Lightlike solitons with spin

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Abstract. New exact solution class of Born – Infeld type nonlinear scalar field model is obtained. These solutions are three-dimensional solitons propagating with speed of light. The appropriate twisted solitons are considered. Energy and absolute value of momentum for such soliton are proportional to absolute value of its angular momentum or spin in some approximation. The model of ideal gas for the twisted lightlike solitons with lowest twist parameter is considered. Explicit conditions provide that the spin of each soliton equals Planck constant. It is shown that these solitons look like photons.

1. Introduction

A nonlinear space-time scalar field model considered here is known for a long time sufficiently. This model is related to well known Born – Infeld nonlinear electrodynamics[1]. It is sometimes called Born – Infeld type scalar field model.

This model is attractive because it has relatively simple and geometrically clear form. It can be considered as a relativistic generalization of the minimal surface or minimal thin film model in three-dimensional space. In this generalization we have an extremal four-dimensional hypersurface or film in five-dimensional space-time. Thus this model can be called the extremal space-time film one.

The model under consideration has much in common with Born – Infeld nonlinear electrodynamics. It can be shown that the methods for description of soliton interaction used in nonlinear electrodynamics [2, 3] applying to the scalar model give the similar results.

Here we present a short report of investigation for the lightlike soliton solutions of the extremal space-time film model. More details can be found in article [4].

2. Extremal space-time film

We consider the action in the following world volume form:

$$\mathcal{A} = \int\limits_{\overline{V}} \sqrt{|\mathfrak{M}|}\,(\mathrm{d}x)^4\ , \quad \text{where} \quad \mathfrak{M} \doteqdot \det(\mathfrak{M}_{\mu\nu})\ , \quad \mathfrak{M}_{\mu\nu} = \mathfrak{m}_{\mu\nu} + \chi^2\,\frac{\partial\Phi}{\partial x^\mu}\,\frac{\partial\Phi}{\partial x^\nu}\ , \qquad (1)$$

$(\mathrm{d}x)^4 \doteqdot \mathrm{d}x^0\mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3$, $\overline{V}$ is a space-time volume, $\mathfrak{m}_{\mu\nu}$ are components of metric tensor for flat four-dimensional space-time, Φ is scalar real field function, χ is a dimensional constant. The Greek indices take values $\{0, 1, 2, 3\}$.

The tensor $\mathfrak{M}_{\mu\nu}$ can be called also the world tensor. The variational principle $\delta\mathcal{A} = 0$ with the world volume action (1) corresponds to extremal four-dimensional film $\Phi(\{x^\mu\})$ in five-dimensional space-time $\{\Phi, x^0,\, x^1,\, x^2,\, x^3\}$.

It is evident that the model under consideration keeps invariance for space-time rotation and scale transformation. Thus any solution of the model field equation give birth to the appropriate class of solutions with the following transform:

$$\Phi(\{x^\mu\}) \;\to\; a\,\Phi(\{L^\mu_{\cdot\nu}\,x^\nu/a\}) \;, \tag{2}$$

where $L^\mu_{\cdot\nu}$ are components of space-time rotation matrix, a is scale parameter.

Here we use the Minkowski metric $\underline{\mathfrak{m}}^{00} = 1$, $\underline{\mathfrak{m}}^{11} = \underline{\mathfrak{m}}^{22} = \underline{\mathfrak{m}}^{33} = -1$. For the case of another metric signature $\{-,+,+,+\}$ in (1) we have the field model with some distinctions [4]. But the appropriate details are inessential here.

3. Lightlike soliton

We consider solutions in the form of wave propagating along x^3 axis of Cartesian coordinate system with the speed of light. Thus we can write

$$\Phi = \Phi\left(\theta, x^1, x^2\right) \;, \quad \theta = \omega\,x^0 - k_3\,x^3 \;, \quad k_3 = \pm\omega \;, \quad \omega > 0 \;. \tag{3}$$

The substitution this function (3) to the model field equation gives the following one:

$$\left(1 - \chi^2\left(\frac{\partial\Phi}{\partial x^2}\right)^2\right)\frac{\partial^2\Phi}{(\partial x^1)^2} + 2\,\chi^2\,\frac{\partial\Phi}{\partial x^1}\,\frac{\partial\Phi}{\partial x^2}\,\frac{\partial\Phi^2}{\partial x^1\partial x^2} + \left(1 - \chi^2\left(\frac{\partial\Phi}{\partial x^1}\right)^2\right)\frac{\partial^2\Phi}{(\partial x^2)^2} = 0\;. \tag{4}$$

We introduce new independent variables $\xi = x^1 + \imath\,x^2$, $\overset{*}{\xi} = x^1 - \imath\,x^2$ ($\imath^2 = -1$) and obtain the general solution of the transverse equation (4) in the form

$$\Phi = \Xi_1(\tilde{\xi}_1) + \Xi_2(\tilde{\xi}_2) \;, \tag{5a}$$

where the functions $\Xi_1(\tilde{\xi}_1)$ and $\Xi_2(\tilde{\xi}_2)$ are arbitrary with restriction by reality of the field function Φ. The connection between variables $\{\tilde{\xi}_1, \tilde{\xi}_2\}$ and $\{\xi, \overset{*}{\xi}\}$ is defined by relations

$$\begin{pmatrix} \mathrm{d}\xi \\ \mathrm{d}\overset{*}{\xi} \end{pmatrix} = \begin{pmatrix} 1 & \chi^2\left(\Xi_2'\right)^2 \\ \chi^2\left(\Xi_1'\right)^2 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{\xi}_1 \\ \mathrm{d}\tilde{\xi}_2 \end{pmatrix} \;. \tag{5b}$$

According to (2) we have the following solution class from solution (5):

$$\Phi(\xi, \overset{*}{\xi}) \;\to \sqrt{\overline{o}\,\overset{*}{\overline{o}}}\;\Phi\left(\xi/\overline{o}, \overset{*}{\xi}/\overset{*}{\overline{o}}\right) \;, \tag{6}$$

where $\overline{o} = \overline{\rho}\,\mathrm{e}^{\imath\,\overline{\varphi}}$ is arbitrary complex constant with respect to coordinates $\{\xi, \overset{*}{\xi}\}$. Here $\overline{\rho}$ and $\overline{\varphi}$ are real constants with respect to coordinates $\{x^1, x^2\}$. But in general case these constants can be depend on the phase θ of soliton: $\overline{\rho} = \overline{\rho}(\theta)$, $\overline{\varphi} = \overline{\varphi}(\theta)$.

We take the arbitrary functions Ξ_1 and Ξ_2 in (5) which are appropriate to the solution

$$\Phi = \frac{\rho\!\!\!/^{\,m+1}}{\chi\,m}\left(\tilde{\xi}^{-m} + \overset{*}{\tilde{\xi}}{}^{-m}\right) \;, \tag{7a}$$

where m is natural number and constant $\rho\!\!\!/$ has a physical dimension of length.

We consider the twisted Gaussian lightlike soliton with the following phase functions

$$\overline{\varphi} = \pm\frac{\theta}{m}\,, \quad \overline{\rho} = \exp\!\left(-\frac{\theta^2}{2\,\overline{\theta}^2}\right)\,, \tag{7b}$$

where the signs '+' and '−' correspond to right and left twisted soliton accordingly, $\overline{\theta}$ is non-dimensional characteristic length of the soliton.

For slowly varying scale function $\overline{\rho}(\theta)$ ($\overline{\theta} \gg 1$) and sufficiently high frequency ω ($\not\rho\,\omega \gg 1$) we have the following notable relations between energy, momentum, and angular momentum of the twisted lightlike soliton:

$$\mathbb{E} = \mathbb{P} = \tilde{\omega}\,\mathbb{J}\,, \quad \text{where} \quad \mathbb{P} \doteqdot |\boldsymbol{\mathbb{P}}|\,, \quad \mathbb{J} \doteqdot |\boldsymbol{\mathbb{J}}|\,, \quad \tilde{\omega} \doteqdot \frac{\omega}{m}\,, \tag{8}$$

$\tilde{\omega}$ is the angular velocity of the twisted soliton.

The reversibility condition for the variable transformation (5b) is violated on a singular line which is a singular surface in three-dimensional space. Thus we have a shell soliton with excluded inner area.

Some graphics relating to solution (7) with twist parameter $m = 1$ are shown on Figures 1 – 3. The twist of soliton is well seen on these figures. We see two-sheeted helical surface with excluded cavity on Fig. 2 for $m = 1$. In general case we have the helical surfaces with $2\,m$ sheets.

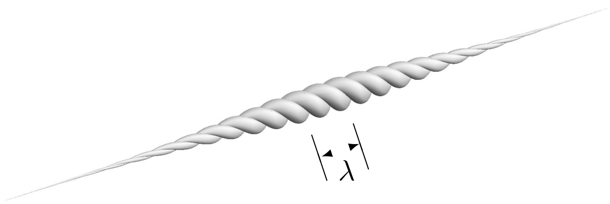

Figure 1. The shell of Gaussian twisted soliton for $m = 1$ and $\overline{\theta} = 10$.

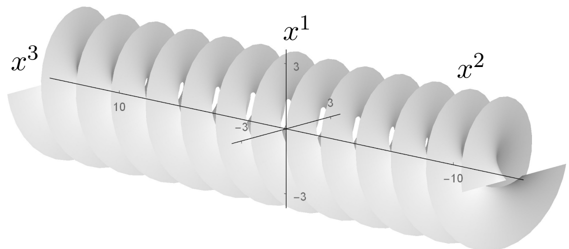

Figure 2. Zero level surfaces of the field function Φ for the Gaussian twisted soliton with $m = 1$ and $\overline{\theta} = 10$.

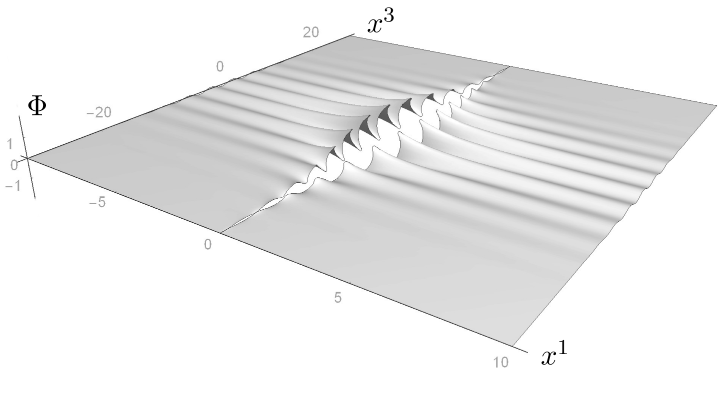

Figure 3. The field function Φ of Gaussian twisted soliton for $m = 1$ and $\overline{\theta} = 10$ on the plane $\{x^1, x^3\}$ for $x^2 = 0$.

4. Twisted solitons and photons

Let us consider ideal gas of the twisted lightlike solitons in a bounded three-dimensional volume V. Let us suppose also that absorptive and emissive capacities of the volume walls are provided by soliton-particles having the constant absolute value of angular momentum $\mathbb{J}_e = \hbar/2$, where $\hbar$ is Planck constant.

It can be shown [4] that in this case the absolute value of angular momentum or spin of the twisted lightlike solitons in the volume V must be equal to $\hbar$, because of the angular momentum conservation in absorption and emission events.

Because of notable connection between energy, momentum, and spin of the twisted lightlike solitons (8), for the case $m = 1$ we obtain the equilibrium distribution function by soliton frequencies which has the form of Planck formula for photons in high-frequency approximation.

A beam of the twisted lightlike solitons with $m = 1$ can provide the polarization property of light, because of an appropriate symmetry of the soliton solution. But the solitons with higher twist parameters $m \geqslant 2$ can not provide this polarization property [4].

Thus the lightlike solitons with twist parameter $m = 1$ can be considered as usual photons in some approximation. But the solitons of higher twist $m \geqslant 2$ have qualitative differences from the solitons of the lowest twist $m = 1$.

5. Conclusions

Thus the field model of extremal space-time film which is sometimes called Born – Infeld type scalar field model has considered in the presented investigation.

We have obtained new exact solution class for this model that is lightlike solitons. The soliton under consideration has a singularity which is a moving two-dimensional surface or shell. The general lightlike soliton can have a set of tubelike shells with the appropriate cavities which are excluded from the space.

We have considered the appropriate significant solution subclass that is twisted lightlike solitons. It is notable that energy and absolute value of momentum for the twisted lightlike soliton is proportional to absolute value of its angular momentum or spin in high-frequency approximation and for slowly varying scale function. We have the appropriate relations which are characteristic for photon in the case of lowest twist parameter $m = 1$ for explicit conditions.

Then we have investigated relations of the twisted lightlike solitons with $m = 1$ to photons. The model of ideal gas of the twisted lightlike solitons in a bounded volume has considered for this purpose. Planck formula for the soliton energy spectral density in the volume has obtained with explicit assumptions in some approximation.

An experimental check for a validity of the obtained soliton energy exact formula for real photon is proposed.

A beam of twisted lightlike solitons has considered. We have shown that this beam provides the effect of mechanical angular momentum transfer to absorbent by circularly polarized beam. This effect well known for photon beam. It has been found that the twisted lightlike soliton beam with $m = 1$ can provide polarization property of light as well as photon beam.

Thus we have a correspondence between photon and lightlike twisted soliton with the minimal value of twist parameter.

References

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