

Non-singlet Q^2 -evolution by inverse Mellin transform in the analytic perturbation theory

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Abstract. We discuss the application of the analytic approach called the fractional Analytic Perturbation Theory (APT) to the QCD analysis of the non-singlet structure function $xF_3(x, Q^2)$. The inverse Mellin transform method applied for the fit of experimental data and for estimation of the Jacobi polynomial method accuracy in extraction of values of the scale parameter Λ_{QCD} and the form of the xF_3 structure function. Our estimates give the accuracy of the Jacobi polynomials method for the x -shape of the structure function about 10%, and accuracy for the scale parameter Λ_{QCD} 4%.

1. Introduction

Recently application of the APT approach to the QCD analysis of the nucleon structure function data with use of the well known method of the expansion of structure functions in a set of the system of orthogonal Jacobi polynomials (see [1–8]) was done. In this work, using the APT approach [9, 10], we apply the inverse Mellin transform method [11] to the QCD analysis of the xF_3 structure function experimental data. It should be noted that the application of the APT to the QCD analysis of the DIS data required a generalization of the APT on the case of non-integer power of QCD running coupling. Such a generalization was proposed in [12]. The inverse Mellin transform method rather precise and gets an accuracy about five significant digits in our case. We compare the results of both methods in order to estimate the accuracy of the Jacobi polynomial method results. In our analysis, we focus on values of the scale parameter Λ_{QCD} and the form of the $xF_3(x, Q^2)$ structure function.

2. Description of the methods

Let us briefly discuss two methods of the QCD analysis of the structure functions: the inverse Mellin transform method and the method base on the expansion of the structure function on a set of the Jacobi polynomials.

2.1. Jacobi polynomials method

This method based on the expansion of the structure function on a set of the Jacobi polynomials. One can express a shape of the structure function on the x -space to values of the Mellin moments



of the structure function. Then the xF_3 structure function can be presented as [2]

$$xF_3^{LT\ N_{max}}(x, Q^2) = x^\alpha(1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) M_3(j+2, Q^2) \quad \text{for PT}, \quad (1)$$

$$= x^\alpha(1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) \mathcal{M}_3(j+2, Q^2) \quad \text{for APT}. \quad (2)$$

Here $\Theta_n^{\alpha,\beta}$ are the Jacobi polynomials, $\alpha = 0.7$ and $\beta = 3.0$ fix the weight function of the Jacobi polynomials.

The perturbative renormalization group Q^2 evolution of the Mellin moments is well known (see, e.g., [13]) and in the leading order reads as

$$M_3^{pQCD}(N, Q^2) = \frac{[\alpha_s(Q^2)]^\nu}{[\alpha_s(Q_0^2)]^\nu} M_3(N, Q_0^2), \quad \nu(N) = \gamma_{NS}^{(0),N}/2\beta_0, \quad N = 2, 3, \dots, \quad (3)$$

where $\alpha_s(Q^2)$ is the QCD running coupling, $\gamma_{NS}^{(0),N}$ are the non-singlet one-loop anomalous dimensions, $\beta_0 = 11 - 2n_f/3$ is the first coefficient of the renormalization group β -function, and n_f denotes the number of active flavors ($n_f = 4$ in our analysis).

In the framework of the APT, the expression (3) is converted to:

$$\mathcal{M}_3^{APT}(N, Q^2) = \frac{\mathcal{A}_\nu(Q^2)}{\mathcal{A}_\nu(Q_0^2)} \mathcal{M}_3(N, Q_0^2), \quad (4)$$

where the analytic function \mathcal{A}_ν is derived from the spectral Källén–Lehmann representation and corresponds to the discontinuity of the ν -th power of the PT running coupling.

In the leading order (LO), the analytic function \mathcal{A}_ν has a rather simple form (see, e.g., [14])

$$\mathcal{A}_\nu^{LO}(Q^2) = [\alpha_{PT}^{LO}]^\nu - \left(\frac{4\pi}{\beta_0}\right)^\nu \frac{\text{Li}_\delta(t)}{\Gamma(\nu)}, \quad (5)$$

$$\text{Li}_\delta(t) = \sum_{k=1}^{\infty} \frac{t^k}{k^\delta}, \quad t = \frac{\Lambda^2}{Q^2}, \quad \delta = 1 - \nu, \quad (6)$$

where the PT running coupling $\alpha_{PT}^{LO} = 4\pi/[\beta_0 \ln(Q^2/\Lambda_{PT}^2)]$ and Li_δ is the polylogarithm function. Note that the function $\mathcal{A}_{\nu=1}(Q^2)$ defines the APT running coupling, $\alpha_{APT}(Q^2)$ (see [15, 16]).

Unknown quantity $M_3(N, Q_0^2)$ in (3) could be parameterized as the Mellin moments of the structure function xF_3 at some point Q_0^2 :

$$M_3(N, Q_0^2) = \int_0^1 dx x^{N-2} xF_3(x, Q_0^2) = \int_0^1 dx x^{N-2} A x^a (1-x)^b (1+\gamma x), \quad N = 2, 3, \dots \quad (7)$$

In our analysis we taken into account the higher twist contribution and therefore

$$xF_3^{exp}(x, Q^2) = xF_3^{LT\ N_{max}}(x, Q^2) + \frac{h(x)}{Q^2}, \quad (8)$$

where $h(x)$ is a shape of the higher twist contribution in the x space.

Table 1. The results of the QCD fit for the scale parameter Λ_{QCD} obtained in the framework of the PT and APT approaches using different methods.

Method of analysis	Λ_{PT} (MeV)	Λ_{APT} (MeV)
Inverse Mellin transform	378 ± 49	422 ± 77
Jacobi polynomials expansion	363 ± 49	407 ± 74

2.2. Inverse Mellin transform method

One can calculate the structure function at some Q^2 -value using the inverse Mellin transform [11]:

$$xF_3(x, Q^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn x^{-n} M_3(n, Q^2), \quad \text{PT}, \quad (9)$$

$$= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn x^{-n} \mathcal{M}_3(n, Q^2), \quad \text{APT}. \quad (10)$$

In the PT, it is well known the analytic continuation of the anomalous dimensions on the complex plane of Mellin moments, n . In the APT case, we calculate the function $\mathcal{A}_\nu(Q^2)$ for the complex Mellin moments by the numerical summation of the set in (6). The path of integration for the inverse Mellin transform lies to the right of all singularities of the analytic continuation of moments $M_3(n, Q^2)$ and $\mathcal{M}_3(n, Q^2)$.

3. Results of fit and discussion

The shape of the function $h(x)$ in (8) as well as the parameters A, a, b, γ in (7), and the scale parameter Λ_{QCD} are found by fit of a combined set of the xF_3 -data (see details in [7]). The description of the fitting procedure can be found in [17]. The target mass corrections are taken into account up to the terms M_N^2/Q^2 (see [18]).

As seen from Table 1, the accuracy of the determination of the scale parameter Λ_{QCD} by the Jacobi polynomials method is about 15 MeV: $\Delta\Lambda^{M-J} = \Lambda_{APT/PT}^M - \Lambda_{APT/PT}^J \approx 15$ MeV. Both methods give the same difference for a value of the parameter Λ_{QCD} in the APT and PT approaches: $\Lambda_{APT}^M - \Lambda_{APT}^J = \Lambda_{PT}^M - \Lambda_{PT}^J = 44$ MeV.

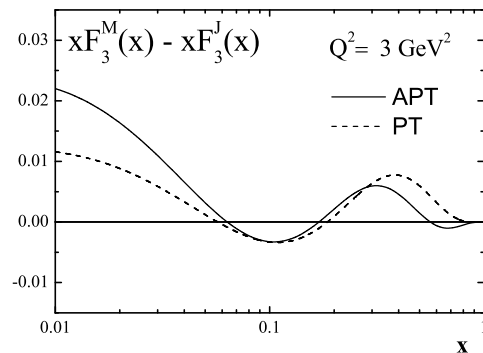


Figure 1. The difference between fitting results for xF_3 structure function data by the inverse Mellin and Jacobi methods presented for the APT and PT approaches.

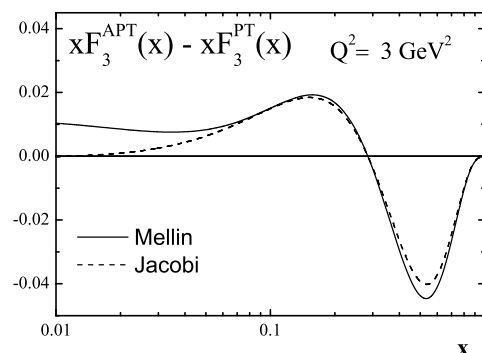


Figure 2. The difference for xF_3 structure function form in the APT and PT approaches for the inverse Mellin and Jacobi methods.

Figure 1 shows the difference in x -space between fitting results for xF_3 structure function data by using the inverse Mellin transform and the Jacobi polynomials expansion methods: $\Delta xF_3^{M-J} = xF_3^M - xF_3^J$ presented for the APT (solid line) and the PT (dashed line) approaches. The number of Jacobi polynomials is $N_{max} = 11$ and $Q_0^2 = 3 \text{ GeV}^2$. One can see that the value of $\Delta xF_3^{M-J} < 0.025$, which corresponds to the accuracy of the Jacobi method better than 10% for both theoretical approaches. This estimation is in qualitative agreement with the result obtained for the non-singlet xF_2 structure function [19].

Figure 2 shows the difference for the xF_3 -shape obtained in the APT (solid line) and the PT (dashed line) approaches using the inverse Mellin transform and the Jacobi polynomials methods: $\Delta xF_3^{\text{APT-PT}} = xF_3^{\text{APT}} - xF_3^{\text{PT}}$. As can be seen from this figure, the Jacobi method gives the same difference with the Mellin one at both theoretical approaches: the APT and the PT. But at small x the Jacobi method is not sensitive to the difference of results in the APT and PT approaches. While the inverse Mellin transform method reveals this difference. We found that at low $Q^2 = 1 \text{ GeV}^2$ the accuracy of the Jacobi method in the APT is two times better in comparison to the PT.

In conclusion we stress, that the Jacobi polynomials method is fast, but gives about 10% accuracy for x -shape of the structure function and 4% accuracy for the scale parameter Λ_{QCD} .

4. Acknowledgments

It is a pleasure for the authors to thank D. V. Shirkov and O. V. Teryaev for interest in this work and helpful discussions. This research was supported by the JINR-BelRFFR grant F14D-007, JINR-Bulgaria Collaborative Grant and by the RFBR Grants (Nrs 13-02-01005 and 14-01-00647).

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