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Acoustic disturbances in a gas with an axial temperature gradient

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Abstract. Linear analysis acoustic disturbances in one-dimensional gas flow with a longitudinal gradient of the sound speed provided. Known wave equation for the acoustic velocity is used. In the case of linear distribution of the sound speed in the hot part of the flow equation has an exact analytic solution. This allows to define the expression describing the propagation acoustic disturbances in a gas with varying mean temperature and density. The results can be used to calculate the resonance frequencies of the gas oscillations in the laboratory and industrial combustors.

1. Introduction

It was shown [1,2], that the resonant frequency of the oscillation in the gas distribution channels are dependent on the mean temperature, density, and sound speed. It is known [3,4] that the boundaries of combustion stability, the amplitude of steady acoustic oscillations depend significantly on the frequency. Expressions describing acoustic disturbances in the gas at a constant mean temperature, density and speed of sound, known and widely used [5-7]. If the mean temperature of the gas is distributed along the channel, the wave equation becomes non-linear and its analytic solution - complex problem.

The purpose of this work - linear analysis acoustic disturbances propagating in the channel in the case of longitudinal linear distribution of the sound speed.

2. Wave equation and its solution

The wave equation for the acoustic velocity in the one-dimensional flow of an ideal gas with low Mach number is known [8] and is given by:

$$\frac{\partial^2 u'}{\partial t^2} - c^2(x)\frac{\partial^2 u'}{\partial x^2} = 0 \tag{1}$$

where u' - acoustic velocity c - sound speed.

In general, this equation can be solved by approximate or numerical methods. Consider the case where the mean temperature of the gas is reduced so that distribution of the sound speed is linear, i.e.:

$$c(x) = a - bx \tag{2}$$

The solution is sought in the form $u' = F_u(x)\exp(i\omega t)$. After substituting in the expression (1) we obtain equation:

$$\frac{d^2F_u}{dx^2} + \left(\frac{\omega}{c}\right)^2 F_u = 0 \qquad (3)$$

where ω - cyclic frequency.

We introduce a new variable:

$$y_u = \int \frac{\omega dx}{c}$$

After the conversion equation (3) takes the form:

$$\frac{d^2F_u}{dy_u^2} + \frac{b}{\omega}\cdot\frac{dF_u}{dy_u} + F_u = 0$$

The solution is sought in the form $F_u = F^* \exp(\alpha' y)$. Defining values α', obtain:

$$F_u(y_u) = e^{-\frac{by_u}{2\omega}}\left(F_1^* e^{i\beta y_u} + F_2^* e^{-i\beta y_u}\right),$$

$$\beta = \sqrt{1-\left(\frac{b}{2\omega}\right)^2}.$$

Going back to the original variable, believing $2F_1^* = C\exp i\phi$, $2F_2^* = C\exp(-i\phi)$, we have:

$$u'(x,t) = \exp\left(-\frac{b}{2}\int\frac{dx}{c}\right)C\cos\left[\left(\omega\beta\int\frac{dx}{c}\right)+\phi\right]\exp(i\omega t) \qquad (4)$$

For distribution (2):

$$\int\frac{dx}{c} = \int\frac{dx}{a-bx} = -\frac{1}{b}\ln(a-bx) + const$$

If the sound speed gradient is absent, acoustic velocity is known [9] and has the form:

$$u'(x,t) = C\cos(\omega x/a + \phi)\exp(i\omega t) \qquad (5)$$

Analysis showed that the expression (4) has the form (5), if $b = 0$, $\beta = 1$, $const = (1/b)\ln a$. Then:

$$u'(x,t) = Ce^{i\omega t}\left(1-\frac{bx}{a}\right)^{1/2}\cos\left[\phi - \frac{\omega\beta}{b}\ln\left(1-\frac{bx}{a}\right)\right] \qquad (6)$$

Acoustic pressure can be determined from the linearized continuity equation:

$$p'(x,t) = -\rho_0 c^2\int\frac{\partial u'}{\partial x}dt = -\frac{\rho_0 c^2}{i\omega}\cdot\frac{dF_u}{dx}e^{i\omega t}$$

where ρ_0 – the mean gas density.

Finally, we have:

$$p'(x,t) = -i\rho_0 cCe^{i\omega t}\left(1-\frac{bx}{a}\right)^{1/2}\left\{\frac{b}{2\omega}\cos\left[\phi - \frac{\omega\beta}{b}\ln\left(1-\frac{bx}{a}\right)\right] + \beta\sin\left[\phi - \frac{\omega\beta}{b}\ln\left(1-\frac{bx}{a}\right)\right]\right\}. \qquad (7)$$

In excretions (6), (7) phase angle ϕ depends on the boundary conditions at the ends of the channel.

3. Conclusion

The simplest approach - linear distribution of the speed of sound was used. An exact analytical solution of the wave equation for acoustic velocity was obtained. It was possible to find the expressions describing the velocity and pressure perturbations in the gas flow. The results of analysis allow calculating the resonance frequency of the gas in different systems with heat sources such as industrial combustors.

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