

Determination of space-dependent radiative properties in diffuse optical tomography using a wavelet multi-scale method

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Abstract. This paper deals with the estimation of radiative property distributions of participating media from a set of light sources and sensors located on the boundaries of a medium. This is the so-called diffuse optical tomography problem. Such a non-linear ill-posed inverse problem is solved through the minimization of a cost function which depends on the discrepancy, in a least-square sense, between some measurements and associated predictions. In the present case, predictions are based on the diffuse approximation model in the frequency domain while the optimization problem is solved by the L-BFGS algorithm. To cope with the local convergence property of the optimizer and the presence of numerous local minima in the cost function, a wavelet multi-scale method associated with the L-BFGS method is designed.

1. Introduction

The diffuse optical tomography (DOT) problem is an inverse problem in which the spatial distributions of absorption and scattering coefficients of a participating medium are sought from a set of light sources and sensors located on the frontier ($\partial\Omega$) of the probed medium (Ω). Such a non-linear ill-posed inverse problem is solved through the minimization of a cost function which depends on the discrepancy, in a least-square sense, between some measurements and associated predictions. Among employed methods to solve this problem, the Limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) method has proven its worth [1, 2]. However, some problems remain to be overcome such as the stability with respect to the initial guesses and the blurring effect of the reconstructed images due to the need for relatively strong regularization tools. In recent years, the wavelet multi-scale method (WMM) has emerged in the field of inversion [3]. This method relies on a reformulation of the original inverse problem into a sequence of sub-inverse problems of different scales using wavelet transform, from the largest scale to the smallest one. It has been shown that the WMM can enhance stability of inversion, accelerate convergence and cope with the presence of local minima to reach the global minimum. The rationale behind this success is that the cost function shows stronger convexity and has less minima at larger scale such that the global minimum can be achieved. Best results can thus be obtained, using this solution to the initialization of the optimization problem at the lower scale until finding the minimum at the original scale. This paper is dedicated to the application of a WMM associated with the L-BFGS method to the specific inverse problem of the DOT and aims to improve the efficiency of the ordinary L-BFGS algorithm.



2. Inverse model

The forward model used in this study is the bidimensional diffuse approximation (DA) model in the frequency domain [4]. This equation leads to compute the photon density $\varphi : \Omega \mapsto \mathbb{C}$ that depends on space-dependent absorption (κ) and reduced scattering (σ') coefficients, and which values at the nodes of the sensors will be called predictions. Let $\partial\Omega_s = \partial\Omega_s^1 \cup \dots \cup \partial\Omega_s^L$ be a set of L surfacic sensors on $\partial\Omega$ and let us consider K collimated sources at different locations along $\partial\Omega$. For each source, let $\check{\varphi}_i : \Omega \mapsto \mathbb{C}$ be equal to measurements of the transmitted photon density. Predictions φ_i and measurements $\check{\varphi}_i$ are integrated to the cost function to be minimized:

$$j(\kappa, \sigma') = \frac{1}{2} \sum_{i=1}^K \int_{\partial\Omega_s} \left| \frac{\varphi_i - \check{\varphi}_i}{\check{\varphi}_i} \right|^2 d\zeta \quad (1)$$

Because of the different orders of magnitude between the two radiative properties κ and σ' , a strategy should be developed to facilitate the simultaneous reconstruction of these properties if a gradient-based method is used to perform this minimization. Indeed, the cost function gradient parts associated with both radiative properties also differ by roughly the same order of magnitude, which is very bad for the convergence. In order to speed-up the iterative convergence to the local minimum, the proposed strategy consists in performing a scaling on the parameters at the beginning of the optimization problem. Choosing an a priori function for both radiative properties, say κ_{ap} and σ'_{ap} , one searches parameters that fluctuate about unity. This adimensionalization leads to recover both $\varkappa(\mathbf{x}) = \kappa(\mathbf{x})/\kappa_{ap}(\mathbf{x})$ and $\varsigma(\mathbf{x}) = \sigma'(\mathbf{x})/\sigma'_{ap}(\mathbf{x})$ for which magnitude is of order one approximately for both coefficients. The inverse problem is then formulated as a constrained optimization problem of the form:

$$\text{Find } (\bar{\varkappa}, \bar{\varsigma}) = \arg \min_{\varkappa, \varsigma} j(\varkappa, \varsigma) \text{ subject to } \varphi_i \text{ solution of the DA model, } \forall i \quad (2)$$

The main tool used for the development of the WMM is the projection onto wavelet bases which relies on Mallat's forward and inverse fast wavelet transforms (FWT) [6]. Let us consider a discrete signal a_N at scale N , i.e. whose length is 2^{-N} , $N \leq 0$. The forward FWT computes the vectors of scale and detail coefficients at scale $N+1$ of the signal a_N , a_{N+1} and b_{N+1} of length 2^{-N-1} , with the help of discrete convolutions between a_N and conjugate mirror filters h and g , respectively. The inverse FWT allows to reconstruct the original signal a_N from discrete convolutions between a_{N+1} and h , and between b_{N+1} and g . The projection of the signal a_N , for example from scale N to scale $N+2$, consists in the following operations: (i) compute a_{N+2} using the forward FWT with a_N then a_{N+1} ; (ii) reconstruct a_N using the inverse FWT with a_{N+2} then a_{N+1} , assuming the vectors of detail coefficients b_{N+2} and b_{N+1} are equal to 0. Note that only the filter h , which fully determines the wavelet basis and may be interpreted as a low pass discrete filter which smooths the data, is used when projecting the signal.

The L-BFGS method [5] associated to a quadratic line-search method is used to solve the optimization problem (2). For CPU time considerations, cost function gradients are computed by the very fast adjoint-state method from the state (φ_i) and adjoint (φ_i^*) variables. For the sake of simplicity, the WMM is presented with the domain Ω discretized on a grid of $2^{-N} \times 2^{-N}$ points, $N \leq 0$, with each sensor constituted of 2^{-M} pointwise measurements on the grid points, $N \leq M \leq 0$, and with all functions involved in the inverse method discretized by P1-finite elements. Thus, the vectors (images) of scale coefficients at the smallest scale associated to the predictions and measurements on sensors (state and adjoint variables) can be simply built from the values of these functions on the grid points. The WMM consists in applying the L-BFGS method until convergence, from the largest scale ($N-M$) to the smallest one (N), with, at a given scale, initializing the radiative properties to the solution of the optimizer at the upper scale. Ultimately, this method is essentially based on careful computations of the cost function and cost function gradient at different scales using the projection operator.

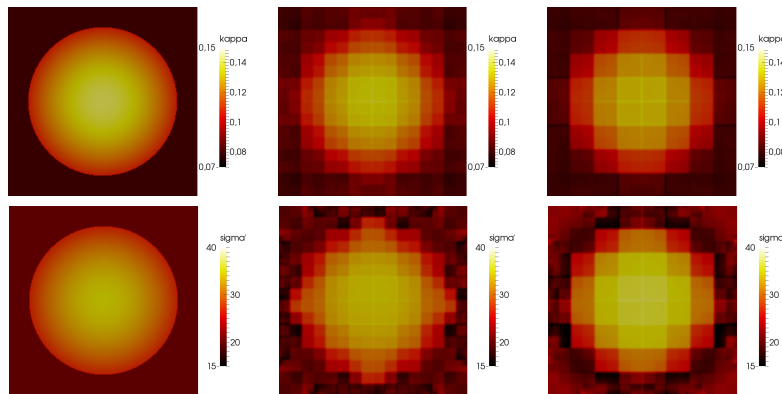


Figure 1. Reconstructions with the Haar wavelet. Top, results for κ ; bottom, results for σ' . From left to right: target properties; $\kappa_{ap} = 0.08$, $\sigma'_{ap} = 20$; $\kappa_{ap} = 0.08$, $\sigma'_{ap} = 22.5$.

3. Numerical results

A selected numerical result is presented in this section to assess the validity of the proposed method. The domain, Ω , is a 4 cm square. The regular grid associated to Ω is composed of 128×128 points ($N = -7$). On each side of the square, one source and two sensors are located. Each sensor contains 16 pointwise photon density measurements ($M = -4$). Synthetic data are considered for this numerical test. These data, $\check{\varphi}$, representing the pseudo-experimental measurements, are built on a finer mesh (255×255 points) than that of generating the predictions φ in order to avoid the inverse crime. Then, a multiplicative white gaussian noise is applied to $\check{\varphi}$ at the nodes of the sensors to simulate the noise inherent to experimental devices, i.e. $\check{\varphi}_{\text{noisy}} = \check{\varphi} (1 + 0.01 \times m)$ where $m \sim \mathcal{N}(0, 1)$. The FWT is applied with the Haar wavelet ($h = [2^{-1/2}, 2^{-1/2}]$) to project the synthetic data at scales $-3, \dots, 0$ and thus, to obtain a series of inverse problems for scales $-7, \dots, -3$. Target properties are given in Figures 1a and 1d. Simultaneous reconstructions of κ and σ' with a priori functions $(\kappa_{ap}, \sigma'_{ap}) = (0.08, 20)$ and $(\kappa_{ap}, \sigma'_{ap}) = (0.08, 22.5)$ were investigated with the L-BFGS method and the WMM using the Haar wavelet. It was observed that, for these specific initial guesses, the L-BFGS method failed to converge unlike the WMM which provided quite accurate results (see Fig. 1). Moreover, for other initial guesses where the two methods converged, it is found that the WMM is faster and provides more accurate reconstructions than the L-BFGS method. For example, with $(\kappa_{ap}, \sigma'_{ap}) = (0.09, 22.5)$, the L-BFGS method converged in 1 049 s while the WMM converged in 235 s (Intel i3 2.4 GHz, RAM 6 GB). Although the per-iteration CPU times are slightly longer for the WMM, fewer iterations are required to reach the convergence. Finally, to avoid the “square-shaped” aspect of the obtained reconstructions due to the use of the Haar wavelet, other wavelets were considered. However, because an edge problem appears when considering the FWT with another wavelet than the Haar wavelet, better results are not obtained. An extension of the proposed WMM with the wavelet extrapolation method [7], which does not produce undesirable edge effects, is in progress to obtain an efficient WMM using other wavelets.

References

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