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CPT breaking and electric charge non-conservation

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Abstract. We demonstrate that CPT-violation due to the e^+e^- mass difference breaks an electric current conservation and generates a photon mass. Cosmological bounds on the photon mass lead to the bound for e^+e^- mass difference that is 10-15 orders of magnitude stronger than any direct experimental bounds.

My talk is based on a paper written in collaboration with A D Dolgov[1].

There is a good old tradition to parametrize CPT violation by attributing different masses to particle and antiparticle (see PDG [2]). This tradition is traced to the theory of $K-\bar{K}$-mesons oscillation, that is equivalent to a non-hermitian Quantum Mechanics (QM) with two degrees of freedom. Diagonal elements of 2×2 Hamiltonian matrix represent masses for particle and antiparticle. Their unequality breaks CPT-symmetry. Such strategy has no explicit loop-holes and is still used for parametrization of CPT-symmetry violation in D and B meson oscillations.

Quantum Field Theory deals with infinite number d.o.f. The very straightforward generalization of CPT-conserving QFT to CPT-violating QFT was made by Barenboim *et al.* (2001) [3]. They represented complex scalar field as an infinite sum over modes and attributed different masses for particle and antiparticle

$$\phi(x)=\sum_{\mathbf{q}}\left\{a(\mathbf{q})\frac{1}{\sqrt{2E}}e^{-i(Et-\mathbf{qx})}+b^+(\mathbf{q})\frac{1}{\sqrt{2\tilde{E}}}e^{i(\tilde{E}t-\mathbf{qx})}\right\}\quad, \tag{1}$$

where(m,E) and $(\tilde{m},\tilde{E})$ are masses and energies for particle and antiparticle with momentum $\mathbf{q}$ respectively. Here $a(\mathbf{q})$ and $b(\mathbf{q})$ and $a^+(\mathbf{q})$ and $b^+(\mathbf{q})$ are the creation and annihilaton operators for particles and antiparticles respectively. They obey the standard Bose commutator relations. In this formalism one can calculate v.e.v. of field operators, i.e. the Wightman functions:

$$<\phi(x),\phi(y)^+>=D^+(x-y;m)\,, \tag{2}$$

$$<\phi(x)^+,\phi(y)>=D^-(x-y;\tilde{m})\,. \tag{3}$$

They are given by the standard Lorentz-invariant Pauli-Jordan functions but with different masses.

Greenberg (2002) [4] noticed that such theory is nonlocal and acausal. The commutator of two fields is equal to the difference $D^+(x-y;m)-D^-(x-y;\tilde{m})$ and does not vanish for space-like separation! In this sense the theory is not a Lorentz-invariant one. Moreover the Feynman propagator is explicitly non-invariant:

$$D_F(q)=\frac{1}{(2E(\mathbf{q}))}\frac{1}{(q_0-E(\mathbf{q}))}-\frac{1}{(2\tilde{E}(\mathbf{q}))}\frac{1}{(q_0+\tilde{E}(\mathbf{q}))} \tag{4}$$

and can be rewritten in invariant form only if $m = \tilde{m}$.These arguments demonstrate that any local fields theory that violates CPT symmetry necessarily violates Lorentz invariance.

We have shown recently that the theories with different masses for particle and antiparticle break also a conservation law for a local electric current [6]. In the theory where photon interacts with non-conserving electric current nothing protects photon from being massive and at the first loop one expects non-zero m_γ:

$$m_\gamma^2 = C\,\frac{\alpha}{\pi}\,\Delta m^2\,. \tag{5}$$

The coefficient C can be calculated for any particilar convention about QFT with different masses for particle and antiparticle. In paper [3] we argued that there is no reasonable model for local QFT where $m \neq \tilde{m}$ and no reliable theoretical frameworks for calculations of C. Still even with uncertain coefficient C the relation (5) is extremely interesting. Indeed an upper bound on the photon mass produces a bound on the mass difference for electron and positron. As it is follows from equation (5):

$$\Delta m_e < 20\, m_\gamma/\sqrt{C}\,, \tag{6}$$

where we have to substitute for m_γ the upper limit on the photon mass.

These limits and discussion of their validity are presented in the review [7]. The Earth based experiments give for the Compton wave length of the photon $\lambda_C > 8 \cdot 10^7$ cm, i.e. $m_\gamma < 3 \cdot 10^{-13}$ eV, and respectively $\Delta m < 6 \cdot 10^{-12}$ eV, nine orders of magnitude stronger than PDG bound. From the measurement of the magnetic field of the Jupiter it follows that the Compton wave length of photon is larger than $5{\cdot}10^{10}$ cm or $m_\gamma < 4{\cdot}10^{-16}$ eV, and respectively $\Delta m < 8{\cdot}10^{-15}$ eV. The strongest solar system bound is obtained from the analysis of the solar wind extended up to the Pluto orbit [8]: $\lambda_C > 2 \cdot 10^{13}$ cm, i.e. $m_\gamma < 10^{-18}$ eV. This is an "official" limit present by the Particle Data Group [2]. The corresponding bound on the electron-positron mass difference is $\Delta m < 2 \cdot 10^{-17}$ eV, which is almost 14 orders of magnitude stronger than the direct bound on Δm.

The strongest existing bound follows from the the existence of the large scale magnetic fields in galaxies [9]: $\lambda_C > 10^{22}$ cm and $m_\gamma < 2 \cdot 10^{-27}$ eV. Correspondingly $\Delta m < 4 \cdot 10^{-26}$ eV, which is 23 orders of magnitude stronger than the direct limit on the electron-positron mass difference.

It is instructive to present a sample of actual calculations of m_γ. We stress again that there is no one example of a local Lorentz invariant Field Theory with non-zero Δm. Here we simply start with 'a la Barenboim-Greenberg decomposition for an electron-positron spinor field operator $\Psi(x)$.

$$\Psi(x) = \sum_{\mathbf{p}} \left\{ a(\mathbf{p}) \frac{u(p)e^{-ipx}}{\sqrt{2\omega(p)}} + b^+(\mathbf{p}) \frac{u(-\mathbf{p})e^{i\tilde{p}x}}{\sqrt{2\tilde{\omega}(p)}} \right\} \tag{7}$$

$$\{a(\mathbf{p}), a^+(\mathbf{p}')\} = \delta_{\mathbf{p},\mathbf{p}'}\ , \quad \text{etc.} \tag{8}$$

The first term in this decomposition annihilates electron with mass m, while the second term creates positron with mass $\tilde{m}$. Creation and annihilation operators obey the standard anti-commutation relations. We also assume the validity of the usual local product of field operators for the electric current

$$j_\mu(x) = \bar{\Psi}(x)\gamma_\mu\Psi(x)\ \ .$$

Because of the electron-positron mass difference this current is not conserved, $\partial j(x) \neq 0$.

Electron-positron pair contribute into the photon polarization operator.

$$\Pi_{\mu\nu} = (ie^2)\int \frac{d^Dp}{(2\pi)^D}\mathrm{Tr}\frac{1}{\hat{p}-m_1}\gamma_\nu\frac{1}{\hat{p}-\hat{q}-m_2}\gamma_\mu = \tilde{g}_{\mu\nu}\Pi_T(q^2) + g_{\mu\nu}\Pi_L(q^2), \tag{9}$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2$. This divergent integral has to be regularized and we choose covariant dimensional regularization. For non-conserving currents a longitudinal function $\Pi_L(q^2)$ has to be generated and nonzero $\Pi_L(0) \neq 0$ corresponds to non-zero photon mass

$$q_\mu \Pi_{\mu\nu} = q_\nu \Pi_L(q^2) = ie^2 \int \frac{d^D p}{(2\pi)^D} \mathrm{Tr} \left[\frac{1}{\hat{p} - m_1} \right] \gamma_\nu \left[\frac{1}{\hat{p} - \hat{q} - m_L} \right] \hat{q} \ . \tag{10}$$

In the standard case of equal masses the $\hat{q}$ is a difference of two inverse propagators and we reproduce the standard Ward identity. In our case $\hat{q}$ is a difference of inverse propagators plus mass difference:

$$\hat{q} = (\hat{p} - m_1) - (\hat{p} - \hat{q} - m_2) + (m_1 - m_2) \ . \tag{11}$$

Substituting this formula into (10) we get

$$\begin{aligned} q_\nu \Pi_L(q^2) &= ie^2(m_1 - m_2) \int \frac{d^D p}{(2\pi)^D} \frac{\mathrm{Tr}\left[(\hat{p} + m_1)\gamma_\nu(\hat{p} - \hat{q} + m_2\right]}{(p^2 - m_1^2)[(p-q)^2 - m_2^2]} = \\ &= 4e^2(m_1 - m_2) q_\nu \int_0^1 dx \int \frac{d^D p}{(2\pi)^D} \frac{m_2 x - m_1(1-x)}{[p^2 + \Delta^2]^2} \ , \end{aligned} \tag{12}$$

where $\Delta^2 = m_1^2(1-x) + m_2^2 x - q^2 x(1-x)$. For $q^2 = 0$ the integral is trivial and one gets that

$$m_\gamma^2 = \Pi_L(0) = \frac{\alpha}{2\pi} [m_2 - m_1]^2 \left[\ln \frac{\Lambda^2}{m^2} - \frac{5}{3} \right] \ , \tag{13}$$

where Λ is a cut-off. Thus the photon mass is divergent and has to be renormalized. Formally it can be an arbitrary number, even zero. But if loop calculations have any physical sense for such theories this number has to be proportional to fine coupling constant and disappear for equal mass, i.e. we arrive to equation (5).

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