

Visible, invisible and trapped ghosts as sources of wormholes and black universes

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Abstract. We construct explicit examples of globally regular static, spherically symmetric solutions in general relativity with scalar and electromagnetic fields, describing traversable wormholes with flat and AdS asymptotics and regular black holes, in particular, black universes. (A black universe is a regular black hole with an expanding, asymptotically isotropic space-time beyond the horizon.) Such objects exist in the presence of scalar fields with negative kinetic energy (“phantoms”, or “ghosts”), which are not observed under usual physical conditions. To account for that, we consider what we call “trapped ghosts” (scalars whose kinetic energy is only negative in a strong-field region of space-time) and “invisible ghosts”, i.e., phantom scalar fields sufficiently rapidly decaying in the weak-field region. The resulting configurations contain different numbers of Killing horizons, from zero to four.

The so-called exotic matter, which violates the weak and null energy conditions (WEC and NEC), is well known to be a necessary ingredient for the construction of wormholes in general relativity and some of its extensions (see, e.g., [1–4] for reviews), although such matter is not observed under usual physical conditions, and its possible existence meets serious theoretical objections mostly related to quantum phenomena (see [5] and references therein). On the other hand, phantom fields naturally appear in some models of string theory [6], supergravities [7] and theories in more than 11 dimensions [8]. One more viewpoint of interest is [9] that a phantom field, being a source of gravity, can exist without its own dynamics and therefore escape the related problems with quantum particle creation.

An important, though not very confident, support from modern cosmological observations allowing for the ratio of pressure to energy density $w = p/\rho < -1$ (see, e.g., [10] and references therein) is one of the reasons for the recent interest in possible phenomena with exotic matter including wormhole construction and properties.

It has also been found that in the presence of exotic matter, say, in the form of a phantom scalar field, not only wormholes are possible but also different types of regular black holes, including the so-called black universes. The latter look from their static regions as “ordinary” black holes in general relativity, but instead of a singularity beyond the horizon, there is an expanding universe which can at large times become isotropic, in particular, de Sitter [11, 12].

In the present note, we briefly describe different extensions of the scalar field solutions of [11]



to configurations containing electric or magnetic fields in the framework of general relativity. One of the motivations for their inclusion is that by modern observations there can exist a global magnetic field up to 10^{-15} Gauss, causing correlated orientations of sources remote from each other [13], and some authors admit a possible primordial nature of such a magnetic field.

The most straightforward extension [14] assumes the action

$$S = \frac{1}{2} \int \sqrt{-g} d^4x \left[R + \varepsilon (\partial\phi)^2 - 2V(\phi) - F_{\mu\nu} F^{\mu\nu} \right], \quad (1)$$

where R is the scalar curvature, $g = \det(g_{\mu\nu})$, and $F_{\mu\nu}$ is the electromagnetic field tensor, and $\varepsilon = \pm 1$ distinguishes usual, canonical ($\varepsilon = +1$) and phantom ($\varepsilon = -1$) scalar fields. We consider static, spherically symmetric space-times with the general metric

$$ds^2 = A(u) dt^2 - \frac{du^2}{A(u)} - r^2(u) (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (2)$$

written in the so-called quasiglobal gauge $g_{00}g_{11} = -1$ convenient for describing Killing horizons which occur at regular zeros of the function $A(u)$ [2]. Assume that the metric is asymptotically flat as $u \rightarrow \infty$, then a *wormhole* is a geometry in which $u \in \mathbb{R}$, $A > 0$ at all u , and the metric is asymptotically flat or AdS as $u \rightarrow -\infty$. If again $u \in \mathbb{R}$ but at large negative u the function $A(u) \sim -r^2(u) \rightarrow -\infty$ (a de Sitter asymptotic), we are dealing with a *black universe*.

In both wormhole and black universe metrics, the area function $r(u)$ has a regular minimum at some $u = u_0$. The WEC and NEC are necessarily violated at such minima due to the Einstein equations: indeed, one of them reads (see equation (4) below) $2Ar''/r = -(T_t^t - T_u^u)$, where T_μ^ν is the stress-energy tensor (SET) and the prime stands for d/du . In terms of the density ρ and radial pressure p_r , the condition $r'' > 0$ near $u = u_0$ implies $\rho + p_r < 0$, which manifests NEC violation. This is true both for an R-region ($A > 0$) where $u = u_0$ is a throat and for a T-region ($A < 0$) where $u = u_0$ is a bouncing point in a Kantowski-Sachs cosmology [2, 15].

The electromagnetic fields can be radial electric (Coulomb) and magnetic (monopole) ones. There is, however, no need to introduce specific electric or magnetic charges (or monopoles): in both wormholes and black universes a radial field can exist without sources due the space-time geometry. In the wormhole case it perfectly conforms to Wheeler's idea of a "charge without charge", in a black universe the situation is basically the same but looks more involved.

The electromagnetic field equations are solved in a general form, leading to the SET $T_\mu^\nu[e] = q^2 r^{-4}(u) \text{diag}(1, 1, -1, -1)$, where $q^2 = q_e^2 + q_m^2$ and the constants q_e and q_m are the effective electric and magnetic charges. The remaining independent Einstein equations read

$$(A'r^2)' = -2r^2V + 2q^2/r^2, \quad (3)$$

$$r''/r = -\varepsilon\phi'^2, \quad (4)$$

$$A(r^2)'' - r^2 A'' = 2 - 4q^2/r^2. \quad (5)$$

Being interested in wormhole and black universe solutions, we take $\varepsilon = -1$ and, using the inverse problem method, assume for the area function $r = \sqrt{u^2 + b^2} \equiv b\sqrt{x^2 + 1}$, $b = \text{const} > 0$ (the length scale). Then equation (5) is integrated giving for $B(x) \equiv A(x)/r^2(x)$

$$B(x) = B_0 + \frac{1 + q^2 + px}{1 + x^2} + \left(p + \frac{2q^2x}{1 + x^2} \right) \arctan x + q^2 \arctan^2 x, \quad (6)$$

where p and B_0 are integration constants. The metric is thus completely known while ϕ and V are now easily found from the field equations. Assuming that our system is asymptotically flat at large x , so that $A \rightarrow 1$, $r \rightarrow \infty$ and $B \rightarrow 0$ as $x \rightarrow \infty$, the constants are related by

$$B_0 = -\pi p/2 - \pi^2 q^2/4, \quad p = 3m - \pi q^2. \quad (7)$$

The whole solution is parametrized by two constants m (Schwarzschild mass at $x \rightarrow \infty$) and q , the charge. It turns out that even if we restrict ourselves to solutions asymptotically flat as $x \rightarrow \infty$ and $m > 0$, there are as many as 10 classes of globally regular space-times which differ in the type of the second asymptotic $x \rightarrow -\infty$ (M — Minkowski, dS — de Sitter, AdS — anti-de Sitter) and the number and kinds of horizons, see table 1 (where $n = 1$ refers to simple horizons, $n = 2$ to extremal ones, R- and T-regions are disposed from left to right along the x axis). Two examples of the corresponding Carter-Penrose diagrams are shown in figure 1: one of them, with two horizons, cannot be drawn on a single plane due to region overlapping while the other, with three horizons, occupies the whole plane except the grey triangles. Some of non-asymptotically flat solutions contain four horizons. We refer to [14] for a detailed description of the global causal structures and Carter-Penrose diagrams for different branches of the general solution.

Table 1. Types of asymptotically flat solutions with $m > 0$.

Configuration type, asymptotics ($x \rightarrow +\infty$) — ($x \rightarrow -\infty$)	Horizons: number, order n	disposition of R- and T-regions
M – M wormhole	none	R
M – M extremal black hole	1 hor., $n = 2$	RR
M – M black hole	2 hor., $n = 1$ (both)	RTR
M – dS black universe	1 hor., $n = 1$	TR
M – dS black universe	2 hor., $n = 2$ and $n = 1$	TTR
M – dS black universe	2 hor., $n = 1$ and $n = 2$	TRR
M – dS black universe	3 hor., $n = 1$ (each)	TRTR
M – AdS black hole	2 hor., $n = 1$ (both)	RTR
M – AdS extremal black hole	1 hor., $n = 2$	RR
M – AdS wormhole	none	R

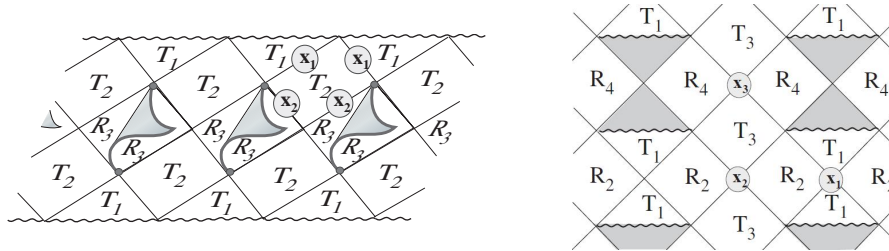


Figure 1. Carter-Penrose diagrams of two space-times presented in table 1, 5th line (left panel) and 7th line (right panel). The letters R and T mark R- and T-regions, x_n mark the horizons, and both are ordered left to right along the x axis.

Since exotic matter or phantom fields are so far not detected, it makes sense to try to avoid their emergence in asymptotic weak-field regions. To do so, we considered a special kind of fields, named “trapped ghosts” [16–18], which have phantom properties only in some restricted strong-field region and become canonical outside it. This happens if we substitute in (1), instead of ε , a function $h(\phi)$ changing its sign at some ϕ value. Choosing, instead of $r = \sqrt{u^2 + b^2}$, the function [17, 18]

$$r(u) = a \frac{x^2 + 1}{\sqrt{x^2 + n}}, \quad a = \text{const} > 0, \quad n = \text{const} > 2. \quad (8)$$

we obtain examples of such a behaviour. Indeed, with (8), $r'' > 0$ (corresponding to a ghost field) at $x^2 < n(2n-1)/(n-2)$ and $r'' < 0$ at larger $|x|$, as required. As before, all other quantities are found from the field equations, and the qualitative features of solutions are the same as described above, including the ten classes of regular solutions presented in table 1.

A weak point of such models is the transition surface from normal to phantom fields. It turns out, for example, that if we consider spherically symmetric perturbations of solutions with such a field, the corresponding effective potential has a singularity which should in general lead to a violent instability [15].

Another approach is to consider what may be called an “invisible ghost”, i.e., a phantom field that decays sufficiently rapidly in the weak-field region. In [15] this idea was realized with the same ansatz (8) on $r(u)$ but with a sigma-model-like combination of two scalar fields, a phantom one, $\psi(u)$, and a canonical one, $\phi(u)$, where ψ decays at large u much more rapidly than ϕ . Since the function $A(u)$ is, as before, found from equation (5) with known $r(u)$, the whole geometry remains the same, changes only the field content of the system.

A configuration with an “invisible ghost” can also be obtained with a single field $\phi(u)$, but then one should choose a function $r(u)$ like $r(u) = a(1+x^8)^{1/8}$, closer than before approaching $r = u$ at large u and preserving the condition $r'' > 0$ to ensure the phantom nature of ϕ . The resulting geometries should be qualitatively the same as those discussed in [14, 18]. Indeed, since $r(x) \approx |u|$ at both infinities, the causal structure and the corresponding Carter-Penrose diagrams are completely determined by zeros of $B(x)$ and its asymptotic behaviour.

Summarizing, we have obtained numerous examples of wormhole, regular black hole and black universe solutions to the Einstein equations with scalar and electromagnetic fields as sources, where the scalar fields are phantom in nature at least in the strong field region.

Acknowledgments

The work of KB was performed within the framework of the Center FRPP supported by MEPhI Academic Excellence Project (contract No. 02.a03.21.0005, 27.08.2013).

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