 - n_0^2) \left[\frac{\pi h}{\arcsin(F^{-0.5})} \right]^2 + \lambda_0^2 \right] \right\}^{\frac{1}{2}} \frac{1}{\lambda_0} \Delta\theta \\ & + [4(n_a^2 - n_0^2) \left[\frac{\pi h}{\arcsin(F^{-0.5})} \right]^2 + \lambda_0^2] \frac{1}{2\lambda_0} \Delta\theta^2 \\ & \left(\frac{\lambda_0}{2n_0} < \frac{\pi h}{\arcsin(F^{-0.5})} < \frac{\lambda_0}{2(n_0^2 - n_a^2)^{\frac{1}{2}}} \right) \end{aligned} \quad (19)$$

When $\theta_0 \in [0.1, \pi/2\text{rad})$, their relationship can be expressed as

$$\begin{aligned} \Delta\lambda = & \left\{ \left(4n_0^2 \left[\frac{\pi h}{\arcsin(F^{-0.5})} \right]^2 - \lambda_0^2 \right) \left[4(n_a^2 - n_0^2) \left[\frac{\pi h}{\arcsin(F^{-0.5})} \right]^2 + \lambda_0^2 \right] \right\}^{\frac{1}{2}} \frac{1}{\lambda_0} \Delta\theta \\ & \left(\frac{\lambda_0}{2n_0} < \frac{\pi h}{\arcsin(F^{-0.5})} < \frac{\lambda_0}{2(n_0^2 - n_a^2)^{\frac{1}{2}}} \right) \end{aligned} \quad (20)$$

5. Conclusion

According to equation (9), (11), (18) and (19), we can conclude that for ordinary or phase-shifted Rugate coatings, when $\theta_0 \in [0, 0.1\text{rad})$, the bandwidth of the wavelength selectivity is almost squarely proportional to that of the angular selectivity. It means that when the incidence angle approaches or even equals 0 rad, both ordinary and phase-shifted Rugate coatings have fine wavelength selectivity and weak angular selectivity. So they can not be used as the spatial filter for the laser pulse at this situation. When the incidence angle is far away from 0 rad, the bandwidth of the wavelength selectivity is proportional to that of the angular selectivity. In order to analyze the potential substitute for pin-hole spatial filter by Rugate coating, we should design Rugate coating with fine angular selectivity and weak wavelength selectivity. With the similar method to design Rugate coating in the optical spectrum domain, we can use needle algorithm to synthesize the well-performed coating structure in the angular spectrum domain.

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