

# Our Understanding on Landau-Raychaudhuri Cosmology

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**Abstract.** Recent developments to the generalized uncertainty principles (GUPs) and the modified dispersion relations (MDRs) play an important role in estimating the quantum corrections to the cosmic line element. We first apply both approaches to de Sitter-Schwarzschild and Reissner-Nördstrom black holes. Then from the fixed point method, the modified Landau-Raychaudhuri equations are derived in emergent cosmic space. We conclude that (non-)singular big bang solutions are obviously model dependent.

The predictions about minimum length uncertainty and maximum measurable momentum modify Heisenberg uncertainty principle (HUP), the so-called Generalized Uncertainty Principle (GUP), in a straight forward manner. The GUP has been confirmed by various observations such as and quantum optics [1] and seems to provide evidence for the existence of minimal length uncertainty [2]. Also, one of the most puzzling paradoxes in experimental high-energy physics is the time delay observed in the ultra high-energy cosmic rays (UHECRs). When assuming Lorentz invariance violation, this can be interpreted, and the dispersion relation becomes subject to modification (MDR) [3, 4], accordingly. Both approaches are applied to de Sitter-Schwarzschild and Reissner-Nördstrom black holes.

The unique spherically symmetric vacuum solution of General Relativity is the Schwarzschild metric [5]. Details on how the modified Landau-Raychaudhuri equations in emergent cosmic space of de Sitter-Schwarzschild and Reissner-Nördstrom black holes vary with GUP and MDR and depend on the equations of state ( $p = \omega\rho$ ) that characterize the cosmic geometry can be found in [6, 7]. The modified Landau-Raychaudhuri equations in de Sitter-Schwarzschild black holes—due to the minimal length and higher-order GUP and MDR approaches, respectively—can be determined as

$$\dot{H} \left[ 1 + \frac{\alpha^2}{4} H^2 + \dots \right] = -3H^2 \left[ 2(1 + \omega) + \frac{1}{4} \alpha^2 H^2 + \dots \right], \quad (1)$$

$$\dot{H} \left[ 1 + \alpha^2 H^2 - \alpha H + \dots \right] = -3H^2 \left[ 6(1 + \omega) + \frac{3\alpha^2}{8} H^2 - 4\alpha H + \dots \right], \quad (2)$$

$$\dot{H} \left[ 1 - \frac{15}{32} (\alpha_1^2 - 4\alpha_2) H^2 - \frac{3}{8} \alpha_1 H^2 \right] = -3H^2 \left[ 96(1 + \omega) + 18\alpha_1 H^2 + \frac{45}{8} (\alpha_1^2 - 4\alpha_2) H^4 \right]. \quad (3)$$

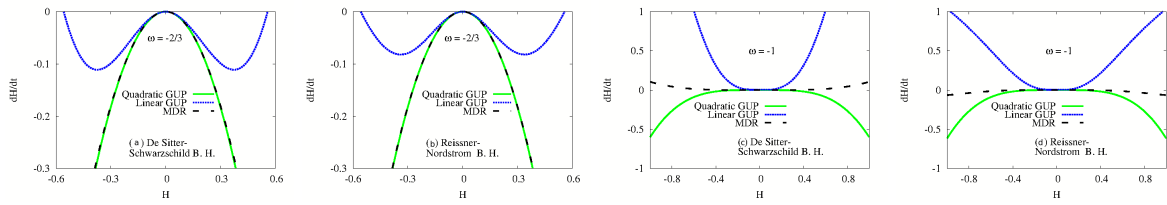


The first equation is related to the linear GUP. From the quadratic GUP, the second equation is obtained. The last equation gives the influence of MDR. From the line element in Reissner-Nordström geometry which has static electrical charged  $Q$  [8], the modified Landau-Raychaudhuri equations for linear and quadratic GUP and MDR approaches, respectively, read

$$\dot{H} \left[ 1 + \frac{\alpha^2}{4} Q H^4 + \alpha^2 \frac{4Q^4}{3\pi} H^6 + \dots \right] = -3H^2 \left[ \frac{2}{3}(1 + \omega) + \frac{2\alpha^2}{3} (QH^2 + Q^2 H^4) H^4 + \dots \right], \quad (4)$$

$$\dot{H} \left[ 1 - \alpha H + \alpha^2(1 + Q)H^2 + \dots \right] = -3H^2 \left[ 2(1 + \omega) - \frac{4}{3}\alpha H + \frac{\alpha^2}{4} H^2 + \dots \right], \quad (5)$$

$$\begin{aligned} \dot{H} \left[ 1 - \frac{5}{128}(\alpha_1^2 - 4\alpha_2)H^4 - \alpha_1(1 + 8Q)H^4 + \dots \right] &= -3H^2 \left[ 2(1 + \omega) + \frac{3\alpha_1}{8}(1 + 3Q)H^2 \right. \\ &\quad \left. - \frac{9\alpha_1 Q^2}{64} H^4 + \frac{5}{6}(\alpha_1^2 - 4\alpha_2)H^4 + \dots \right]. \end{aligned} \quad (6)$$



**Figure 1.** A depiction of  $dH/dt$  as function of  $H$  for de Sitter-Schwarzschild ((a) and (c)) and Reissner-Nördstrom ((b) and (d)) black holes at  $\omega = -2/3$  and  $-1$ , with the minimal length GUP (Linear GUP) (solid curve), higher order GUP (Quadratic GUP) (dotted curve) and modified dispersion relation (MDR) (dashed curve).

The results are given in Fig. 1. The solid curve stands for linear GUP at  $\omega = -2/3$  and  $-1$ . We find two fixed points,  $H_1 = 0$  and  $H_2 \sim 1/\gamma\ell_p = H_P$ , which point to non-singular big bang. Accordingly, the Universe age can be determined from  $\tau = \int_{H_1}^{H_2} dH/\dot{H} \rightarrow \infty$ , resulting in an infinite time-interval to reach the fixed points. It is obvious that these results are not observed from the other approaches and for other equations of state. Thus, we merely conclude that (non-)singular solutions depend on the metric types, the equations of states, and the quantum gravity approaches. These results address the need for further systematic studies and highlight the marginal impacts of the conclusion claimed in Ref. [9]. Last but not least, the absence of a good theory for quantum gravity should not permit advocating conclusions about singular or nonsingular big bang based on model-dependent approaches.

## References

- [1] I. Pikovski, M. R. Vanner, M. Aspelmeyer, M. S. Kim and C. Brukner, *Nature Phys.* **8**, 393 (2012).
- [2] F. Scardigli and R. Casadio, *Int. J. Mod. Phys. D* **18**, 319 (2009).
- [3] A. Tawfik and A. Diab, *Int. J. Mod. Phys. D* **23**, 1430025 (2014).
- [4] A. Tawfik and A. Diab, *Rept. Prog. Phys.* **78**, 126001 (2015).
- [5] S. M. Carroll, "Spacetime and Geometry: An Introduction to General Relativity" (University of Chicago, New York, 2004).
- [6] A. Tawfik and A. Diab, *Int. J. Mod. Phys. A* **30**, 1550059 (2015).
- [7] A. Tawfik and E. Abou El Dahab, *Int. J. Mod. Phys. A* **30**, 1550030 (2015).
- [8] H. Reissner, *Annalen der Physik* **50**, 106 (1916)
- [9] A. F. Ali, *Phys. Lett. B* **732**, 335 (2014)