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Weak waves in multifractional liquids with bubbles

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Abstract. The propagation of weak waves in multifractional mixtures of liquid with vapor–gas and gas bubbles of different sizes and different compositions with phase transitions is studied. The dispersed phase consists of *N+M* fractions having various gases in bubbles and different in the bubbles radii. Phase transitions accounted for *N* fractions. The total bubble volume concentration is small (less than 1%). The dispersion relation is derived and dispersion curves is built. Influence of the mass concentration is shown. It is shown that dispersion and dissipation of acoustic waves depends significantly on presence of different bubbles in fractions of the dispersed phase.

1. Introduction

A propagation of acoustic waves in liquid with gas bubbles is studied intensively both theoretically and experimentally. Nigmatulin [1], Nakoryakov et al. [2], Temkin [3] considered various basic aspects of acoustics of multiphase flows. Hao and Prosperetti [4] obtained the results of study of the vapor bubble dynamics in liquid. Commander and Prosperetti [5] presented a model of the propagation of plane waves with small amplitude in a mixture of liquid and of gas bubbles. This model is adequate for the frequencies far from resonance at the volume content of a disperse phase of 1-2%. Nigmatulin et al. [6] showed the need of allowing for the effect of liquid compressibility for problems of acoustics of bubbly liquids. Shagapov [7] considered a problem of propagation of small disturbances in liquids with polydisperse bubbles. Nigmatulin et al. [8] studied the small radial oscillations of vapor-gas bubbles in liquid under the influence of an acoustic field. They showed that capillary effects and phase transitions lead to the new resonant frequency of small bubbles. This frequency is different from the Minnaert frequency. Vakhitova and Shagapov [9] investigated the propagation of small disturbances in two-component two-phase bubbling media with a three-temperature model. They found that heat and mass transfer rather than an interphase friction mainly determine dispersion. Gubaidullin and Nikiforov [10] obtained a dispersion relation describing a propagation of harmonic spherical and cylindrical disturbances in monodisperse mixture of liquid with the vapor-gas bubbles. They showed strong dependence of an attenuation of pulse waves on a vapor concentration in bubbles. The propagation of acoustic waves in a two-fractional mixture of liquid with bubbles is investigated in [11-13]. The propagation of sound waves in two-fractional liquids with polydisperse bubbles is investigated in [14-17]. A mathematical model of the propagation of sound waves in the two-fractional mixture of liquid with polydisperse vapor-gas and gas bubbles with phase transitions is shown in [18].

The propagation of acoustic waves in multifractional mixtures of liquid with gas and vapor-gas bubbles of different sizes and different compositions is studied in [19].

In the present paper, a propagation of weak waves in multifractional liquids with bubbles is studied.

2. Dispersion relation

Linearized equations for one-dimensional disturbances in a multifractional mixture of liquid with bubbles are obtained from the general equations of motion for bubbly mixtures [1]. Solving this system this dispersion relation is obtained

$$\left(\frac{K_*}{\omega}\right)^2=\frac{1}{C_f^2}+\frac{\rho_{10}}{p_0}\frac{\left(\sum\limits_{i=1}^{M}H_{3i}+\sum\limits_{j=1}^{N}H_{3j}\right)\left(\sum\limits_{i=1}^{M}H_{1i}+\sum\limits_{j=1}^{N}H_{1j}\right)}{\sum\limits_{i=1}^{M}\left(\frac{m_i}{\tau_{T1i}}+H_{2i}\right)+\sum\limits_{j=1}^{N}\left(\frac{m_j}{\tau_{T1j}}+H_{2j}\right)-i\omega}+$$

$$+\frac{\rho_{10}}{p_0}\sum_{i=1}^{M}\left(\frac{\alpha_{20i}}{N_{Ri}}\left(1-\frac{M_{4i}}{M_{3i}}\right)\right)+\frac{\rho_{10}}{p_0}\sum_{j=1}^{N}\left(\frac{\alpha_{20j}}{N_{Rj}}\left(1-\frac{M_{4j}}{M_{3j}}\right)\right) \qquad (1)$$

Here C_f is the frozen velocity of sound ($C_f=C_1/\alpha_{10}$) and the following notations are also accepted:

$$H_{1j}=\frac{m_j}{\tau_{T1j}}\left(\frac{M_{1j}M_{4j}}{M_{3j}}+M_{2j}\right),\ H_{1i}=\frac{m_i}{\tau_{T1i}}\left(\frac{M_{1i}M_{4i}}{M_{3i}}+M_{2i}\right),\ H_{2j}=\frac{m_j}{\tau_{T1j}}\frac{M_{1j}b_j}{M_{3j}},\ H_{2i}=\frac{m_i}{\tau_{T1i}}\frac{M_{1i}b_i}{M_{3i}},$$

$$H_{3j}=\frac{\alpha_{j0}}{N_{Rj}}\frac{b_j}{M_{3j}},\ H_{3i}=\frac{\alpha_{i0}}{N_{Ri}}\frac{b_i}{M_{3i}},\ M_{1j}=G_j-M_{2j}-\frac{L_{1j}N_{3j}}{L_{4j}-\delta N_{2j}},\ M_{2j}=\frac{N_{2j}L_{1j}}{N_{Rj}(L_{4j}-\delta N_{2j})},$$

$$M_{3j}=\frac{N_{3j}\delta-L_{2j}N_{3j}}{L_{4j}-\delta N_{2j}}+L_{3j}+M_{4j},\ M_{4j}=\frac{L_{4j}-L_{2j}N_{2j}}{N_{Rj}(L_{4j}-\delta N_{2j})},\ M_{1i}=-\left(N_{3i}+M_{2i}\right),\ M_{2i}=\frac{N_{2i}}{N_{Ri}},$$

$$M_{3i}=1+N_{3i}\left(1+b_i\right)+M_{4i},\ M_{4i}=\frac{1}{N_{Ri}}\left(1+N_{2i}\left(1+b_i\right)\right),\ N_{1j}=\frac{i\omega\tau_{T1j}}{m_j}-1,\ N_{1i}=1,\ N_{2j}=i\omega\tau_{Tj}-1,$$

$$N_{2i}=i\omega\tau_{Ti}-1,\ N_{3j}=k_{2j}\left(c_j-R_j\right)-1+G_j,\ N_{3i}=k_{2i}\left(c_i-R_i\right)-1,\ L_{1j}=E_j\left(1-i\omega\tau_j\right),$$

$$L_{2j}=-\frac{l_0k_{2j}}{\left(1-k_{Vj0}\right)T_0}+\Delta R_j-L_{1j}\left(1+b_j\right),\ L_{3j}=1-G_j\left(1+b_j\right),\ L_{4j}=L_{1j}+\Delta R_j\,N_{2j},\ k_{2j}=\frac{i\omega\tau_{Tj}}{c_j},$$

$$k_{2i}=\frac{i\omega\tau_{Ti}}{c_i},\ b_j=\frac{c_1\tau_{Tj}}{c_j\tau_{T1j}},\ b_i=\frac{c_1\tau_{Ti}}{c_i\tau_{T1i}},\ N_{Rj}=\frac{-(i\omega)(a_{j0})^2G_{Rj}\overset{\circ}{\rho}_{10}}{3(t_{Aj}G_{Rj}+1)p_0},\ N_{Ri}=\frac{-(i\omega)(a_{i0})^2G_{Ri}\overset{\circ}{\rho}_{10}}{3(t_{Ai}G_{Ri}+1)p_0},$$

$$G_{Rj}=\frac{1}{t_{Rj}}-i\omega,\ G_{Ri}=\frac{1}{t_{Ri}}-i\omega,\ t_{Rj}=\frac{(a_{j0})^2}{4\nu_1},\ t_{Ri}=\frac{(a_{i0})^2}{4\nu_1},\ t_{Aj}=\frac{a_{j0}}{C_1(\alpha_{j0})^{1/3}},\ t_{Ai}=\frac{a_{i0}}{C_1(\alpha_{i0})^{1/3}},$$

$$m_{j0}=\frac{\overset{\circ}{\rho}_{j0}}{\overset{\circ}{\rho}_{10}},\ m_{i0}=\frac{\overset{\circ}{\rho}_{i0}}{\overset{\circ}{\rho}_{10}},\ m_j=\frac{\rho_{j0}}{\rho_{10}},\ m_i=\frac{\rho_{i0}}{\rho_{10}},\ i=\overline{1,M},\ j=\overline{1,N}.$$

Dispersion relation (1) (i.e. the function of the complex wave number K_* on the frequency ω) determines a propagation of weak waves in multifractional mixtures of liquid with vapor-gas and gas bubbles (different initial radii, different initial volume contents and different thermal properties of fractions) with the interphase diffusion mass transfer.

3. Results

Figure 1 shows the phase velocity and attenuation coefficient in three-fraction mixture of water for the volume contents of fractions of $\alpha_{2N1} = \alpha_{2N2} = \alpha_{2M1} = 0.0033$, for the radius of vapor-air bubbles of $a_{0N1} = 1.5 \cdot 10^{-3}$ m, carbon dioxide with vapor bubbles – $a_{0N2} = 10^{-3}$ m, helium bubbles – $a_{0M1} = 0.5 \cdot 10^{-3}$ m, calculated by dispersion relation (1). In unperturbed state a pressure of a two-fractional mixture is $p_0 = 0.1$ MPa. Curves *1* are obtained for value of mass concentration of water vapor in bubbles is $k_{V0} = 0.1$, curves *2* – for value $k_{V0} = 0.5$, curves *3* – for value $k_{V0} = 0.9$.

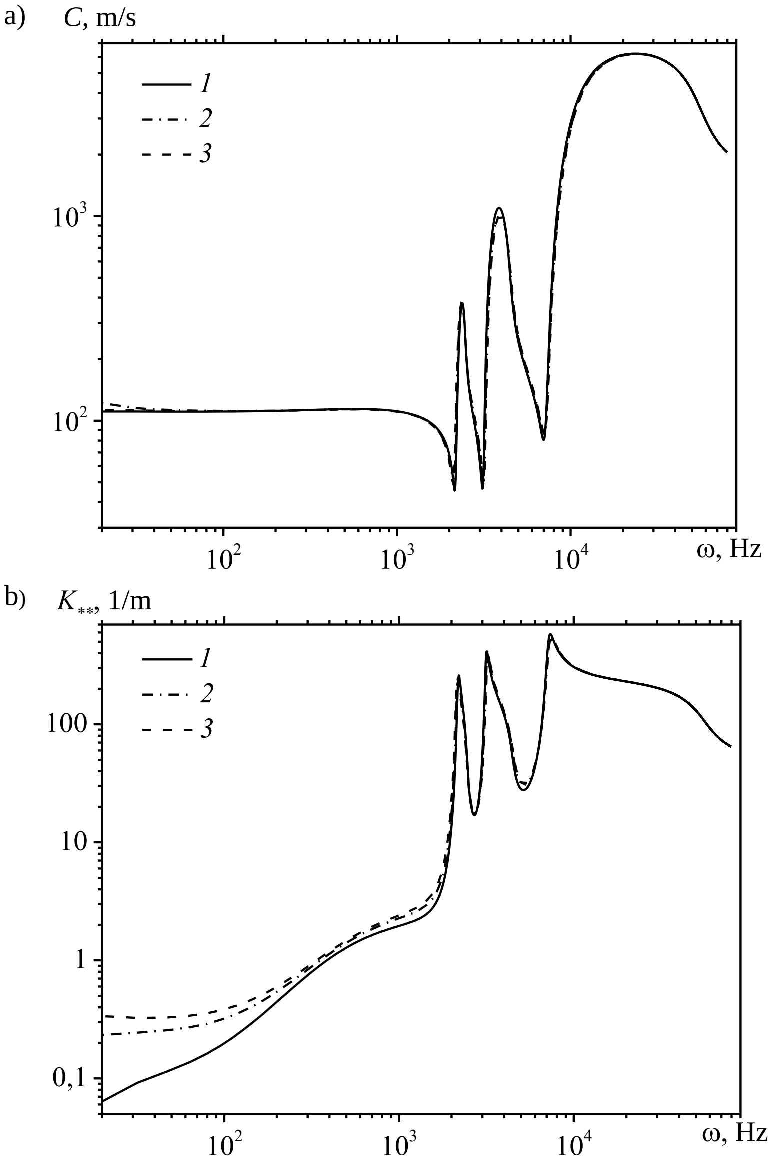

Figure 1. Frequency dependences of the phase velocity (a) and of the attenuation coefficient (b) for the three-fraction mixture of water for three different mass concentration: k_{V0}=0.1 (solid line), k_{V0}=0.5 (dash-dotted line), k_{V0}=0.9 (dashed line).

The difference of mass concentration of water vapor in bubbles almost has no effect on the phase velocity value (figure 1(a)). However, the presence of three different bubble sizes in the mixture is important significantly. This leads to the occurrence of the three minimum points of the phase velocity curve in the near of the resonance frequencies of the oscillations of bubbles and to a characteristic inflection of a phase velocity curve.

Figure 1(b) shows that increasing the mass concentration of vapor in bubbles leads to increasing the value of the attenuation coefficient in low-frequency region. For the same frequencies the difference is for four-five times. And also, three different bubble sizes leads to the occurrence of the three maximum values of the attenuation coefficient curves.

4. Conclusions

The propagation of weak waves in multifractional mixture of liquid with vapor-gas and gas bubbles taking into account the phase transitions is studied. In particular, it is shown that three fractions of bubbles with different radii cause the appearance of three local minimum values of the phase velocity and three local maxima in the dependence of the attenuation coefficient in the vicinity of the resonance frequencies of the natural vibrations of bubbles. It is shown that mass concentration of vapor in bubbles can significantly influence on dispersion curves.

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