

Estimation of reliability of linear point structures revealed in two-dimensional distributions of experimental data

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Abstract. In the experiments at the FOBOS spectrometer [1] dedicated to study the spontaneous fission of the ^{248}Cm and ^{252}Cf nuclei in the mass correlation distribution of fission fragments new unusual structures bounded by magic clusters were observed for the first time. The structures were interpreted as a manifestation of a new exotic decay called collinear cluster tri-partition (CCT). These pioneer results were confirmed and detailed later in the series of experiments at different time-of-flight spectrometers [2]. Interpretation of the results obtained needs estimation of the statistical reliability of the structures mentioned above. The report presents the results of the solution to the problem of statistical reliability estimation on the basis of morphological image analysis [3].

1. Introduction

In this work based on the results of RFBR grants 14-02-93960, 14-07-00409-a we continue to study a previously unknown type of nuclear transformations found by project authors – collinear cluster tri-partition (CCT). In series of experiments at different time-of-flight spectrometers we have observed multiple manifestations of a new type of multibody decay of low excited heavy nuclei called by us collinear cluster tri-partition (CCT) [1], [2]. The results were obtained predominantly in the frame of the “missing mass” approach. It means that only two from at least three decay partners were actually detected whereas a total mass of these fragments being less the mass of mother system serves a signature of a multibody decay. Thus analyzing of the mass-mass distribution of the fission fragments detected in coincidence let reveal us the decay mode absolutely unknown in the past. Unfortunately there is an essential random background linked with the scattered fragments from the conventional binary fission in the mass region of possible manifestation of the CCT. We have find that due to the physics behind (clustering) the CCT events form some regular predominantly linear or/and almost rectangular structures in the mass-mass plots (see the example of “rectangular” structures at figure 1). The main goal of this work is to estimate the statistical reliability of these structures in the mass correlation distribution of fission fragments or, in other words, to estimate probability of “random” realization of the



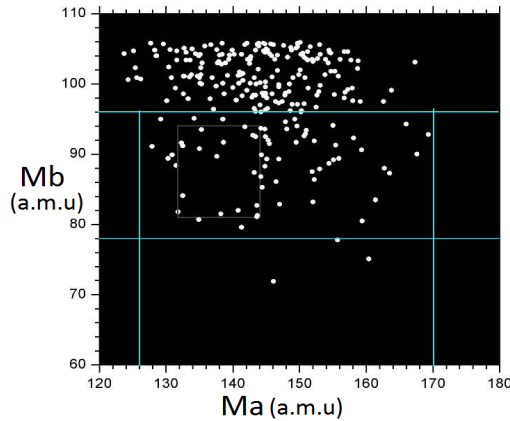


Figure 1. Example of a “rectangular” structure for estimating of its reliability. The chosen “field of view” is shown by a gray line: Ma varies from 125 a.m.u. to 169 a.m.u, Mb varies from 79 a.m.u. to 96 a.m.u.

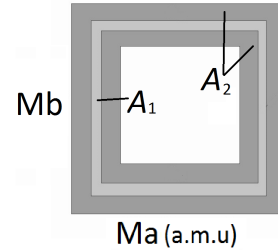


Figure 2. Example of domains $A_1, A_2 \subset \Omega$ of constant brightness in a model image of a “rectangular” structure.

corresponding structures. Previously we have developed an approach [4] based on morphological image analysis [3] which will be used to solve this task.

2. Method of morphological image analysis

Let us briefly consider some notions of the method of morphological image analysis [4]. Let \tilde{f} be an experimentally obtained signal (mass-mass distribution), which can be represented in the form

$$\tilde{f} = f + \nu, \quad (1)$$

where f is a signal possibly containing several structures of our interest, and ν is additive noise. An image of a signal $f(\cdot)$ is understood as a numerical square-integrable function defined on a subset X of the Euclidean plane \mathcal{R}^2 . The domain X is termed the field of view, and the value $f(x)$ of the function $f(\cdot)$ at the point $x \in X$ is termed the brightness of the point x of the field of view X . In the case under consideration, $X = \{x_1, \dots, x_n\}$ and, correspondingly, the images $\tilde{f}(\cdot)$, $f(\cdot)$, and $\nu(\cdot)$ from (1) are defined at the same points and are elements of the Euclidean plane \mathcal{R}^n . As to the error $\nu \in \mathcal{R}^n$, we will assume that it is a random image with zero mathematical expectation $\mathbf{E}\nu = 0$ and the correlation operator $\sigma^2 I$, where $I \in \{\mathcal{R}^n \rightarrow \mathcal{R}^n\}$ is the unit operator and σ^2 is unknown.

Let us denote the “rectangular” structure image by $\omega(\cdot)$ and assume it to be defined on a variable-size mobile subset Ω of the field of vision X . The form of the image $\omega(\cdot)$ will be understood as the set of images

$$V_\omega = \{\omega(\cdot), \omega(x) = c_1 \chi_{A_1}(x) + c_2 \chi_{A_2}(x), c_1 \geq c_2, c_1, c_2 \in \mathcal{R}^1, x \in \Omega\}, \quad (2)$$

where $\chi_{A_i}(x) = \begin{cases} 1, & x \in A_i, \\ 0, & x \notin A_i, \end{cases} \quad x = 1, 2.$ V_ω is a convex closed cone in \mathcal{R}^2 and in \mathcal{R}^n . In

this definition A_1 and A_2 are different subsets of constant brightness in Ω . According to this definition, the form of the image of an object contains all images of this object differing in the brightnesses on subdomains of constant brightness in Ω .

Figure 2 shows domains A_1 and $A_2 \subset \Omega$ with a constant brightness in the image of a “rectangular” structure. In this figure, the field of view is divided into subdomains A_1 and A_2 .

The “rectangular” structure directly corresponds to the domain A_1 , and the points surrounding it correspond to the domain A_2 . The form (in the common sense of this word) and size of the domains A_1 and A_2 are specified (postulated) a priori by the researcher. The proposed method enables one to verify the correctness of this postulate. In the given case, the form of the linear structure was determined on the basis of figure 1. The brightnesses over the domains A_1 and A_2 are assumed to be constant. The fact that the brightness at the image point belonging to the “rectangular” structures must be higher than that at the points surrounding it is reflected in the condition $c_1 \geq c_2$ in expression (2).

The projection (see below) of some image $g(\cdot)$ defined on Ω onto a form V_ω is understood as the image $(P_{V_\omega}g)(\cdot)$, which uniquely exists, because V_ω is a convex closed cone (see [3]): $(P_{V_\omega}g)(x) = \hat{c}_1\chi_{A_1}(x) + \hat{c}_2\chi_{A_2}(x)$, $x \in \Omega$, where \hat{c}_1 and \hat{c}_2 are solutions to the following minimization problem:

$$\int_{\Omega} (g(x) - \hat{c}_1\chi_{A_1}(x) - \hat{c}_2\chi_{A_2}(x))^2 dx = \min_{c_1, c_2 \in \mathcal{R}^1, c_1 \geq c_2} \int_{\Omega} (g(x) - c_1\chi_{A_1}(x) - c_2\chi_{A_2}(x))^2 dx.$$

Below, we will omit, for brevity, the sign (\cdot) in the notation of images.

Let us consider the problem of separating the “rectangular” structure in the framework of the above-formulated model of signal detection as a problem of testing the statistical hypothesis H that the image f contains a fragment f_ω that can be represented in the form

$$H : \exists f_\omega = g + \nu, \exists t \in T, g \in t(V_\omega), \nu \in \mathcal{N}(0, \sigma^2 I), \|\nu\|^2 \ll \|g\|^2, \quad (3)$$

where the form of g coincides with (2) within a shift and scale transformation, $t \in T$ is a shift and scale transformation, T is the class of such transformations, $\|z\|^2 = \int_{\Omega} z(x) dx$, and z is some image. The alternative K is that such fragments are absent. For solving the problem of testing this hypothesis, the following functional [3] is used:

$$j(z) = \frac{\|(I - P_{V_\omega})z\|^2}{\|(P_{V_\omega} - P_{V_U})z\|^2}. \quad (4)$$

In (4) z is some image and $P_{V_U}z$ is a projection of image z onto the form U of the uniform field of view: $U = \{u(\cdot), u(x) = \text{const}\chi_\omega(x), x \in \Omega\}$. Functional (4) possesses the following properties [4]. If there is a fragment f_ω satisfying condition (3) but not representable in the form of the “noised uniform field of view”

$$f_\omega = g + \nu, \exists t \in T, g = t(U), \quad (5)$$

then the value of functional (4) is small. If there is a fragment f_ω satisfying condition (5), then the functional $j(f_\omega)$ has the order $O(1)$. If, at last, fragments f_ω satisfying condition (4) or (5) are absent, then the functional $j(f_\omega)$ again has the order $O(1)$.

The decision rule has the following form: a hypothesis H is accepted if, by a shift and scale transformation, a fragment f_ω can be found such that $j(f_\omega) \leq A$, where A is an empirically determined constant, and it is rejected if such a fragment is absent. The value of functional (4) characterizes the “distance” between an image z and an image of the form (2). Functional (4) is invariant with respect to brightness and contrast transformations. In order to determine the constant A , the value of the constant was found (based on real data) at which the image of a determined “rectangular” structure satisfied the researcher. In our case, that was $A = 1.35$. Then, the reliability of the obtained value was tested by means of a model experiment. We used 100000 model images of an additive noise with the parameters corresponding to the real experiment. Then an empirical distribution of the values of functional (4) at the specified

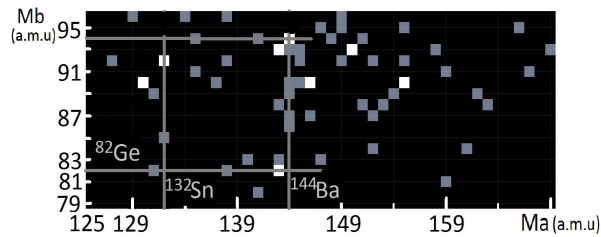


Figure 3. Image for “rectangular” searching taken from figure 1. Gray lines were added manually to show “magic” numbers.

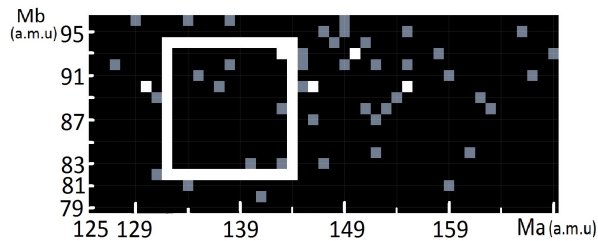


Figure 4. The “rectangular” structure found by morphological method.

noise level was constructed. On the basis of this distribution, the probability $P(j \leq A) \approx 0.02$ was found. This probability is the probability of false acceptance of a hypothesis against the nearest “homogeneous field of view” alternative. Properties of the functional (4) imply that this probability estimates an upper bound for the probability of false acceptance of the hypothesis against the alternative that such a fragment is absent. This criterion is analogous to the principle of the locally homogeneous strongest criterion [5]. According to the statistical analysis made above, the probability, that the “rectangular” structure found on real data (see figures 3, 4), is generated by the noise is small and approximately equals to 2%.

3. Conclusion

In order to obtain the quantitative estimates of the “rectangular” structures statistical reliability an approach based on morphological image analysis has been developed. In the framework of this approach, the probability of a random realization (due to the presence of noise) of a structure or its scaled copy is estimated. Only the presence of such an estimate justifies the physical interpretation of the revealed structures, removing the question of whether these structures are statistical artifacts or not.

Acknowledgments

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