

# Self-similarity of proton spin and $z$ -scaling

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**Abstract.** The concept of  $z$ -scaling previously developed for analysis of inclusive reactions in proton-proton collisions is applied for description of processes with polarized particles. Hypothesis of self-similarity of the proton spin structure is discussed. The possibility of extracting information on spin-dependent fractal dimensions of hadrons and fragmentation process from the cross sections and asymmetries is justified. The double longitudinal spin asymmetry  $A_{LL}$  of jet and  $\pi^0$ -meson production and the coefficient of polarization transfer  $D_{LL}$  measured in proton-proton collisions at  $\sqrt{s} = 200$  GeV at RHIC are analyzed in the framework of  $z$ -scaling. The spin-dependent fractal dimension of proton is estimated.

## 1. Introduction

Spin is one of the most fundamental properties of elementary particles. The proton spin structure is studied for a long time in processes with polarized leptons and protons. The goal is to understand complete picture of the proton spin in terms of quark and gluon degrees of freedom. Microscopic scenario of proton structure implies knowledge of momentum and spin distributions of its constituents at different scales. In the framework of QCD, the non-linear Yang-Mills equations taking into account gauge invariance and Lorentz covariance regulate dynamics of the constituent interactions both at hard and soft regimes.

There is a convincing evidence that inclusive reactions with unpolarized particles reveal self-similarity over a wide scale range [1, 2]. The scaling features of hadron production in  $p + p$  and  $\bar{p} + p$  collisions ( $z$ -scaling) are treated as a manifestation of self-similarity of the structure of colliding protons, interaction mechanism of their constituents, and fragmentation process into real hadrons. The parameters of the scaling,  $c$ ,  $\delta$  and  $\epsilon_F$ , have physical interpretation of a heat capacity of the produced matter, fractal dimension of the proton structure and fractal dimension of the fragmentation process, respectively. Extension of the method for analysis of polarization phenomena and verification of self-similarity of particle production in polarized  $p + p$  collisions is an interesting problem which could give new insight into our understanding the origin of proton spin. The spin-dependent fractal dimensions are new parameters of the  $z$ -scaling approach. They characterise polarization properties of proton structure, constituent interactions and hadronization process.

In this paper we analyze the double longitudinal spin asymmetry  $A_{LL}$  of jet and meson production and the coefficient of polarization transfer  $D_{LL}$  measured by the STAR and PHENIX collaborations at RHIC to extract information on spin-dependent fractal dimensions of proton and verify the hypothesis of self-similarity of proton spin in the  $z$ -scaling approach.



## 2. $z$ -Scaling

The  $z$ -scaling [1, 2] manifests itself in the fact that the inclusive spectra of various types of particles are described with a universal scaling function. The function  $\Psi(z)$  depends on single variable  $z$  in a wide range of the transverse momentum, registration angles, collision energies, and centralities. The scaling variable has the form:

$$z = z_0 \cdot \Omega^{-1}. \quad (1)$$

The quantity  $z_0$  is ratio of the transverse kinetic energy of the selected binary sub-process required for production of the inclusive particle with its partner (antiparticle) and the charged multiplicity density at midrapidity raised to a power of  $c$ . The parameter  $c$  is interpreted as a heat capacity of the produced matter. The selected binary sub-process is given by maximum of the function  $\Omega(x_1, x_2, y_a, y_b) = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2} (1 - y_a)^{\epsilon_a} (1 - y_b)^{\epsilon_b}$  with the condition  $(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2$ , where  $M_X = x_1 M + x_2 M + m/y_b$  is the minimal mass of the recoil system. The constraint expresses locality of the interaction at the constituent level. It restricts momentum fractions via kinematics of the constituent interactions and the 4-momenta  $P_1, P_2$  and  $p$  of the colliding protons and the inclusive particle with masses  $M$  and  $m$ , respectively.

The function  $\Omega(x_1, x_2, y_a, y_b)$  is proportional to the relative number of the configurations at the constituent level which include the binary sub-processes corresponding to the momentum fractions  $x_1$  and  $x_2$  of colliding protons and to the momentum fractions  $y_a$  and  $y_b$  of the secondary objects directly produced in the constituent interactions. The parameters  $\delta_1$  and  $\delta_2$  are fractal dimensions of the protons and  $\epsilon_a$  and  $\epsilon_b$  are fractal dimensions of the fragmentation process in the scattered and recoil direction, respectively. For the reactions with no polarizations we have  $\delta_1 = \delta_2 = \delta$  and  $\epsilon_a = \epsilon_b = \epsilon_F$ . The value of  $\epsilon_F$  depends on type of the inclusive hadron. The parameter dependence of the maximum of  $\Omega(x_1, x_2, y_a, y_b)$  used in (1) is briefly indicated as  $\Omega \equiv \Omega_{0000} =: \{\delta_1, \delta_2, \epsilon_a, \epsilon_b\}$  in the next. The low index (0000) corresponds to unpolarized particles in the initial and final states. Similar notation applies for processes with polarizations.

The scaling function  $\Psi(z)$  is expressed in terms of the inclusive cross section  $Ed^3\sigma/dp^3$ , multiplicity density  $dN/d\eta$ , and total inelastic cross section  $\sigma_{in}$  for the inclusive reaction  $P_1 + P_2 \rightarrow p + X$ . The function  $\Psi(z)$  is determined by the following expression:

$$\Psi(z) = -\frac{\pi}{(dN/d\eta)\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3}. \quad (2)$$

Here  $J$  is Jacobian for the transition from the variables  $\{p_T^2, y\}$  to  $\{z, \eta\}$ . The value of  $\Psi(z)$  is probability density to produce the inclusive particle with the corresponding value of the self-similarity parameter  $z$ .

## 3. Self-similarity for processes with polarizations

Below we use the  $z$ -scaling approach to formulate self-similarity hypothesis for reactions with polarized particles and discuss a possibility for studying inclusive particle production in polarized  $p + p$  collisions to extract information on spin-dependent fractal dimensions of proton [3, 4].

The reaction  $\vec{p} + \vec{p} \rightarrow h + X$  with two longitudinally polarized protons in the initial state is described by the spin-dependent cross sections  $\sigma_{++}, \sigma_{--}, \sigma_{+-}, \sigma_{-+}$ . The signs (+) and (−) denote positive and negative helicities of the protons, respectively. The double spin asymmetry  $A_{LL}$  of the process is expressed via combination of the cross sections in the form:

$$A_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}. \quad (3)$$

The corresponding quantities  $\Omega$  are written as follows:  $\Omega_{++00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \epsilon_F, \epsilon_F\}$ ,  $\Omega_{--00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \epsilon_F, \epsilon_F\}$ ,  $\Omega_{+-00} =: \{\delta, \delta + \Delta\delta, \epsilon_F, \epsilon_F\}$ ,  $\Omega_{-+00} =: \{\delta + \Delta\delta, \delta, \epsilon_F, \epsilon_F\}$ .

The process  $\vec{p} + p \rightarrow \vec{h} + X$  with one longitudinally polarized proton in the initial state and one longitudinally polarized particle (e.g.  $\Lambda$  hyperon) in the final state is described by the coefficient of polarization transfer. It is written in the following form:

$$D_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}. \quad (4)$$

The symbols with (+) and (−) denote cross sections corresponding to the parallel and antiparallel spin orientations relative to the respective momenta of the polarized particles (positive and negative helicities). The polarization in the initial state is related to the spin-dependent correction ( $\Delta\delta$ ) of the proton fractal dimension. Let us consider (+) helicity in the initial state only and denote  $\epsilon_F$  as the fractal dimension for hadronization of an unpolarized particle ( $h$ ) in the final state. This corresponds to  $\Omega_{+000} = \{\delta - \Delta\delta/4, \delta + \Delta\delta/4, \epsilon_F, \epsilon_F\}$ . If the inclusive particle is polarized, ( $\vec{h}$ ), the spin-dependent correction  $\Delta\epsilon_F$  to the value of  $\epsilon_F$  is included. The notations for such process are as follows:  $\Omega_{+0+0} = \{\delta - \Delta\delta/4, \delta + \Delta\delta/4, \epsilon_F - \Delta\epsilon_F/2, \epsilon_F\}$ ,  $\Omega_{+0-0} = \{\delta - \Delta\delta/4, \delta + \Delta\delta/4, \epsilon_F + \Delta\epsilon_F/2, \epsilon_F\}$ .

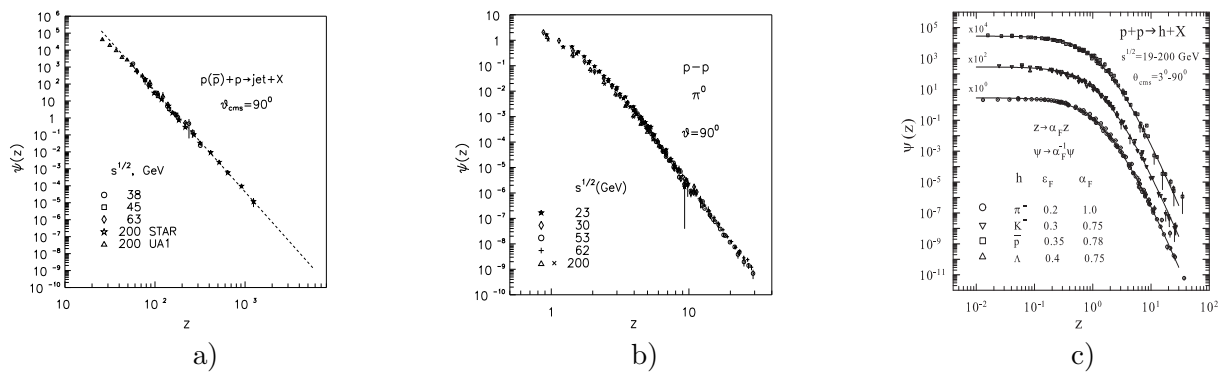
Using experimental information on the spin asymmetries and the unpolarized cross section, the spin-dependent functions  $\Psi_{++}, \Psi_{--}, \Psi_{+-}, \Psi_{-+}$  can be constructed [3, 4]. The functions have different arguments which we denote as  $z_{++}, z_{--}, z_{+-}, z_{-+}$ , respectively. They are expressed (1) by spin-dependent fractal dimensions. Based on existence of the  $z$ -scaling in unpolarized  $p + p$  collisions, we consider self-similarity of polarization processes at a constituent level in the form:

$$\Psi_{++} = \Psi(z_{++}), \quad \Psi_{+-} = \Psi(z_{+-}), \quad \Psi_{00} = \Psi(z_{00}). \quad (5)$$

This hypothesis assumes universality of  $\Psi(z)$  for different spin orientations. The relations include corrections  $\Delta\delta$  and  $\Delta\epsilon_F$  to the fractal dimensions  $\delta$  and  $\epsilon_F$  found for unpolarized reactions. Information on both the polarized and unpolarized cross sections are necessary to extract the spin-dependent fractal dimensions from the polarization characteristics ( $A_{LL}, D_{LL}$ ) of a given process. Such data allows us to obtain restrictions on the parameters  $\Delta\delta$  and  $\Delta\epsilon_F$  of the model.

#### 4. $z$ -Scaling and polarized $p + p$ collisions at RHIC

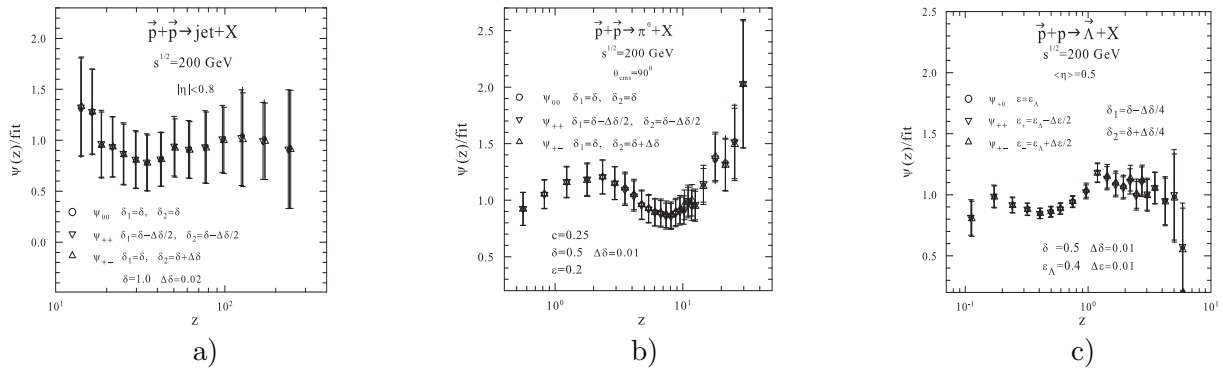
Figure 1 shows the scaling function  $\Psi(z)$  for unpolarized  $p + p$  collisions. The  $z$ -presentation of spectra of jets measured at  $\sqrt{s} = 38 - 200$  GeV and  $\pi^0$  mesons measured at  $\sqrt{s} = 23 - 200$  GeV and  $\theta_{cms} = 90^\circ$  is depicted in Figs. 1(a) and 1(b), respectively. The  $z$ -scaling of other hadrons is illustrated in Fig. 1(c). The data were obtained at the accelerators ISR, Sp̄pS and RHIC. As



**Figure 1.** The data  $z$ -presentation of inclusive spectra of jet (a)  $\pi^0$  meson (b) and hadron (c) production in  $p + p$  and  $\bar{p} + p$  collisions at ISR, Sp̄pS and RHIC energies.

seen from the figure, similarity of the inclusive cross sections is valid in a wide range of kinematic variables. The function  $\Psi(z)$  exhibits a power behavior at high  $z$ .

Extension of the method for reactions with polarized particles was tested with data obtained at RHIC. We have constructed the functions  $\Psi_{++}$ ,  $\Psi_{+-}$  and  $\Psi_{+0}$  in dependence on corresponding scaling variables to verify the hypothesis (5). Figure 2 demonstrates coincidence of the ratios of spin-dependent and spin-independent functions in dependence on  $z$ . The symbols shown in Fig. 2(a) correspond to the inclusive cross section [5] and the spin asymmetry  $A_{LL}$  [6] of jet production in proton-proton collisions measured by the STAR collaboration at  $\sqrt{s} = 200$  GeV. The data allows us to estimate the spin-dependent fractal dimension of proton from the process.



**Figure 2.** The ratios of scaled spin-dependent and spin-independent functions for  $\vec{p} + \vec{p} \rightarrow jet + X$  (a)  $\vec{p} + \vec{p} \rightarrow \pi^0 + X$  (b) and  $\vec{p} + p \rightarrow \vec{\Lambda} + X$  (c) processes at  $\sqrt{s} = 200$  GeV.

Figure 2(b) demonstrates the scaled spin-dependent functions  $\Psi(z)/fit$  for the reaction  $\vec{p} + \vec{p} \rightarrow \pi^0 + X$ . The double-longitudinal spin asymmetry  $A_{LL}$  of  $\pi^0$  mesons measured by the PHENIX collaboration at  $\sqrt{s} = 200$  GeV and  $\theta_{cms} = 90^\circ$  [7] and corresponding unpolarized cross section were used in the analysis. The correction to the fractal dimension  $\delta$  of unpolarized proton is found to be  $\Delta\delta = 0.01$ . Figure 2(c) shows ratios of the spin-dependent and spin-independent functions for the process  $\vec{p} + p \rightarrow \vec{\Lambda} + X$ . The comparison exploits data on the longitudinal spin transfer coefficient  $D_{LL}$  measured by the STAR collaboration at  $\sqrt{s} = 200$  GeV and  $\langle \eta \rangle = 0.5$  [8]. Using the value of  $\Delta\delta = 0.01$  obtained from the analysis of  $\pi^0$ -meson production, the spin-dependent correction to  $\epsilon_\Lambda$  is found to be  $\Delta\epsilon_\Lambda = 0.01$ .

Performed analysis justifies application of the  $z$ -scaling approach to study the scale properties of spin structure in hadron interactions with polarized particles. The coincidence of functional ratios shown in Fig. 2 indicates self-similarity of spin-dependent processes expressed by (5). This condition was used to estimate corrections  $\Delta\delta$  and  $\Delta\epsilon_\Lambda$  to the fractal dimensions  $\delta$  and  $\epsilon_\Lambda$ . The suggested procedure of data analysis is applicable to a wide class of polarization reactions. A systematic phenomenological investigation of the processes with polarizations based on the self-similarity principle would contribute to further development of theory and understanding of spin as one of the most important and basic property of particles.

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