Optical conductivity and string theory

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Abstract. In this note we summarize some facts about optical conductivity and the corresponding holographic models attempting to describe it.

1. Optical conductivity

The optical properties of solids provide an important tool to study their microscopic dynamics. In certain materials with strong electron correlations a quantum critical state of matter can occur. In this situation the system looks on average the same, regardless of the time- and length scale on which it is observed. This implies, that scale invariant correlations should drive the response functions, revealing some universal patterns determined by the quantum mechanical nature of the fluctuations. Such regularities were reported for the mid-infrared frequency range of the optical conductivity of optimally doped copper-oxide superconductors [1, 2]. In more detail in ref. [1] two interesting regions of frequencies were identified from the data.

The low frequency region ($\omega < T$), where measurement time was large compared to the time scale set by inverse temperature. In this region system follows classical relaxation dynamics with a relaxation time τ. Conductivity in this case is given by the usual Drude formula

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} \ . \tag{1}$$

The second region ($T < \omega < \Omega$) directly probes the scale invariance of the quantum critical state. A non-universal cut-off Ω is called the optical gap and is required by the f-sum rule [1]. In this region the optical conductivity $\sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega)$ follows the universal scaling law

$$|\sigma(\omega)| \sim \omega^{2/3} \ , \tag{2}$$

with a frequency independent phase angle $\arctan(\sigma_2(\omega)/\sigma_1(\omega)) \approx 60^o$. The experimental data are shown in figure 1. A remarkable thing is the quite robust temperature independence of this scaling relation.

2. Holographic models

Up to date we have no microscopic understanding of how quantum criticality emerges from the strong correlations of the normal state near optimal doping. An emergent scale invariance implies that the low energy effective theory takes a form of a strongly coupled conformal field theory (CFT) [3]. A widely used tool to analyze certain strongly coupled CFT systems is the anti-de Sitter conformal field theory (AdS/CFT) correspondence, a setup in which a

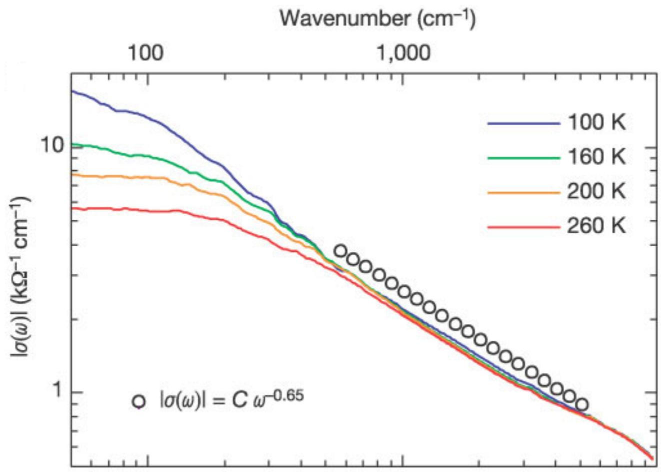

Figure 1. Universal power law of the optical conductivity from ref. [1].

nongravitational system is mapped to a theory with gravity in a higher dimensional spacetime. Over the past few years, it has been applied to model many interesting phenomena, including superconductivity and superfluidity, Fermi surfaces and non-Fermi liquids (see [4] for a review). The main advantage of this method is that it is based on conserved currents, rather than on a quasi-particle description, which makes it more suitable for understanding the relation between the infrared and the ultraviolet degrees of freedom. A natural question to ask is whether any form of power-like scaling can occur in the optical conductivity computed in a holographic model?

The simplest bulk description of a translationally invariant system at finite temperature and charge density is given by a charged black hole solution [4]. This was then improved by the coupling to a bulk scalar field, which could provide a model for a solid state lattice and in turn account for a realistic description of the charge transport [5]. Because the relevant physics is in two dimensional layers, the CFT in question lives in $(2+1)$ spacetime dimensions. Thus, the gravitational dual is a theory in four dimensional spacetime.

In more detail, the action used in [5] in $d = 3+1$ dimensions reads

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_a\phi\nabla^a\phi + 6 + \phi^2\right] . \tag{3}$$

States in a CFT are mapped to solutions of the equations of motion following from the action (3), which have some specified boundary conditions. In Poincare coordinates $\{\mathbf{x}, z\} = \{t, x, y, z\}$ any solution of the Klein-Gordon equation for chosen parameters has the near boundary asymptote

$$\phi(\mathbf{x}, z) \sim \phi_0(\mathbf{x})z + \phi_1(\mathbf{x})z^2 . \tag{4}$$

In particular, in the boundary theory this corresponds to the operator $\hat{O}$ of conformal dimension $\Delta = 2$ that has a source $\phi_0(\mathbf{x})$ and corresponding expectation value $\langle\hat{O}(\mathbf{x})\rangle = \phi_1(\mathbf{x})$. The simplest model of a lattice was to impose a one dimensional source of the form of a single Fourier mode, i.e.

$$\phi_0(x) = A_0\cos(kx) . \tag{5}$$

Such a choice causes explicit translational symmetry breaking in a chosen direction. The claim of ref. [5] is that with this assumption both frequency regions described in the previous section are reproduced. This has motivated further research and explorations of more realistic and transparent models of holographic lattices [6, 7, 8].

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To analyze the data, rather than using log-log plots another quantity was chosen for the study:

$$\alpha = 1 + \omega \frac{|\sigma|''}{|\sigma|'} \; . \tag{6}$$

A value of $\alpha = -2/3$ would correspond to the mid-infrared conductivity of the cuprates. In contrast to the claim of [5] no constant α as a function of ω was found. Sample results, taken from ref. [8], are shown in figure 2. This leads to the conclusion, that holographic lattices do not encode the power-law optical conductivity.

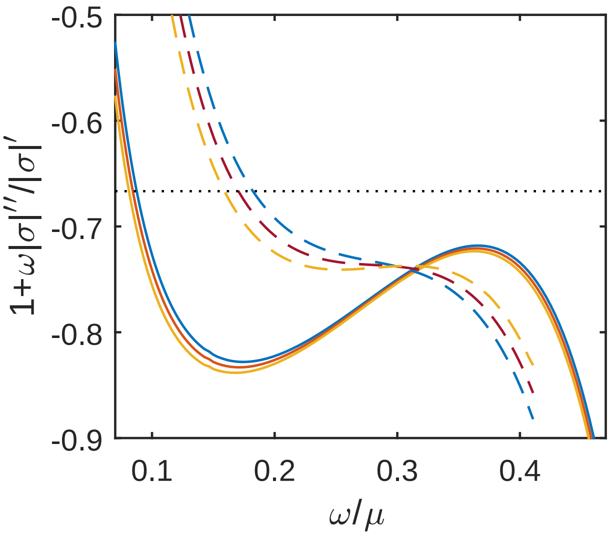

Figure 2. The exponent as a function of frequency. Figure taken from ref. [8]. Dashed and solid lines correspond to two slightly different lattice models.

However, one point which could be improved, is to replace an unrealistic, spatially delocalized lattice of a single Fourier mode with a lattice of localized atoms in the form of (possibly regularized) Dirac delta functions. A case where the source was set to be

$$\phi_0(t, x, y) = A_0 \, \delta(x) \; , \tag{7}$$

i.e., the Dirac delta supported on a $x = 0$ line was considered in ref. [9]. The solution turned out to impose strong constraints on the choice of matter interactions of the dual gravitational theory. Only the potential

$$V(\phi) = -6 \cosh\left(\frac{\phi}{\sqrt{3}}\right) \; , \tag{8}$$

for the scalar filed allows solutions with such boundary conditions. This form of interactions is one of possibilities which come from consistent compactifications of ten dimensional string theories [10]. This shows, that seemingly innocent modifications of the source causes highly non-trivial consequences. This might be an indication that similar impact would be imprinted on a conductivity computed in such a background.

Having a theory in which one can consistently study localized atoms we can go a step ahead and consider a regularized lattice of atoms, e.g., in a form motivated by linearized solution of a regularized Dirac delta source

$$\phi_0(x) = \frac{A_0 \sinh(z_0)}{\cosh(z_0) - \cos(x)} \; . \tag{9}$$

The parameter z_0 regularizes the configuration and the limit $z_0 = 0$ gives exact delta functions placed at positions $x = 2\pi n, (n \in \mathbb{Z})$. Sample configurations are shown in fig. 3.

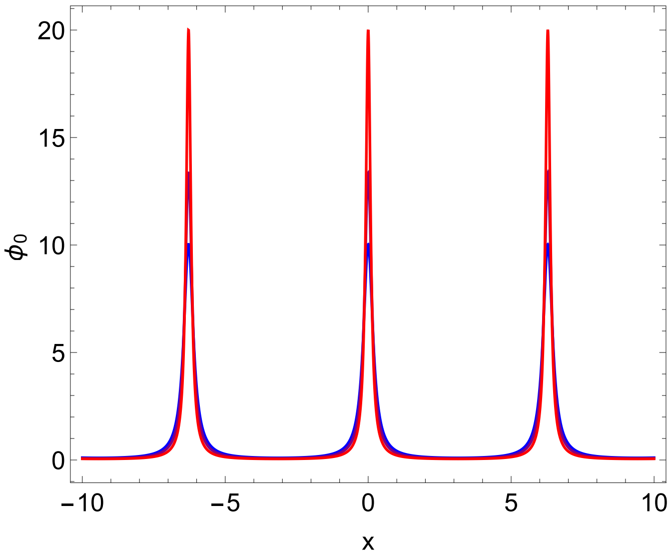

Figure 3. Sample lattices which can be considered in the holographic models. Parameters: $A_0 = 1,\ z_0 = 0.1,\ 0.15,\ 0.2$.

Another possible direction is to formulate a holographic dual of a system in which translational symmetry is broken spontaneously, not explicitly. This is closer to the cases of actual systems which exhibit this form of symmetry breaking.

Up to date the only setup which successfully reproduced the $\omega^{-2/3}$ law is based on unparticles, i.e., scale-invariant excitations [11]. Given the fact that the radial direction in the AdS space describes dual RG flow and contains excitations at all energy scales, in principle it contains the correct ingredients to capture the unparticle dynamics. This suggests that in some construction of holographic models, perhaps different that the one discussed above, should be able to reproduce the power law.

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