

Pion and kaon in the Beth-Uhlenbeck approach

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Abstract. In the present work the Mott effect for pions and kaons is described within a Beth-Uhlenbeck approach on the basis of the PNJL model. The contribution of these degrees of freedom to the thermodynamics is encoded in the temperature dependence of their phase shifts. A comparison with results from $N_f = 2 + 1$ lattice QCD thermodynamics is performed.

1. Introduction

One of the central problems in the investigation of the transition from hadronic to quark matter is a microphysical description of the dissociation of hadrons into their quark constituents. This Mott transition occurs under extreme conditions of high temperatures and densities as they are provided, e.g., in ultrarelativistic heavy-ion collisions or in the interiors of compact stars. Since an ab-initio description of QCD thermodynamics within simulations of lattice QCD (LQCD) is yet limited to finite temperatures and low chemical potentials only, the development of effective model descriptions is of importance. Here, we develop a relativistic Beth-Uhlenbeck approach to the description of mesonic bound and scattering states in a quark plasma [1, 2] further by including the strange sector. To this end we employ the PNJL model which is particularly suitable for addressing the appearance of pions and kaons as both, quasi Goldstone bosons of the broken chiral symmetry and pseudoscalar meson bound states. Within this framework the confinement of colored quark states is effectively taken into account by coupling the chiral quark dynamics to the Polyakov loop and its effective potential. The model is widely used to describe quark-gluon thermodynamics in the meanfield approximation [3, 4], but has also been developed to address mesonic correlations [5, 6, 7, 8, 9, 10, 11, 12]. The relativistic Beth-Uhlenbeck approach is the appropriate tool to develop a unified description of quark-gluon and hadron thermodynamics including the transition between both asymptotic regimes of QCD.

In the next section the basic formulae for the thermodynamic potential, the phase shifts and the pressure in the Beth-Uhlenbeck approach are presented, followed by a discussion of numerical results compared to LQCD data in Sect. 3 and conclusions in Sect. 4.

2. Relativistic Beth-Uhlenbeck approach for the PNJL model

The Lagrangian for the 3 flavor PNJL model is given by

$$\mathcal{L} = \bar{q}[\mathbf{i}\not{\partial} - m_0 + \gamma_0(\mu - \mathbf{i}A_4)]q + G_S \sum_{a=0}^8 [(\bar{q}\lambda^a q)^2 + (\bar{q}\mathbf{i}\gamma_5\lambda^a q)^2] - \mathcal{U}(\Phi, \bar{\Phi}; T) .$$



Here, q denote the quark fields with 3 colors and flavors, λ^a are the Gell-Mann matrices in flavor space ($a = 0, 1, 2, 3 \dots 8$), G_S is dimensionful coupling constant. The Polyakov-loop potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$ is chosen in the polynomial form [4], with the parameters taken from that reference. The gluon background field in the Polyakov gauge is a diagonal matrix in color space $A_4 = \text{diag}(\phi_3 + \phi_8, -\phi_3 + \phi_8, -2\phi_8)$. The Polyakov loop field Φ is defined via the color trace over the gauge-invariant average of the Polyakov line $L(\vec{x})$ [4].

The thermodynamic potential in Gaussian approximation has the form [11]

$$\Omega_{\text{Gauß}} = \mathcal{U}(\Phi, \bar{\Phi}; T) + \frac{\sigma_{\text{MF}}^2}{4G_S} + \Omega_Q + \Omega_M, \quad (1)$$

where in the quark (Ω_Q) and meson (Ω_M) contributions the zero-point energy terms are removed ("no sea" approximation). The quark contribution is given by

$$\Omega_Q = -\frac{2N_c N_f}{3} \int \frac{dp}{2\pi^2} \frac{p^4}{E_p} [f_\Phi^+(E_p) + f_\Phi^-(E_p)], \quad (2)$$

where $f_\Phi^\pm(E_p)$ are the generalized Fermi distribution functions

$$f_\Phi^+(E_p) = \frac{(\bar{\Phi} + 2\Phi Y)Y + Y^3}{1 + 3(\bar{\Phi} + \Phi Y)Y + Y^3}, \quad f_\Phi^-(E_p) = \frac{(\Phi + 2\bar{\Phi}\bar{Y})\bar{Y} + \bar{Y}^3}{1 + 3(\Phi + \bar{\Phi}\bar{Y})\bar{Y} + \bar{Y}^3}. \quad (3)$$

with the abbreviation $Y = e^{-(E_p - \mu)/T}$ and $\bar{Y} = e^{-(E_p + \mu)/T}$ (cf. Ref. [5]). The meson contribution reads [2]

$$\Omega_M = -d_M \int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{2\pi} [g^+(\omega + \mu_M) + g^-(\omega - \mu_M)] \delta_M(\omega, \mathbf{q}).$$

Here d_M are the meson degeneracy factors: $d_\pi = 3$ for pions and $d_K = 4$ for kaons; $g(E) = (e^{\beta E} - 1)^{-1}$ is the Bose function, $\mu_M = \mu_i - \mu_j$ is the chemical potential of a meson M composed of quark i with chemical potential μ_i and antiquark j with chemical potential $-\mu_j$. The phase shift at rest $\delta_M(\omega, \mathbf{q} = \mathbf{0}) = \delta_M(\omega)$ is determined by

$$\delta_M(\omega) = -\text{Im} \ln [\beta^2 S_M^{-1}(\omega - \mu_M + i\eta)] . \quad (4)$$

The propagator and the polarization function in the present work should be understood as to be evaluated at the shifted energy $z = \omega - \mu_M + i\eta$

$$\begin{aligned} S_M^{-1}(z) &= G_S^{-1} - \Pi_M(z), \\ \Pi_M(z) &= 4 \left\{ (I_1^i + I_1^j) - [(\omega - \mu_M)^2 - (M_i - M_j)^2] I_2^{ij}(\omega - \mu_M) \right\}, \end{aligned} \quad (5)$$

where the 1-loop integrals are given by

$$I_1^i = -N_c \int \frac{dp}{4\pi^2} \frac{p^2}{E_i} [f_\Phi^+(E_i) + f_\Phi^-(E_i)], \quad (6)$$

$$\begin{aligned} I_2^{ij} = -N_c \int \frac{d^3p}{(2\pi)^3} & \left[\frac{1}{2E_i} \frac{1}{(E_i + \omega)^2 - E_j^2} f_\Phi^+(E_i) - \frac{1}{2E_i} \frac{1}{(E_i - \omega)^2 - E_j^2} f_\Phi^-(E_i) \right. \\ & \left. + \frac{1}{2E_j} \frac{1}{(E_j - \omega)^2 - E_i^2} f_\Phi^+(E_j) - \frac{1}{2E_j} \frac{1}{(E_j + \omega)^2 - E_i^2} f_\Phi^-(E_j) \right]. \end{aligned} \quad (7)$$

3. Results

The parameters used for the numerical studies in the present work are the bare quark masses $m_{u,d} = 5.5$ MeV and $m_s = 138.6$ MeV, the three-momentum cutoff $\Lambda = 602$ MeV and the scalar coupling constant $G_S \Lambda^2 = 2.317$. With these parameters one finds in vacuum a constituent quark mass of 367 MeV, a pion mass of 135 MeV and pion decay constant $f_\pi = 92.4$ MeV. We report here results for the case of vanishing chemical potentials.

The solution for the pion phase shift is shown in Figure 1 as function of the energy variable ω . The jump of the phase shift from 0 to π indicates the position of a bound state in the spectrum below the threshold of the continuum states which is located where the phase shift starts decreasing towards zero. With increasing temperature the threshold moves to lower ω -values and the phase shift jumps from π to 0 when the Mott temperature is reached where the bound state gets dissociated. This behavior is in accordance with the Levinson theorem, for details see [11]. In the kaon phase shift arises one more threshold at low energy because

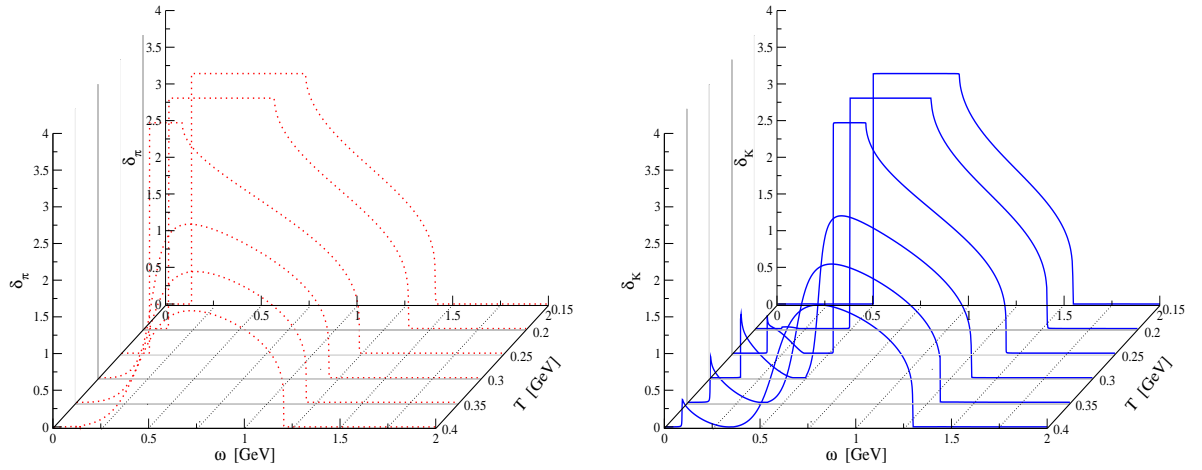


Figure 1. Phase shift of pions (left panel) and kaons (right panel) as function of the energy for different temperatures from $T = 150$ MeV to 400 MeV.

the dynamical masses of the quarks composing the kaon are different, for more information see Ref.[9, 10]. In the left panel of Figure 2 the pressure of pions and kaons is presented. It shows a typical behavior: first increasing with temperature towards the Stefan-Boltzmann limit which isn't reached since due to the chiral phase transition the continuum threshold lowers and this induces a reduction of the meson gas pressure already before the Mott temperature is reached. Above the Mott temperature, the growing meson width leads to a stronger reduction of the pressure with a rather sharp onset of this effect. In the right panel of Figure 2 we compare the total pressure of the $N_f = 2 + 1$ flavor PNJL model with recent LQCD data from [13, 14]. While our results agree with LQCD below $T \simeq 150$ MeV, a discrepancy opens up beyond this temperature which is mainly due the fact that the chiral transition temperature in the PNJL model is too high (~ 250 MeV) when compared to the pseudocritical temperature of $T_c = 154 \pm 9$ MeV from LQCD [15]. Above a temperature of $T \simeq 300$ MeV the model pressure reaches towards the Stefan-Boltzmann limit while the LQCD pressure stays below it.

4. Conclusions

The main result this work is the addition of the strange sector to the relativistic Beth-Uhlenbeck approach which allows to compare with $N_f = 2 + 1$ LQCD data. At the present level of description, there is a discrepancy between our results and the LQCD ones for temperatures

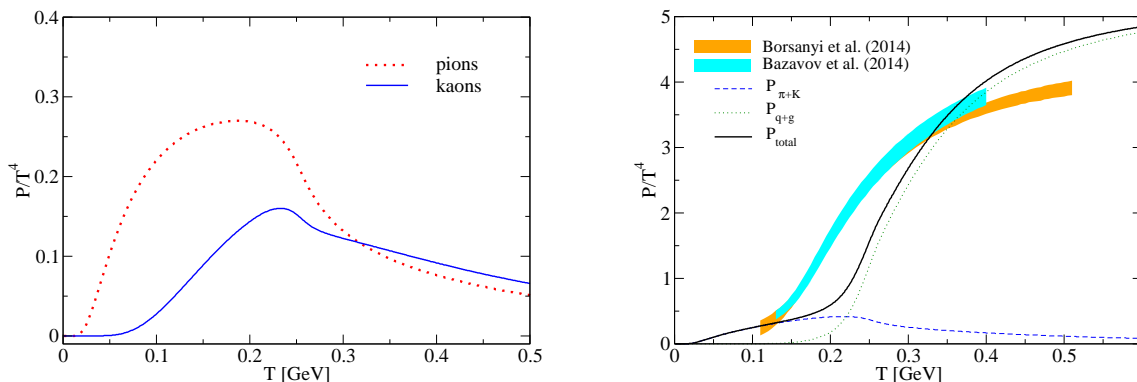


Figure 2. Left panel: pressure of pions (red dotted line) and kaons (blue solid line). Right panel: total pressure of $N_f = 2 + 1$ flavor PNJL model with pseudoscalar meson correlations as a function of temperature (black solid line) and LQCD data (colored bands [13, 14]).

$T \gtrsim 150$ MeV which can be reduced as follows. First, by including higher lying hadronic states into the calculation of the total pressure. Second, by lowering the Mott temperature. Third, by accounting for virial corrections (perturbative) to the quark-gluon pressure corresponding to residual interactions of quarks and gluons in the plasma. These improvements are subject of our current work.

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5. References

- [1] Hüfner J, Klevansky S P, Zhuang P and Voss H 1994 *Annals Phys.* **234** 225
- [2] Blaschke D, Buballa M, Dubinin A, Röpke G and Zablocki D 2014 *Annals Phys.* **348** 228
- [3] Fukushima K 2004 *Phys. Lett. B* **591**, 277
- [4] Ratti C, Thaler M A and Weise W 2006 *Phys. Rev. D* **73**, 014019
- [5] Hansen H, Alberico W M, Beraudo A, Molinari A, Nardi M and Ratti C 2007 *Phys. Rev. D* **75**, 065004
- [6] Radzhabov A, Blaschke D, Buballa M and Volkov M K 2008 *Phys. Atom. Nucl.* **71**, 1981
- [7] Radzhabov A, Blaschke D, Buballa M and Volkov M K 2011 *Phys. Rev. D* **83**, 116004
- [8] Wergieluk A, Blaschke D, Kalinovsky Y L and Friesen A 2013 *Phys. Part. Nucl. Lett.* **10**, 660
- [9] Yamazaki K and Matsui T 2013 *Nucl. Phys. A* **913**, 19
- [10] Yamazaki K and Matsui T 2014 *Nucl. Phys. A* **922**, 237
- [11] Blaschke D, Dubinin A and Buballa M 2015 *Phys. Rev. D* **91**, 125040
- [12] Blaschke D, Dubinin A and Turko L 2015 *Phys. Part. Nucl.* **46**, 732
- [13] Borsanyi A, Fodor Z, Hoelbling C, Katz S D, Krieg S and Szabo K K 2014 *Phys. Lett. B* **730**, 99
- [14] Bazavov A, *et al.* [HotQCD Collaboration] 2014 *Phys. Rev. D* **90**, 094503
- [15] Kaczmarek O, *et al.* 2011 *Phys. Rev. D* **83**, 014504