

Equation of State in Non-Zero Magnetic Field

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Abstract. The Polyakov linear-sigma model (PLSM) and Hadron Resonance Gas (HRG) model are considered to study the hadronic and partonic equation(s) of state, the pressure, and response to finite magnetic field, the magnetization. The results are compared to recent lattice QCD calculations. Both models are in fairly good agreement with the lattice.

In heavy-ion collisions, especially the peripheral ones, the magnetic field likely has significant effects, which can be coupled to experimental observables, for instance, in the STAR experiment at the relativistic heavy-ion collider (RHIC) and ALICE experiment at the large hadron collider (LHC). Influences of such huge magnetic fields, $\mathcal{O}(m_\pi^2)$, on the hadronic and partonic matter and on the structure of the quark-gluon phase was analyzed in various models, such as the hadron resonance gas (HRG) [1] and also estimated in lattice QCD simulations [2, 3]. Coupling Polyakov loops to the linear-sigma model (PLSM) introduces color charge interactions to the pure gauge field. PLSM has revealed interesting features from studying the responses of QCD equation(s) of state to non-zero magnetic field [4]. In the present work, we compare our calculations to pressure and magnetization from HRG and PLSM at vanishing and finite magnetic field with recent lattice QCD calculations [3].

The influence of finite magnetic field to HRG and PLSM can be divided into two parts :

- The first one, the dispersion relation, becomes subject of modification [5, 6]. Assuming that the magnetic field is directed along z -axis, then in HRG, $E_{\ell s_z}(B) = [m^2 + P_\parallel^2 + 2g_s(\ell + s_z + 1/2)B]^{1/2}$, and in PLSM, $E_{B,f}(B) = [p_z^2 + m_f^2 + |q_f|(2n + 1 - \sigma)B]^{1/2}$, where n is Landau quantum number, and $\sigma = \pm S/2$ is related to the spin quantum number.
- The second part is the phase space, which gets modifications because of the effects of the magnetic catalysis,

$$\int \frac{d^3p}{(2\pi)^3} \longrightarrow \frac{|q_f|B}{2\pi} \sum_{\nu=0}^{\infty} \int \frac{dp_z}{2\pi} (2 - 1\delta_{0\nu}), \quad (1)$$

where $2 - 1\delta_{0\nu}$ stands for degenerate Landau levels.

Treating hadron resonances as a collision-free gas [7, 8, 9, 10] allows us to estimate various thermodynamic quantities from the grand canonical partition function [7, 8]. In non-zero magnetic fields, One has

$$\ln Z(T, V, \mu) = \frac{Vg_i}{(2\pi)^3} |Q_i| eB_z \sum_n \sum_{s_z} \int_0^\infty \pm dp_z \ln \left\{ 1 \pm \exp[(\mu_i - E_i^{B_z})/T] \right\}, \quad (2)$$



where \pm stands fermions and bosons, respectively. V is the volume and g_i is the degeneracy for i^{th} hadron.

In the mean field approximation of PLSM, the partition function expresses the exchange of energy between particle and antiparticle at temperature (T) and baryon chemical potential (μ_f). At finite magnetic field and assuming Landau quantization [12], the quark-antiquark potential is given by

$$\Omega_{\bar{\psi}\psi}(T, \mu_f, B) = -2 \sum_f \frac{|q_f|BT}{(2\pi)^2} \sum_{\nu=0}^{\infty} (2 - \delta_{0\nu}) \int_0^{\infty} dp_z \left\{ \ln \left[1 + 3 \left(\phi + \phi^* e^{-\frac{E_{B,f}-\mu_f}{T}} \right) e^{-\frac{E_{B,f}-\mu_f}{T}} + e^{-3\frac{E_{B,f}-\mu_f}{T}} \right] \right. \\ \left. \ln \left[1 + 3 \left(\phi^* + \phi e^{-\frac{E_{B,f}+\mu_f}{T}} \right) e^{-\frac{E_{B,f}+\mu_f}{T}} + e^{-3\frac{E_{B,f}+\mu_f}{T}} \right] \right\}, \quad (3)$$

where antiquarks can be taken into consideration by adding a second integral with ϕ and $-\mu$.

The magnetization gives one of the most fundamental properties of the QCD matter $\mathcal{M} = (T/V)\partial \ln \mathcal{Z} / \partial eB$, where $e \neq 0$ is the elementary electric charge. Positive magnetization refers to para-, while negative \mathcal{M} refer to diamagnetism. Left two panels (a) and (b) of Fig. 1 present the magnetization calculated from HRG model and PLSM, respectively, at different values of magnetic fields as function of temperature (curves). The results are compared to recent lattice QCD simulations (symbols) [3]. The agreement is fairly good. We conclude that the QCD matter possesses paramagnetic property for a wide range of temperatures. The right two panels, (c) and (d), illustrate the thermodynamic pressure. Again, the agreement between HRG and PLSM calculations, and lattice QCD simulations is good.

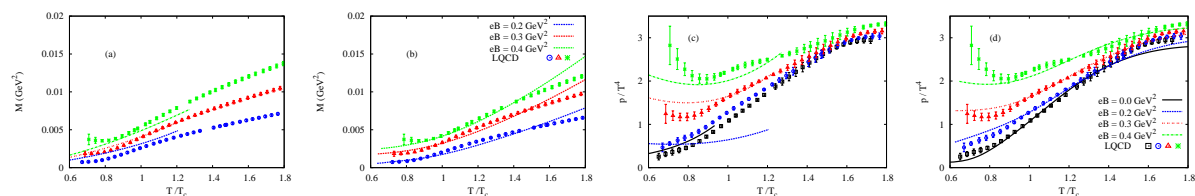


Figure 1. The temperature dependence of the magnetization (left two panels) and pressure (right two panels) is calculated from HRG and from PLSM at different magnetic fields, $eB = 0.0 - 0.4 \text{ GeV}^2$ (curves) and compared with recent lattice QCD calculations (symbols) [13].

References

- [1] G. Endrodi, JHEP **1304**, 023 (2013).
- [2] G. S. Bali, F. Bruckmann, G. Endrodi, and A. Schafer, PoS **LATTICE2013**, 182 (2014).
- [3] G. Bali, F. Bruckmann, G. Endrodi, S. Katz, and A. Schaefer, JHEP **1408**, 177 (2014).
- [4] A. Tawfik, and N. Magdy, Phys. Rev. C **90**, 015204 (2014).
- [5] J. F. Letessier, and J. Rafelski, "Hadrons and Quark-Gluon Plasma", (Cambridge university press, Cambridge, 2004).
- [6] L. D. Landau, and E. M. Lifshitz, *Quantum Mechanics non-relativistic theory*, (Butterworth Heinemann, Heidelberg, 1965).
- [7] F. Karsch, K. Redlich, and A. Tawfik, Eur. Phys. J. **C29**, 549 (2003).
- [8] K. Redlich, F. Karsch, and A. Tawfik, J. Phys. **G30**, S1271 (2004).
- [9] A. Tawfik, Phys. Rev. **D71**, 054502 (2005).
- [10] A. Tawfik, J. Phys. **G31**, S1105 (2005).
- [11] R. Venugopalan, and M. Prakash, Nucl. Phys. **A546**, 718 (1992).
- [12] A. Tawfik, A. Diab, and M. T. Hussein, "Chiral Magnetic Effects from Extended $SU(3)$ Linear-sigma Model", submitted to Phys. Rev. C (2015).
- [13] G. S. Bali, F. Bruckmann, G. Endrodi, S. D. Katz, A. Schafer, JHEP **177** (2014).