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Continuum radiative heat transfer modeling in multi-component anisotropic media in the limit of geometrical optics

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Abstract. Continuum-scale equations of radiative transfer and corresponding boundary conditions are derived for a multi-component anisotropic medium consisting of components in the range of geometrical optics. The derivations are obtained by employing the volume-averaging theory. This study generalizes the previous derivations obtained for multi-component isotropic media.

1. Introduction

Analysis of radiative heat transfer in heterogeneous media is pertinent to a variety of technological and scientific applications including combustion, solar energy, thermal insulations and space systems, and natural environments such as porous soils, snow, and wood. These media often consist of morphological features with the characteristic size significantly larger than the radiation wavelength of interest and, therefore, optical and radiative analyses can be based on the laws of geometrical optics. The radiative transfer equation (RTE) is typically employed as the theoretical basis for radiative characterization of such media assuming the media are homogeneous and described by a set of radiative properties [1]. The radiative properties—the extinction and scattering coefficients, and the scattering phase function—can be obtained theoretically or based on experimental measurements. A phenomenological extension of the RTE to heterogeneous media is based on the introduction of apparent radiative properties properties, which account for the actual morphology and composition of the multi-component medium. Theoretical and experimental approaches to determination of the apparent radiative characteristics of heterogeneous media are discussed in [1–3]. Volume-averaging approaches directly account for the multi-component composition and complex morphology of the medium. They result in coupled volume-averaged RTEs, one for each component or phase [4–6]. The derivation are based on the assumption that the media and their components are isotropic. The latter assumption limits the applicability of the volume-averaging approaches to multi-component anisotropic media, found in many applications such as described in [7].

Previous pertinent studies on directional radiative characterization of anisotropic heterogeneous media focused on ordered, layered structures [8] and fibrous materials [9, 10]. In this paper, the continuum-scale RTEs for multi-component anisotropic media with components in the limit of geometrical optics are derived along with the associated boundary conditions and radiative property definitions by employing the volume-averaging theory. The theory presented is an extention of the previous derivations for multi-component isotropic media presented in [5,6].

2. Continuum-scale equations of radiative transfer

Consider radiative transfer in a multi-component anisotropic medium. The medium consists of $i = 1, \ldots, M_1$ and $i = M_1 + 1, \ldots, M$ semi-transparent and opaque components, respectively, each of an arbitrary shape. Each component i is adjacent to $j = 1, \ldots, N_{i,1}$ and $j = N_{i,1} + 1, \ldots, N_i$ semi-transparent and opaque components, respectively. The present analysis is subject to the following assumptions: (i) all components are non-polarizing and the state of polarization can be neglected; (ii) all components are at local thermodynamic equilibrium; (iii) characteristic dimensions of all components are much larger than the radiation wavelengths of interest so that laws of geometrical optics are valid in each component; (v) diffraction effects are negligible; (vi) dependent-scattering effects due to inter-component radiative interactions are negligible; (vii) all components are at rest as compared to the speed of light; ($viii$) radiative transfer in each component and the whole medium is quasi-steady.

Each component i is characterized by the set of the discrete-scale optical and radiative properties: the effective refractive index $n_i(\hat{\mathbf{s}})$, the absorption and scattering coefficients, $\kappa_{\lambda,\mathrm{d},i}(\vec{\mathbf{r}}, \hat{\mathbf{s}})$ and $\sigma_{\lambda,\mathrm{s},\mathrm{d},i}(\vec{\mathbf{r}}, \hat{\mathbf{s}})$, respectively, and the scattering phase function $\Phi_{\lambda,\mathrm{d},i}(\vec{\mathbf{r}}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}})$. Note that the refractive indices are assumed to be indepdenent of wavelength and position for all components. Furthermore, each component i is characterized by its temperature $T_i(\vec{\mathbf{r}})$, i.e. the components are allowed to be at thermal non-equilibrium with respect to each other.

The quasi-steady discrete-scale intensity in each component i can be determined by solving the corresponding quasi-steady discrete-scale RTEs [1]:

$$\hat{\mathbf{s}} \cdot \nabla_{\vec{\mathbf{r}}} L_i\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}\right) = -\beta_{\mathrm{d},i}\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}\right) L_i\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}\right) + \kappa_{\mathrm{d},i}\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}\right) L_{\mathrm{b},i}\left(\vec{\mathbf{r}}\right) + \frac{\sigma_{\mathrm{s},\mathrm{d},i}\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}\right)}{4\pi} \int\limits_{\Omega_\mathrm{i}=0}^{4\pi} L_i\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}_\mathrm{i}\right) \Phi_{\mathrm{d},i}\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) \mathrm{d}\Omega_\mathrm{i}, \quad i = 1, \ldots, M, \tag{1}$$

where the spectral subscript λ has been omitted for brevity. $L_{\mathrm{b},i}$ is the spectral blackbody intensity inside the component i. Equation (1) is subject to the boundary conditions at $A_{ij,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}>0}$:

$$L_i\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}\right) = \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\cdot\hat{\mathbf{n}}_{ji}>0}} \tau''_{ji}\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) L_j\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{ji}\, \mathrm{d}\Omega_\mathrm{i} - \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\cdot\hat{\mathbf{n}}_{ji}<0}} \rho''_{ij}\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) L_i\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{ji}\, \mathrm{d}\Omega_\mathrm{i}, \quad j = 1, \ldots, N_{i,1}, \tag{2a}$$

$$L_i\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}\right) = \varepsilon'_{ji}\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}\right) L_{\mathrm{b},j}\left(\vec{\mathbf{r}}_{ij}\right) - \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\cdot\hat{\mathbf{n}}_{ji}<0}} \rho''_{ij}\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) L_i\left(\vec{\mathbf{r}}_{ij}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{ji}\, \mathrm{d}\Omega_\mathrm{i}, \quad j = N_{i,1} + 1, \ldots, N_i, \tag{2b}$$

where $\vec{\mathbf{r}}_{ij}$ is a position vector at the interface A_{ij}, and ε'_{ji} and $L_{\mathrm{b},j}\left(\vec{\mathbf{r}}_{ij}\right)$ are the directional spectral emissivity of the interface between components j and i and the blackbody intensity emitted by the component j into the component i, respectively. A superficial average is taken for each term in Eq. (1). Applying the spatial averaging theorem (SAT) [11],

$$\left\langle \nabla_{\vec{\mathbf{r}}}\, L_i\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}\right)\right\rangle = \nabla_{\vec{\mathbf{x}}} I_i\left(\vec{\mathbf{x}}, \hat{\mathbf{s}}\right) - \frac{1}{V\left(\vec{\mathbf{x}}\right)} \sum_{j=1}^{N_i} \int\limits_{A_{ij}} L_i\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}\right) \hat{\mathbf{n}}_{ji}\, \mathrm{d}A, \qquad I_i\left(\vec{\mathbf{x}}, \hat{\mathbf{s}}\right) \stackrel{\mathrm{def}}{=} \left\langle L_i\left(\vec{\mathbf{r}}, \hat{\mathbf{s}}\right)\right\rangle \tag{3}$$

to the resulting term containing the intensity gradient, and following the further derivation steps as elaborated for the case of isotropic multi-component media [6] leads to

$$\begin{aligned}
\hat{\mathbf{s}}\cdot\nabla_{\vec{\mathbf{x}}} I_i\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right) = &-\left[\beta_{\mathrm{d},i}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)+\sum_{j=N_{i,1}+1}^{N_i}\kappa_{ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)+\sum_{j=1}^{N_{i,1}}\sigma_{\mathrm{s,t},ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)+\sum_{j=1}^{N_i}\sigma_{\mathrm{s,r},ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)\right]I_i\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)\\
&+\kappa_{\mathrm{d},i}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)I_{\mathrm{b},i}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)+\sum_{j=N_{i,1}+1}^{N_i}\kappa_{ji}I_{\mathrm{b},j}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)+\frac{\sigma_{\mathrm{s,d},i}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)}{4\pi}\int\limits_{\Omega_i=0}^{4\pi}I_i\left(\vec{\mathbf{x}},\hat{\mathbf{s}}_{\mathrm{i}}\right)\Phi_{\mathrm{d},i}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}_{\mathrm{i}},\hat{\mathbf{s}}\right)\mathrm{d}\Omega_{\mathrm{i}}\\
&+\sum_{j=1}^{N_{i,1}}\frac{\sigma_{\mathrm{s,t},ji}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)}{4\pi}\int\limits_{\Omega_i=0}^{4\pi}I_j\left(\vec{\mathbf{x}},\hat{\mathbf{s}}_{\mathrm{i}}\right)\,\Phi_{\mathrm{t},ji}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}_{\mathrm{i}},\hat{\mathbf{s}}\right)\mathrm{d}\Omega_{\mathrm{i}}+\sum_{j=1}^{N_i}\frac{\sigma_{\mathrm{s,r},ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)}{4\pi}\int\limits_{\Omega_i=0}^{4\pi}I_i\left(\vec{\mathbf{x}},\hat{\mathbf{s}}_{\mathrm{i}}\right)\Phi_{\mathrm{r},ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}_{\mathrm{i}},\hat{\mathbf{s}}\right)\mathrm{d}\Omega_{\mathrm{i}},\\
&\qquad i=1,\ldots,M_1.
\end{aligned}\tag{4}$$

The continuum-scale absorption and scattering coefficients, and the scattering phase functions, associated with the superficial average intensities I_i and $I_{\mathrm{b},j}$ appearing in Eq. (1), are defined as

$$\kappa_{ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)\stackrel{\mathrm{def}}{=}-\frac{\int\limits_{A_{ij,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}<0}\alpha'_{ij}\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}\right)L_i\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}\right)\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}\,\mathrm{d}A}{I_i\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)V\left(\vec{\mathbf{x}}\right)},\tag{5a}$$

$$\kappa_{ji}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)\stackrel{\mathrm{def}}{=}\frac{\int\limits_{A_{ij,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}>0}\varepsilon'_{ji}\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}\right)L_{\mathrm{b},j}\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}\right)\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}\,\mathrm{d}A}{I_{\mathrm{b},j}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)V\left(\vec{\mathbf{x}}\right)},\tag{5b}$$

$$\sigma_{\mathrm{s,r},ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)\stackrel{\mathrm{def}}{=}-\frac{\int\limits_{A_{ij,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}<0}\rho'^{\frown}_{ij}\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}\right)L_i\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}\right)\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}\,\mathrm{d}A}{I_i\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)V\left(\vec{\mathbf{x}}\right)},\tag{5c}$$

$$\sigma_{\mathrm{s,t},ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)\stackrel{\mathrm{def}}{=}-\frac{\int\limits_{A_{ij,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}<0}\tau'^{\frown}_{ij}\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}\right)L_i\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}\right)\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}\,\mathrm{d}A}{I_i\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)V\left(\vec{\mathbf{x}}\right)},\tag{5d}$$

$$\Phi_{\mathrm{r},ij}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}_{\mathrm{i}},\hat{\mathbf{s}}\right)\stackrel{\mathrm{def}}{=}-\frac{\int\limits_{A_{ij,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}>0}\rho''_{ij}\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}_{\mathrm{i}},\hat{\mathbf{s}}\right)L_i\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}_{\mathrm{i}}\right)\hat{\mathbf{s}}_{\mathrm{i}}\cdot\hat{\mathbf{n}}_{ji}\,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}\,\mathrm{d}A}{\left(4\pi\right)^{-1}\sigma_{\mathrm{s,r},ij}I_i\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)V\left(\vec{\mathbf{x}}\right)},\quad \hat{\mathbf{s}}_{\mathrm{i}}\cdot\hat{\mathbf{n}}_{ji}<0,\tag{5e}$$

$$\Phi_{\mathrm{t},ji}\left(\vec{\mathbf{x}},\hat{\mathbf{s}}_{\mathrm{i}},\hat{\mathbf{s}}\right)\stackrel{\mathrm{def}}{=}\frac{\int\limits_{A_{ij,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}>0}\tau''_{ji}\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}_{\mathrm{i}},\hat{\mathbf{s}}\right)L_j\left(\vec{\mathbf{r}}_{ij},\hat{\mathbf{s}}_{\mathrm{i}}\right)\hat{\mathbf{s}}_{\mathrm{i}}\cdot\hat{\mathbf{n}}_{ji}\,\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}_{ji}\,\mathrm{d}A}{\left(4\pi\right)^{-1}\sigma_{\mathrm{s,t},ji}I_j\left(\vec{\mathbf{x}},\hat{\mathbf{s}}\right)V\left(\vec{\mathbf{x}}\right)},\quad \hat{\mathbf{s}}_{\mathrm{i}}\cdot\hat{\mathbf{n}}_{ji}>0.\tag{5f}$$

The set of Eqs. (4) presents generalization of Eqs. (18) and (27) in [5], and Eqs. (33) in [6] for a multi-component anisotropic medium consisting of any number of semi-transparent and opaque components. The definitions of continuum-scale radiative properties, Eqs. (5a)–(5f) provide the mathematical basis for development of numerical techniques for determination of continuum-scale radiative properties utilizing the exact geometry of multi-component media. They require the knowledge of the complete actual and blackbody discrete-scale radiative intensity fields in each component obtained for a selected model problem.

3. Continuum-scale boundary conditions

Equations (4) are subject to boundary conditions at the wall-medium interface at $\hat{\mathbf{s}} \cdot \hat{\mathbf{n}}_\mathrm{w} > 0$, where $\hat{\mathbf{n}}_\mathrm{w}$ is a unit normal vector pointing from the wall into the medium. The wall is assumed to consist of only a single component that can be either semi-transparent or opaque. The discrete-scale boundary conditions at the boundary of the multi-component medium are formulated analogously to the boundary conditions (2a) and (2b). They read for the semi-transparent and opaque walls, respectively:

$$L_i\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}\right) = \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\,\cdot\,\hat{\mathbf{n}}_{\mathrm{w}i}>0}} \tau''_{\mathrm{w}i}\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) L_\mathrm{w}\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{\mathrm{w}i}\,\mathrm{d}\Omega_\mathrm{i} - \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\,\cdot\,\hat{\mathbf{n}}_{\mathrm{w}i}<0}} \rho''_{i\mathrm{w}}\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) L_i\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{\mathrm{w}i}\,\mathrm{d}\Omega_\mathrm{i}, \tag{6a}$$

$$L_i\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}\right) = \varepsilon'_{\mathrm{w}i}\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}\right) L_{\mathrm{b,w}}\left(\vec{\mathbf{r}}_{i\mathrm{w}}\right) - \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\,\cdot\,\hat{\mathbf{n}}_{\mathrm{w}i}<0}} \rho''_{i\mathrm{w}}\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) L_i\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{\mathrm{w}i}\,\mathrm{d}\Omega_\mathrm{i}, \tag{6b}$$

The variation of the discrete-scale radiative properties and the curvature of the wall–medium interface are assumed to be negligible over the interface area associated with the averaging volume V adjacent to said boundary, $A_{i\mathrm{w},\hat{\mathbf{s}}\,\cdot\,\hat{\mathbf{n}}_{\mathrm{w}i}>0}$. The continuum-scale boundary conditions are obtained by surface averaging of the boundary intensity $L_i\left(\vec{\mathbf{r}}_{i\mathrm{w}}, \hat{\mathbf{s}}\right)$ over the portion of the wall-medium interface inside the averaging volume V adjacent to the wall, A_w,

$$I_i\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}\right) = \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\,\cdot\,\hat{\mathbf{n}}_{\mathrm{w}i}>0}} \tau''_{\mathrm{w}i}\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) I_{\mathrm{w}i}\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{\mathrm{w}i}\,\mathrm{d}\Omega_\mathrm{i} - \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\,\cdot\,\hat{\mathbf{n}}_{\mathrm{w}i}<0}} \rho''_{i\mathrm{w}}\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) I_i\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{\mathrm{w}i}\,\mathrm{d}\Omega_\mathrm{i}, \tag{7a}$$

$$I_i\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}\right) = \varepsilon'_{\mathrm{w}i}\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}\right) I_{\mathrm{b,w}i}\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}\right) - \int\limits_{\Omega_{\hat{\mathbf{s}}_\mathrm{i}\,\cdot\,\hat{\mathbf{n}}_{\mathrm{w}i}<0}} \rho''_{i\mathrm{w}}\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}_\mathrm{i}, \hat{\mathbf{s}}\right) I_i\left(\vec{\mathbf{x}}_\mathrm{w}, \hat{\mathbf{s}}_\mathrm{i}\right) \hat{\mathbf{s}}_\mathrm{i} \cdot \hat{\mathbf{n}}_{\mathrm{w}i}\,\mathrm{d}\Omega_\mathrm{i}. \tag{7b}$$

4. Summary and conclusions

A mathematical model for radiative transfer and characterization of multi-component anisotropic participating media with the individual components in the limit of geometrical optics has been formulated. The model includes the set of governing continuum-scale radiative transfer equations, and the associated boundary conditions and radiative property definitions. The formulation presented is a generalization of the previous formulation for multi-component isotropic media with components in the limit of geometrical optics. The model presented finds application in radiative transfer analysis and characterization of a broad range of multi-component anisotropic media encountered in engineered and natural systems.

References

[1] Modest M 2013 *Radiative Heat Transfer* 3rd ed (San Diego: Academic Press)
[2] Baillis D and Sacadura J F 2000 *Journal of Quantitative Spectroscopy and Radiative Transfer* **67** 327–363
[3] Tancrez M and Taine J 2004 *International Journal of Heat and Mass Transfer* **47** 373–383
[4] Gusarov A V 2008 *Physical Review B* **77** 144201
[5] Lipiński W, Petrasch J and Haussener S 2010 *Journal of Quantitative Spectroscopy and Radiative Transfer* **111** 253–258
[6] Lipiński W, Keene D, Haussener S and Petrasch J 2010 *Journal of Quantitative Spectroscopy and Radiative Transfer* **111** 2474–2480
[7] Haussener S and Steinfeld A 2012 *Materials* **5** 192–209
[8] Gusarov A 2013 *Journal of Quantitative Spectroscopy and Radiative Transfer* **116** 156–168
[9] Coquard R and Baillis D 2006 *Journal of Heat Transfer* **128** 1022–1030
[10] Bellet F, Chalopin E, Fichot F, Iacona E and Taine J 2009 *International Journal of Heat and Mass Transfer* **52** 1544–1551
[11] Whitaker S 1999 *The Method of Volume Averaging* (*Theory and Applications of Transport in Porous Media* vol 13) (Dordrecht: Kluwer Academic Publishers)