The study of undulator radiation of transversal oscillator

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Abstract. We study the radiation intensity for a charged transversal oscillator moving around a dielectric cylinder. Similar to the case of coaxial circular motion, under certain conditions for the parameters of the trajectory and dielectric cylinder, strong narrow peaks appear in the angular distribution of the radiation intensity in the exterior medium for a given harmonic. Comparing to the case of a uniform coaxial circular motion, for a transversal oscillator new peaks appear. The oscillations serve as a mechanism for the control of the spectral-angular characteristics of the radiation intensity.

1. Introduction

Wide applications of the synchrotron radiation motivate the importance of investigations for various mechanisms of control for the radiation parameters [1]-[4]. From this viewpoint it is of interest to investigate the influence of medium, boundaries, shift of trajectory from the circular one on the spectral and angular distributions of the synchrotron/undulator emission. The investigation of the radiation from a charged particle, circulating around a dielectric cylinder immersed in a homogeneous medium [5]-[11], has shown that, under the Cherenkov condition for the material of the cylinder and the velocity of the charge projection on the cylinder surface, the strong narrow peaks appear in the angular distribution of the number of quanta emitted into the exterior medium. For some values of the parameters of the problem (energy of the particle, ratio of the radii of the cylinder and of the particle orbit, dielectric permittivity of the cylinder) the angular density of the number of quanta at the peaks exceeds the corresponding quantity for the radiation in the vacuum by several orders of magnitude. In the present paper, we aim to study the sensitivity of the peaks with respect to distortions of the particle's trajectory from circular one and possible new features in the radiation process. Namely, we consider a combination of two types of motion, consisting of a circular motion and transversal oscillations. The paper is organized as follows. In the next section we present the expressions for the radiation intensity on a given harmonic. The features of the radiation and the numerical results are discussed in section 3.

2. Radiation intensity from a transversal oscillator

Consider a point charge q moving along a helical trajectory outside a dielectric cylinder with radius ρ_c and with the dielectric permittivity ε_0. We will assume that this system is immersed in a homogeneous medium with the dielectric permittivity ε_1 (magnetic permeability will be taken to be unit). In cylindrical coordinates the trajectory of the charge is given by the functions $(\rho(t), \phi(t), z(t))$. We will assume that along the radial direction the charge makes an oscillatory motion around a circular trajectory $\rho = \rho_0$, described by

$$\rho(t) = \rho_0 \left[1 + a\cos(n_0\omega_0 t)\right], \tag{1}$$

where n_0 is a positive integer and a is the relative amplitude of the oscillations. For the motion along the cylinder axis z we will take the uniform one with the velocity $v_\parallel$: $z(t) = v_\parallel t$. In order to fix the function $\phi(t)$ for the azimuthal motion, we consider the special case with a constant energy of the charge: $\mathcal{E} = \text{const}$. This is the case when the particle moves under the static magnetic field only. From here it follows that $v^2 = v_\rho^2 + v_\phi^2 + v_\parallel^2 = \text{const}$, where $v_\rho = d\rho(t)/dt$, $v_\phi = \rho(t)d\phi(t)/dt$. For a given $\rho(t)$ from (1), the condition $v^2 = \text{const}$ determines the function $\phi(t)$. The corresponding function is obtained in the integral form. In order to simplify the consideration, we will discuss the case of small oscillations, $a \ll 1$. In this case, to the linear order in a, we get

$$\phi(t) \approx \omega_0 t - \frac{a}{n_0}\sin(n_0\omega_0 t). \tag{2}$$

Examples of the projection of the trajectory on the plane perpendicular to the z-axis are presented in figure 1.

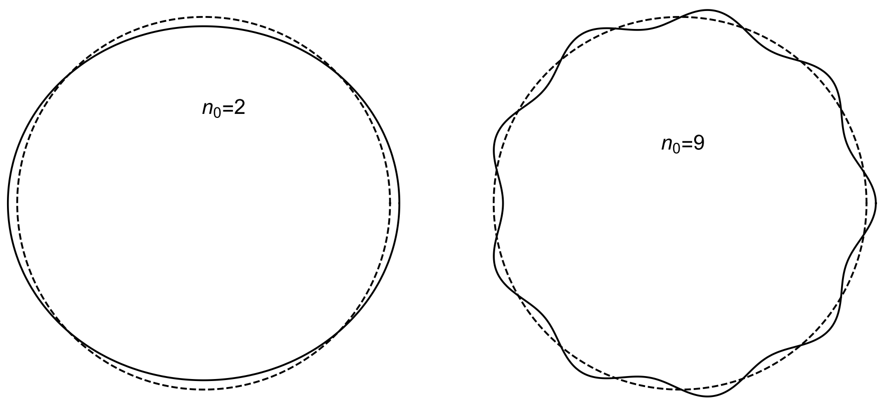

Figure 1. Examples of the particle transversal trajectory for $n_0 = 2, 9$ (numbers near the curves). Dashed curves are the trajectories for the circular motion.

In the same approximation, for the components of the velocity one finds

$$\begin{aligned} v_\rho &= -n_0 v_\perp a \sin(n_0\omega_0 t),\ v_\perp \equiv \omega_0\rho_0, \\ v_\phi &\approx v_\perp, v_z \approx v_\parallel. \end{aligned} \tag{3}$$

We expect that this type of motion with transverse oscillations can be generated by the magnetic field directed along the z-axis and

$$\mathbf{B} = (0, 0, B_0\left[1 + b\cos(n_0\phi)\right]), \tag{4}$$

assuming that $b \ll 1$. A similar motion in the case of a planar undulator is discussed, for example, in [4].

Considering the transversal oscillation as a small perturbation and by using the general formula from [11], the radiation intensity on a given harmonic n can be decomposed into two parts. The first one corresponds to the radiation for a circular helix, $dI_n^{(0)}/d\Omega$ (see [9]), and the second one comes from the transversal oscillations:

$$\begin{aligned}\frac{dI_n}{d\Omega} &= \frac{dI_n^{(0)}}{d\Omega} + \frac{q^2 a}{2\pi^3 c} \frac{n^2\omega_0^2\sqrt{\varepsilon_1}}{\left|1-\beta_{1\|}\cos\theta\right|^3} \\ &\times \sum_{j=\pm 1} \mathrm{Re}\left[\sum_{p=\pm 1} pD_{n+jn_0}^{(0p)} \sum_{p'=\pm 1} p' D_{n+jn_0,n}^{(1p',j)*} + \sum_{p=\pm 1} D_{n+jn_0}^{(0p)} \sum_{p'=\pm 1} D_{n+jn_0,n}^{(1p',j)*}\cos^2\theta\right]. \quad (5)\end{aligned}$$

Here $\beta_{1\|} = v_\| \sqrt{\varepsilon_1}/c$, $d\Omega$ is the element of the solid angle, θ is the angle between the wave vector of the radiated photon and the cylinder axis. The expressions for $dI_n^{(0)}/d\Omega$ and coefficients $D_m^{(0p)}$ are given in [9]. For the coefficients $D_m^{(1p)}$ with $p = \pm 1$ we have

$$\begin{aligned}D_{m,n}^{(1p)} &= \frac{\pi}{2ic}\left[J_{m+p,m}^{(1)}(n,\lambda_1) - H_{m+p,m}^{(1)}(n,\lambda_1)\frac{W(J_{m+p},J_{m+p})}{W(J_{m+p},H_{m+p})}\right] \\ &+ \frac{i\pi\lambda_1}{2ck_z}\left[J_{m,m,z}^{(1)}(n,\lambda_1) - H_{m,m,z}^{(1)}(n,\lambda_1)\frac{W(J_m,J_m)}{W(J_m,H_m)}\right] \\ &+ \frac{pJ_m(\lambda_0\rho_c)}{c\rho_c\alpha_m}\frac{J_{m+p}(\lambda_0\rho_c)}{W(J_{m+p},H_{m+p})} \\ &\times \left[\frac{k_{zn}H_{m,m,z}^{(1)}(n,\lambda_1)}{W(J_m,H_m)} + \frac{\lambda_0}{2}\sum_{l=\pm 1}\frac{H_{m+l,m}^{(1)}(n,\lambda_1)}{W(J_{m+l},H_{m+l})}\right], \quad (6)\end{aligned}$$

where $J_m(x)$ and $H_m(x)$ are the Bessel and Hankel (of the first kind) functions, and

$$W(J_{m+p},F_{m+p}) = J_m(\lambda_0\rho_1)\frac{\partial F_m(\lambda_1\rho_1)}{\partial\rho_1} - F_m(\lambda_1\rho_1)\frac{\partial J_m(\lambda_0\rho_1)}{\partial\rho_1}, \quad (7)$$

with $F = J$ and $F = H^{(1)}$. In (6) we use the following notations

$$\begin{aligned}F_{m+p,m}^{(1,j)}(n,\lambda_1) &= \frac{v_\perp}{2}\left[p\lambda_1\rho_0 F_m(\lambda_1\rho_0) - n\left(p - j/n_0\right)F_{m+p}(\lambda_1\rho_0)\right], \\ F_{m,m,z}^{(1,j)}(n,\lambda_1) &= \frac{v_z}{2}\left[\lambda_1\rho_0 F_m'(\lambda_1\rho_0) + j\frac{m}{n_0}F_m(\lambda_1\rho_0)\right]. \quad (8)\end{aligned}$$

Other notations are defined by the relations

$$\begin{aligned}\lambda_1 &= \frac{\omega_n}{c}\sqrt{\varepsilon_1}\sin\theta,\ \lambda_0 = \frac{\omega_n}{c}\sqrt{\varepsilon_0 - \varepsilon_1\cos^2\theta}, \\ \omega_n &= \frac{n\omega_0}{1-\beta_{1\|}\cos\theta},\ k_{zn} = \frac{\omega_n}{c}\sqrt{\varepsilon_1}\cos\theta, \quad (9)\end{aligned}$$

and

$$\alpha_m = \frac{\varepsilon_0}{\varepsilon_1 - \varepsilon_0} - \frac{\lambda_0 J_m(\lambda_0\rho_1)}{2}\sum_{l=\pm 1} l\frac{H_{m+l}(\lambda_1\rho_1)}{W(J_{m+l},H_{m+l})}. \quad (10)$$

The equation $\alpha_m = 0$ determines the eigenmodes of the dielectric cylinder.

For the special case of a circular helix, the features of the radiation intensity is discussed in [9]. By using Debye's asymptotic expansions for the Bessel and Neumann functions, it can be seen that under the condition $|\lambda_1|\rho_c < n$, at points where the real part of the function α_n, given by formula (10), is equal to zero, the contribution of the imaginary part of this function into the coefficients $D_n^{(0p)}$ can be exponentially large for large values n. This leads to the appearance of strong narrow peaks in the angular distribution for the radiation intensity at a given harmonic n. The condition for the real part of the function α_n is obtained from the equation determining the eigenmodes for the dielectric cylinder by the replacement $H_n \to Y_n$, where $Y_n(x)$ is the Neumann function. This equation has no solutions for $\lambda_0^2 < 0$, which is possible only for $\varepsilon_0 < \varepsilon_1$. Hence, the peaks do not appear for the case $\lambda_0^2 < 0$ which corresponds to the angular region $\cos^2\theta > \varepsilon_0/\varepsilon_1$. As necessary conditions for the presence of the strong narrow peaks in the angular distribution for the radiation intensity one has $\varepsilon_0 > \varepsilon_1$, $\widetilde{v}\sqrt{\varepsilon_0}/c > 1$, were $\widetilde{v} = \sqrt{v_\parallel^2 + \omega_0^2\rho_c^2}$ is the velocity of the charge image on the cylinder surface. The second condition is the Cherenkov condition for the velocity of the charge image on the cylinder surface and dielectric permittivity of the cylinder.

3. Numerical analysis

We have numerically investigated the angular density of the number of the radiated quanta, per period $T = 2\pi/\omega_0$ of the transversal motion,

$$\frac{dN_n}{d\Omega} = \frac{T}{\hbar\,|\omega_n|}\frac{dI_n}{d\Omega}, \tag{11}$$

as a function of the angle θ. In figure 2, an example is plotted for the harmonic $n = 10$. The dashed curves correspond to the radiation in a homogeneous medium with permittivity ε_1 and the full curves correspond to the radiation from an oscillator rotating around a dielectric cylinder with permittivity $\varepsilon_0 = 3.75\varepsilon_1$ immersed into a homogeneous medium with permittivity ε_1. For the transversal oscillations we have taken $n_0 = 2$. For the other parameters one has $\beta_{1\parallel} = 0.28$, $\beta_{1\perp} = 0.95$, $\rho_c/\rho_0 = 0.8$. The left panel corresponds to the radiation in the absence of the transversal oscillations ($a = 0$) and for the right panel $a = 0.05$.

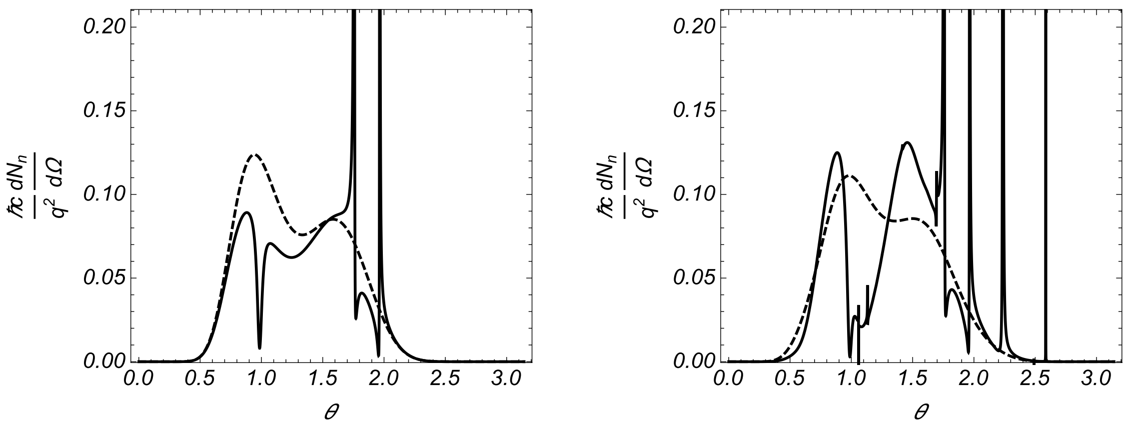

Figure 2. The angular density of the number of the quanta, radiated per period of the transverse motion on the harmonic $n = 10$, as a function of the angle θ. The left panel corresponds to the radiation for a motion along circular helix and the right panel is for the transversal oscillator. The dashed curves correspond to the radiation in a homogeneous medium with dielectric permittivity ε_0 immersed in the same medium. The values of the parameters are given in the main text.

As it is seen from the graphs, in the presence of the cylinder the intensities in these two cases differ notably. The frequency of the emitted quanta at the peaks depends on the angular location of the peak. In the case of spiral motion the frequency increases due to the Doppler effect 9. The wavelength of the radiated quanta is given by the expression

$$\lambda_n = \frac{2\pi c}{\omega_n} = \frac{2\pi\rho_0}{n\beta_{1\perp}}|1 - \beta_{1\parallel}\cos\theta|. \tag{12}$$

In case of waveguide's radius $\rho_0 \sim 1$cm and $n \sim 100$ the frequency of radiated wave is in the terahertz range.

4. Conclusion

We have investigated the radiation intensity from a transversal oscillator rotating around a dielectric cylinder. Periodical distortions of the trajectory serve as an additional mechanism for the control of the radiation intensity. In addition to the peaks for purely circular motion, new peaks appear under certain conditions for the parameters of the trajectory and dielectric cylinder in the angular distribution of the radiation intensity in the exterior medium. However, it should be noted that we have used the perturbation theory. The estimation of the radiation intensity at the peaks, appearing as a consequence of transversal oscillations, have to be done on the base of non-perturbative approach, for example, by using numerical methods. The corresponding results will be presented elsewhere.

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