Radiation from a longitudinal oscillator moving along a circular trajectory

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Abstract. Radiation intensity is investigated for a charged longitudinal oscillator moving along a helical trajectory around a dielectric cylinder immersed into a homogeneous medium. The influence of the longitudinal oscillations one on the characteristics of the angular distribution of the radiation intensity in the exterior medium is discussed. It is shown that the presence of oscillations serves as a mechanism to control the location and the height of the radiation peaks.

1. Introduction

The presence of medium can essentially change the characteristics of the high-energy electromagnetic processes. Moreover, new types of radiation phenomena can arise. Well-known examples are Cherenkov and transition radiations. In particular, by taking into account the wide applications of the synchrotron radiation, it is important to study the influence of medium on the spectral and angular characteristics of the synchrotron radiation.

In references [1]-[4] we have investigated the radiation generated by a charge moving along a helical orbit around/inside a dielectric cylinder. It has been shown that under certain conditions on the material of the cylinder and on the particle velocity, strong narrow peaks appear in the angular density of the radiation intensity. At these peaks the radiated energy exceeds the corresponding quantity in the case of a homogeneous medium by several orders of magnitude. In these investigations it was assumed that the charge moves along a circular helix coaxial with the cylinder axis and the rotation in the plane perpendicular to this axis is uniform. In realistic situations the trajectory of the particle may differ from the coaxial one. In Ref. [5] we have considered the electromagnetic fields and the radiation intensity for a charge moving around a dielectric cylinder along a helical trajectory the projection of which on the plane perpendicular to the cylinder axis is an arbitrary closed curve. In particular, the influence of the trajectory shift from the circular one on the characteristics of the peaks is discussed.

In the present paper, as an application of general results of [5], we consider the radiation from a longitudinal oscillator moving along a circular helical trajectory around a dielectric cylinder. In Section 2, the formula for the angular distribution of the radiation intensity on a given harmonic is presented for a general case of helical motion. The special case of a longitudinal oscillator moving along a circular helix is discussed in Section 3. The main results are summarized in Section 4.

2. Radiation intensity for a general motion

Consider a point charge q moving along a helical trajectory around a dielectric cylinder with radius ρ_c. The dielectric permittivity of the cylinder will be denoted by ε_0 and the cylinder is immersed into a homogeneous medium with permittivity ε_1. In cylindrical coordinates (ρ, ϕ, z), with z-axis along the cylinder axis, the motion of the charge is described by the functions $\rho = \rho_e(t)$, $\phi = \phi_e(t)$, $z = v_{\|}t$. We will denote by T the period of the transverse motion and $\omega_0 = 2\pi/T$. Introducing the angle θ between z-axis and the radiation direction, at large distances from the cylinder, for the angular density of the radiated energy per unit time on a given harmonic $n = 1, 2, \dots$, one has

$$\frac{dI_n}{d\Omega} = \frac{q^2}{2\pi^3 c} \frac{n^2\omega_0^2\sqrt{\varepsilon_1}}{\left|1-\beta_{1\|}\cos\theta\right|^3} \sum_{m=-\infty}^{\infty} \left[\left| D_{m,n}^{(+1)} - D_{m,n}^{(-1)} \right|^2 + \left| D_{m,n}^{(+1)} + D_{m,n}^{(-1)} \right|^2 \cos^2\theta \right], \tag{1}$$

with the notation $\beta_{1\|} = v_{\|}\sqrt{\varepsilon_1}/c$. Here, we have defined

$$\begin{aligned} D_{m,n}^{(p)} &= \frac{\pi}{2ic}\left[J_{m+p,m}(n,\lambda_1) - H_{m+p,m}(n,\lambda_1)\frac{W(J_{m+p},J_{m+p})}{W(J_{m+p},H_{m+p})}\right] \\ &+\frac{i\pi\lambda_1}{2ck_z}\left[J_{m,m,z}(n,\lambda_1) - H_{m,m,z}(n,\lambda_1)\frac{W(J_m,J_m)}{W(J_m,H_m)}\right] \\ &+\frac{pJ_m(\lambda_0\rho_c)}{c\rho_c\alpha_m}\frac{J_{m+p}(\lambda_0\rho_c)}{W(J_{m+p},H_{m+p})}\left[\frac{k_{zn}H_{m,m,z}(n,\lambda_1)}{W(J_m,H_m)} + \frac{\lambda_0}{2}\sum_{l=\pm 1}\frac{H_{m+l,m}(n,\lambda_1)}{W(J_{m+l},H_{m+l})}\right], \end{aligned} \tag{2}$$

where $J_m(x)$ and $H_m(x) = H_m^{(1)}(x)$ are the Bessel function and the Hankel function of the first kind,

$$\begin{aligned} \lambda_1 &= \frac{\omega_n}{c}\sqrt{\varepsilon_1}\sin\theta,\ \lambda_0 = \frac{\omega_n}{c}\sqrt{\varepsilon_0 - \varepsilon_1\cos^2\theta}, \\ \omega_n &= \frac{n\omega_0}{1-\beta_{1\|}\cos\theta},\ k_{zn} = \frac{\omega_n}{c}\sqrt{\varepsilon_1}\cos\theta, \end{aligned} \tag{3}$$

and

$$\alpha_m = \frac{\varepsilon_0}{\varepsilon_1 - \varepsilon_0} - \frac{\lambda_0 J_m(\lambda_0\rho_1)}{2}\sum_{l=\pm 1} l\frac{H_{m+l}(\lambda_1\rho_1)}{W(J_{m+l},H_{m+l})}. \tag{4}$$

Other functions are defined in accordance with

$$W(J_{m+p},F_{m+p}) = J_m(\lambda_0\rho_1)\frac{\partial F_m(\lambda_1\rho_1)}{\partial\rho_1} - F_m(\lambda_1\rho_1)\frac{\partial J_m(\lambda_0\rho_1)}{\partial\rho_1}, \tag{5}$$

where $F = J, H$, and

$$\begin{aligned} F_{m',m,l}(n,\lambda_1) &= \frac{1}{T}\int_0^T dt\, v_l(t)F_{m'}(\lambda_1\rho_e(t))e^{-im\phi_e(t)+in\omega_0 t}, \\ F_{m+p,m}(n,\lambda_1) &\equiv F_{m+p,m,\phi}(n,\lambda_1) - ipF_{m+p,m,\rho}(n,\lambda_1), \end{aligned} \tag{6}$$

with $l = \rho, \phi, z$, $v_z = v_{\|}$. For a given direction, the radiation frequency is given by $|\omega_n|$. For $\beta_{1\|} > 1$, the radiation along the Cherenkov direction $\theta = \theta_C = \arccos(c/(v_{\|}\sqrt{\varepsilon_1}))$ should be considered separately. For a charge rotating near the boundary of the dielectric waveguide, for the wavelength of the radiated quanta one gets

$$\lambda_{(n)} \approx \frac{2\pi\rho_c}{n\beta_{1\perp}}|1-\beta_{1\|}\cos\theta|,\ \beta_{1\perp} = v_{1\perp}\sqrt{\varepsilon_1}/c. \tag{7}$$

If $\beta_{1\|} \cos\theta$ is not too close to 1, for the waveguide radius $\rho_c \sim 1$cm and for harmonics $n \sim 100$ the frequency of the radiated wave is in the terahertz range. By decreasing the radius of the waveguide, the radiation frequency increases. For $\rho_0 \sim 0.1$mm the frequency is near the optical range. Currently dielectric waveguides are available with the radius of the order of nanometer (see, for instance, [6, 7]).

In a special case of a circular helix with $\rho_e(t) = \rho_e = \text{const}$, $\phi_e(t) = \omega_0 t$, one has $F_{m',m,l}(n, \lambda_1) = v_l F_{m'}(\lambda_1 \rho_e)\delta_{nm}$ and in (1) the term $m = n$ contributes only. The radiation intensity in this special case has been discussed in detail in [4]. In particular, the conditions were specified under which strong narrow peaks appear in the angular distribution for the radiation intensity at a given harmonic n.

3. Radiation intensity from a longitudinal oscillator

In this section we specify the general formula for the radiation intensity given above for a special case of the motion along a circular helix with additional longitudinal oscillations of the charge. In this case the components of the velocity are given by the expressions

$$\begin{aligned} v_\rho(t) &= 0,\ v_z(t) = v_\|, \\ v_\phi(t) &= v_\perp[1 + a\cos(n_0\omega_0 t)],\ v_\perp = \omega_0\rho_e, \end{aligned} \tag{8}$$

with n_0 being an integer. For the angular velocity of the rotation and for the angular coordinate one has

$$\begin{aligned} \Omega_\phi(t) &= \omega_0[1 + a\cos(n_0\omega_0 t)], \\ \phi_e(t) &= \omega_0[t + \frac{a}{n_0\omega_0}\sin(n_0\omega_0 t)]. \end{aligned} \tag{9}$$

For the motion (8), by taking unto account that $\rho_e(t) = \rho_e = \text{const}$ and $v_\rho = 0$, we see that $F_{m+p,m,\rho}(n, \lambda_1) = 0$. For the remaining functions, with the help of the formula

$$e^{i\alpha\sin\xi} = \sum_{s=-\infty}^{+\infty} J_s(\alpha)e^{is\xi}, \tag{10}$$

for $m \neq 0$ one gets the following expressions

$$\begin{aligned} F_{m+p,m,\phi}(n, \lambda_1) &= F_{m+p,m}(n, \lambda_1) = \frac{n}{m} v_\perp F_{m+p}(\lambda_1\rho_e) J_{\frac{n-m}{n_0}}\left(\frac{ma}{n_0}\right), \\ F_{m+p,m,z}(n, \lambda_1) &= v_\| F_{m+p}(\lambda_1\rho_e) J_{\frac{n-m}{n_0}}\left(\frac{ma}{n_0}\right). \end{aligned} \tag{11}$$

In these expressions $(n - m)/n_0$ should be an integer which we will denote by s. Hence, for the allowed values of m one has

$$m = n - sn_0,\ s = 0, \pm1, \pm2, \ldots \tag{12}$$

So, for $m \neq 0$ we get

$$D^{(p)}_{m,n} = J_s\left(\frac{ma}{n_0}\right)\overline{D}^{(p)}_{m,n}, \tag{13}$$

where

$$\begin{aligned}
\overline{D}^{(p)}_{m,n} &= \frac{\pi v_\perp n}{2icm}\left[J_{m+p}(\lambda_1\rho_e) - H_{m+p}(\lambda_1\rho_e)\frac{W(J_{m+p},J_{m+p})}{W(J_{m+p},H_{m+p})}\right] \\
&+\frac{i\pi v_\parallel \lambda_1}{2ck_z}\left[J_m(\lambda_1\rho_e) - H_m(\lambda_1\rho_e)\frac{W(J_m,J_m)}{W(J_m,H_m)}\right] + \frac{pJ_m(\lambda_0\rho_c)}{\rho_c\alpha_m}\frac{J_{m+p}(\lambda_0\rho_c)}{W(J_{m+p},H_{m+p})} \\
&\times\left[\frac{k_z H_m(\lambda_1\rho_e)}{W(J_m,H_m)}\frac{v_\parallel}{c} + \frac{\lambda_0}{2}\frac{n}{m}\frac{v_\perp}{c}\sum_{l=\pm1}\frac{H_{m+l}(\lambda_1\rho_e)}{W(J_{m+p},H_{m+p})}\right]. \qquad (14)
\end{aligned}$$

The expression for $\overline{D}^{(p)}_{m,n}$ coincides with the expression of $D^{(p)}_{m,n}$ in the case of a uniform circular motion with $v_\phi(t) = \omega_0\rho_e$ if we replace $v_\perp \to v_\perp n/m$.

The term $m = 0$ contributes for $n = n_0$ only. In this case we have $F_{m+p,m,z}(n,\lambda_1) = 0$ and

$$F_{m+p,m}(n,\lambda_1) = F_{m+p,m,\phi}(n,\lambda_1) = \frac{v_\perp}{2}aF_p(\lambda_1\rho_e)\delta_{n,n_0}. \qquad (15)$$

For the functions $D^{(p)}_{m,n}$ one finds

$$D^{(1)}_{0,n} = -D^{(-1)}_{0,n} = \frac{\pi a v_\perp}{4ic}\left[J_1(\lambda_1\rho_e) - H_1(\lambda_1\rho_e)\frac{W(J_1,J_1)}{W(J_1,H_1)}\right]. \qquad (16)$$

Hence, for the contribution of the $m = 0$ term, which is present only for $n = n_0$, one gets

$$\frac{dI_{n,m=0}}{d\Omega} = \frac{q^2 v_\perp^2 a^2}{4\pi c^3}\frac{n^2\omega_0^2\sqrt{\varepsilon_1}}{\left|1-\beta_{1\parallel}\cos\theta\right|^3}\left|J_1(\lambda_1\rho_e) - H_1(\lambda_1\rho_e)\frac{W(J_1,J_1)}{W(J_1,H_1)}\right|^2. \qquad (17)$$

In the case $n \neq n_0$, for the angular density of the radiation intensity on a given harmonic n we find the expression

$$\frac{dI_n}{d\Omega} = \frac{q^2}{2\pi^3 c}\frac{n^2\omega_0^2\sqrt{\varepsilon_1}}{\left|1-\beta_{1\parallel}\cos\theta\right|^3}{\sum_{s=-\infty}^{+\infty}}' J_s^2\Big(\frac{ma}{n}\Big)\left[\left|\overline{D}^{(+1)}_{m,n} - \overline{D}^{(-1)}_{m,n}\right|^2 + \left|\overline{D}^{(+1)}_{m,n} + \overline{D}^{(-1)}_{m,n}\right|^2\cos^2\theta\right], \qquad (18)$$

where m is given by (12) and the prime means that the term with $s = n/n_0$ should be excluded from the summation. For $a \ll 1$ the dominant contribution comes from the term with $s = 0$ and the contributions of the other terms are small by the factor a^{2s}. The leading term coincides with the corresponding quantity for a charge uniformly rotating around a dielectric cylinder. For $n = n_0$ the radiation intensity is the sum of the expressions given by (17) and (18):

$$\begin{aligned}
\frac{dI_{n_0}}{d\Omega} &= \frac{q^2}{2\pi^3 c}\frac{n_0^2\omega_0^2\sqrt{\varepsilon_1}}{\left|1-\beta_{1\parallel}\cos\theta\right|^3}\left\{\frac{\pi^2 v_\perp^2 a^2}{2c^2}\left|J_1(\lambda_1\rho_e) - H_1(\lambda_1\rho_e)\frac{W(J_1,J_1)}{W(J_1,H_1)}\right|^2\right. \\
&\left. + \sum_{s=-\infty,\neq 0}^{+\infty} J_{s-1}^2(sa)\left[\left|\overline{D}^{(+1)}_{sn_0,n_0} - \overline{D}^{(-1)}_{sn_0,n_0}\right|^2 + \left|\overline{D}^{(+1)}_{sn_0,n_0} + \overline{D}^{(-1)}_{sn_0,n_0}\right|^2\cos^2\theta\right]\right\}. \qquad (19)
\end{aligned}$$

In figures below we have presented the angular density of the number of the radiated quanta, per period T of the transversal motion,

$$\frac{dN_n}{d\Omega} = \frac{T}{\hbar\left|\omega_n\right|}\frac{dI_n}{d\Omega}, \qquad (20)$$

as a function of the angle θ. The graphs are plotted for the harmonic $n = 10$. The dashed curves correspond to the radiation in a homogeneous medium with permittivity ε_1 and the full curves correspond to the radiation from a charge rotating around a dielectric cylinder with permittivity $\varepsilon_0 = 3.75\varepsilon_1$ immersed into a homogeneous medium with permittivity ε_1 (for a charge rotating in the vacuum ($\varepsilon_1 = 1$), $\varepsilon_0 = 3.75$ corresponds to the dielectric permittivity of fuzed quartz). In all cases we have taken $\rho_c/\rho_e = 0.95$. For figure 1 one has $\beta_{1\|} = 0.35$ and $\beta_{1\perp} = 0.9$. The left panel corresponds to the radiation in the absence of the longitudinal oscillations ($a = 0$) and for the right panel $a = 0.1$. Figure 2 presents the same graphs for a purely transverse motion with the same energy, $\beta_{1\|} = 0$, $\beta_{1\perp} = \sqrt{0.35^2 + 0.9^2} \approx 0.966$. In this case the angular density is symmetric with respect to $\theta = \pi/2$. For the angular locations and heights of the left two peaks on the right panel of figure 2, induced by longitudinal oscillations, one has $(0.52, 19.35)$ and $(0.76, 556.2)$. For the left panel of figure 3 we have taken $a = 0.2$ and for the right panel $n_0 = 3$. For both panels the values of the other parameters are the same as those on the right panel of figure 1.

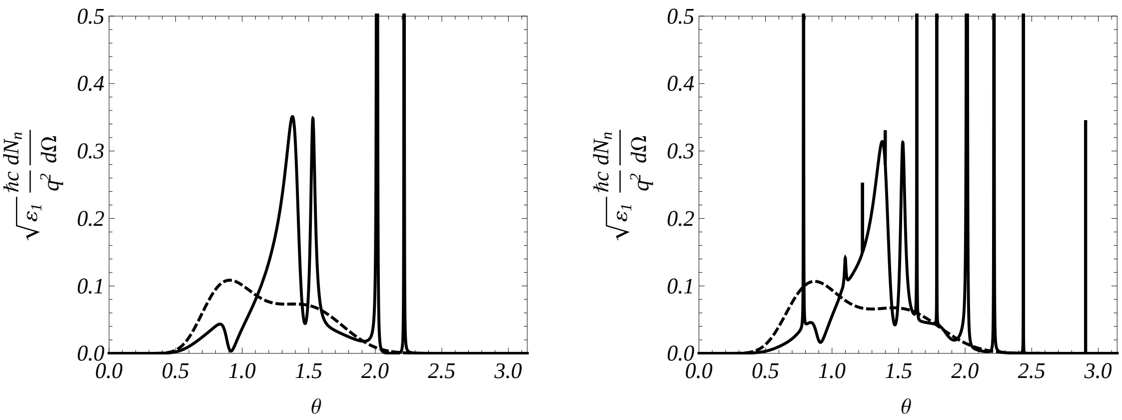

Figure 1. The angular density of the number of the quanta, radiated per period of the transverse motion on the harmonic $n = 10$, as a function of the angle θ. The dashed curves correspond to the radiation in a homogeneous medium with dielectric permittivity ε_1 and the full curves correspond to the radiation from a charge rotating around a dielectric cylinder with permittivity ε_0 immersed in the same medium. The left panel corresponds to the radiation in the absence of the longitudinal oscillations $a = 0$ and for the right panel $a = 0.1$, $n_0 = 2$. For the values of the other parameters we have taken $\rho_c/\rho_e = 0.95$, $\beta_{1\|} = 0.35$, $\beta_{1\perp} = 0.9$.

As it is seen from the graphs, the presence of longitudinal oscillations leads to the appearance of new peaks in the angular density of the radiated quanta. The number of peaks is increasing with the increase of the amplitude and frequency of the longitudinal oscillations.

4. Conclusion

We have investigated the radiation intensity of a charged longitudinal oscillator moving along an helical trajectory around a dielectric cylinder. Similar to the case of coaxial circular motion under certain conditions for the parameters of the trajectory and dielectric cylinder strong narrow peaks appear in the angular distribution of the radiation intensity in the exterior medium. Instead of a single peak in the case of a uniform coaxial circular motion, for an longitudinal oscillator set of peaks appear. The increase of the oscillating amplitude leads to the increase of the number of the peaks and the peaks are shifted to the direction of small angles. Heights of the peaks may either decrease or increase compared with the case of the uniform motion. The frequency

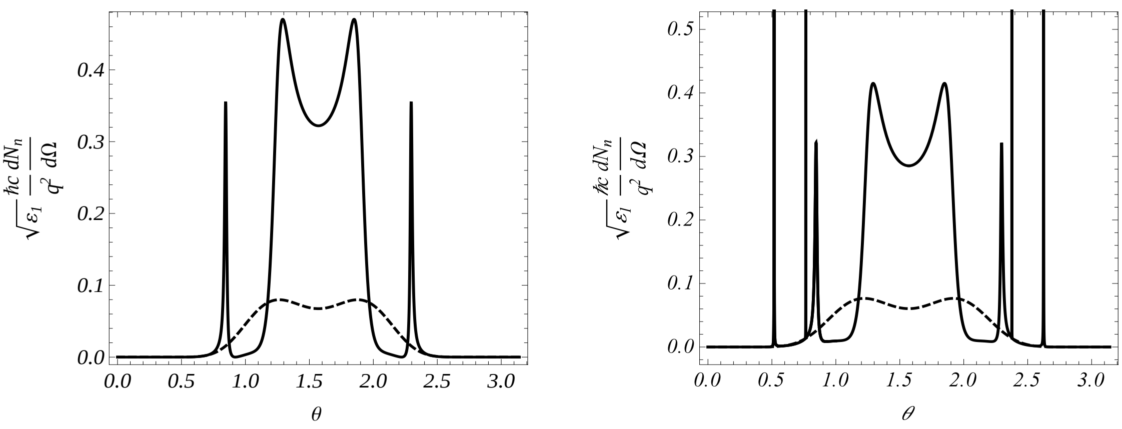

Figure 2. The same graphs as in figure 1 for a purely transverse motion: $\beta_{1\|} = 0$, $\beta_{1\perp} \approx 0.966$.

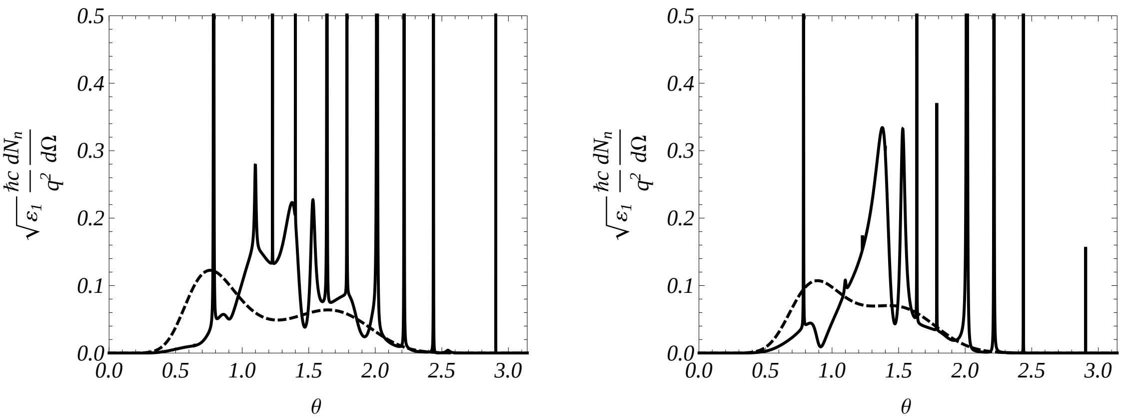

Figure 3. The same as in the right panel of figure 1. For the left panel $a = 0.2$, $n_0 = 2$ and for the right panel $a_0 = 0.1$, $n_0 = 3$. For both panels the values of the other parameters are the same as those on the right panel of figure 1.

of the emitted quanta at the peaks depends on the angular location of the peak. In the case of spiral motion the frequency increases due to the Doppler shift.

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