

Stellar Structure and Evolution With Varying Fundamental Couplings

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Abstract. We discuss the feasibility of using solar-type main-sequence stars as probes of fundamental physics and unification. We use a simple polytropic stellar structure model and study its sensitivity to variations of the gravitational, strong and electroweak coupling constants in the context of unification scenarios. We quantify the sensitivity of the Sun's interior temperature to these variations, finding $|\Delta\alpha/\alpha| < 1.3 \times 10^{-4}$ for a 'canonical' choice of unification scenario, and discuss prospects for future improvements. Further details can be found in [1].

1. Introduction

The advent of computational astrophysics has brought about an arsenal of powerful tools to test fundamental Physics in fairly complex astronomical systems. In particular, these new methods can be specially relevant in the context of the discussion of spacetime-dependent cosmological phenomena.

In this work, such a computational approach is presented for the case of varying fundamental couplings in a solar-type star: describing this star by a simple polytropic model, we are able to assess the sensitivity of the structure of the star to variations of the electromagnetic and gravitational fine-structure constants. The formalism used is valid in a broad class of unification models [1].

2. Phenomenology of Unification

We wish to describe phenomenologically a broad class of models which allow for simultaneous variations of several fundamental couplings, such as the electromagnetic fine-structure constant $\alpha = e^2/(\hbar c)$ and its gravitational analog $\alpha_p = Gm_p^2/(\hbar c)$.

The relations between the couplings are model-dependent. Here we will follow [2], considering a class of grand unification models in which the weak scale is determined by dimensional transmutation and further assuming that relative variation of all the Yukawa couplings is the same. Finally we assume that the variation of the couplings is driven by a dilaton-type scalar field. In this case one finds that the variations of α_p and α are related through

$$\frac{\Delta\alpha_p}{\alpha_p} = [0.8R + 0.2(1 + S)] \frac{\Delta\alpha}{\alpha} \quad (1)$$



where R and S can be taken as free phenomenological parameters. Their absolute value can be anything from order unity to several hundreds, but while R can be positive or negative (with the former case being more likely), physically one expects that $S > 0$. Nevertheless, we can simply treat both as phenomenological parameters to be constrained by astrophysical data.

3. Polytopic Stars

A polytopic star is a very well studied simplified model for the structure of a star in equilibrium, built from the standard equations of continuity and hydrostatic equilibrium as well as a polytopic equation of state, respectively:

$$\frac{dm}{dr} = 4\pi r^2 \rho, \quad \frac{dp}{dr} = -\frac{Gm\rho}{r^2}, \quad p = K\rho^{1+1/n}, \quad (2)$$

where r is the radial distance to the centre of the star, m is the mass within the sphere of radius r (assuming spherical symmetry), ρ is the density, p is the pressure, n is the polytopic index (to be selected), and K is the polytopic constant (to be defined by boundary conditions). The use of this relation avoids the need to include an additional equation for temperature, describing the transport of energy in the interior.

In such a simplified approach the structure of the star is described by the numerical solution of the Lane-Emden Equation, as given by

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\theta}{d\xi} \right) = -\theta^n \quad (3)$$

by making $\theta^n = \rho/\rho_c$ and $\xi = r/a$ where ρ_c is the value of the density at the centre of the star and $a^2 = K(n+1)/(4\pi G\rho_c^{1-1/n})$. The initial conditions that make physical sense are $\theta(\xi=0)=1$ and $\theta'(\xi)=0$. For any $n \in [0, 5]$, there is $\xi_s \geq 6$ such that ξ_s is a zero of $\theta(\xi)$ - in fact, we choose it to be the first such zero. We also define θ'_s to be the derivative of the same function in this point. Then, if we consider a star with total mass M_* and radius R_* (not to be confused with the unification parameter R), we have

$$\rho_c = \left(-\frac{\xi_s}{3\theta'_s} \right) \frac{3M_*}{4\pi R_*^3}, \quad K = \frac{G}{n+1} \left[\frac{4\pi}{\xi_s^{n+1} (-\theta'_s)^{n-1}} \frac{M_*^{n-1}}{R_*^{n-3}} \right]^{1/n}. \quad (4)$$

With this, all of the structural details of our model of a star are defined in terms of n . For a real star, n can be fixed by using its luminosity, L_* , as a boundary condition:

$$L_* = L_n = \frac{M_*}{\xi_s^2 (-\theta'_s)^2} \int_0^{\xi_s} \varepsilon \xi^2 \theta^n d\xi, \quad (5)$$

where ε is the emissivity, or energy production rate per unit mass and time, inside the star. In general, ε is a function of ξ whose expression will depend on the energy production process considered. The version we adopt here are the global fits for the emissivity of the PP chains (relevant for low mass stars, like the Sun or late-type stars).

4. Impact On The Star From α

For the emissivity, ε_{ij} , of the reaction between species i and j , the following family of expressions can be used;

$$\frac{\varepsilon_{ij}}{m_p^{-1} \hbar^1 c^{-1}} = f \sqrt{\frac{8}{3A_{ij}}} \left(\frac{\pi^2 \omega_{ij}}{2} \right)^{1/6} \frac{S_0}{m_p^{-1} \hbar^2} \frac{K^{-2/3} \rho_c^{1-2/3n}}{m_p^4 \hbar^{-3} c^{5/3}} \frac{\alpha^{1/3}}{\theta^{n+2/3}} \exp \left[-3 \left(\frac{\pi^2 \omega_{ij}}{K \rho_c^{1/n} c^{-2}} \frac{\alpha^2}{\theta} \right)^{1/3} \right] \quad (6)$$

where f is the energetic gain of the nuclear reaction in units of $m_p c^2$, A_{ij} is the reduced atomic mass in units of m_p , $w_{ij} = z_i z_j A_{ij}$ where z is the atomic number and S_0 the reference value for the S -factor (the effective cross section). This is the expression that will go into equation (5).

A model of a specific star, when we have (M_*, R_*, L_*) , can be obtained by solving equation (5) together with equation (6), where we impose that $L_n = L_*$. The solution provides the behavior of ρ and p . Any change in the physics will imply a different solution in the interior for the same global parameters. The model accommodates such a change by having a slightly different value of n that fulfills the equation for the luminosity. This approach allows us to obtain quantitative estimates of the relative change of the stellar structure when the underlying physics changes. Although a polytrope is not the best available model of a real star, it is a close enough model for our purposes. The assumption behind this approach is that the relative change from the real star is of the same magnitude as the relative change of the polytropic model, when every other unknown aspect of the physics is not allowed to change.

5. Results and Constraints

From observations of a specific star, the value of its stellar parameters may be obtained, specifically its mass M_* , radius R_* and luminosity L_* . This allows us to build a model, assuming that the set of equations and physics we use are valid, that fits these boundary conditions and describe the structure of the interior. However, if our physics is changed by a small amount, we must find a new model, adjusted to the same boundary conditions, but with a slightly different structure. This approach can also be implemented for polytropic models, which allow us to quantify with great precision the change that one single aspect of the physics can have on the structure of the interior. Specifically, we can determine by how much specific thermodynamic quantities will change in the interior when we introduce a small change in the physical constants entering the basic equations of stellar structure.

In order to have a useful test, we must refer to a quantity that can be obtained from the observation of real stars. The ideal candidate is the value of the temperature near the centre, since this value has a direct effect on sound speed or neutrino production. The former can be expected to be measured with significant precision with asteroseismology (or helioseismology for the solar case) while the latter may be used for the Sun. We do not include the accumulated effect in time, since that requires a proper modeling of the evolution which is beyond the scope of the present work.

Presently, the precision we have for the temperature (or sound speed) in the interior of the Sun, is well below 1%. While far from this, a very high precision is also expected from the asteroseismology of other stars, as is currently being obtained with data from the NASA space mission Kepler.

We thus use the formalism in the previous sections to estimate the impact that variations of α will have in the internal structure of a star for a given unification model (parametrized by specific values of R and S). We consider a polytropic star similar to our Sun (assumed to have a polytropic index n_0), to which will correspond a slightly different value for n . This star will have the same value of M_*/m_p , $R_*/(m_p^{-1} \hbar c^{-1})$, and $L_*/(m_p^2 \hbar^{-1} c^4)$ (M_* , R_* and L_* are the Sun's mass, radius and luminosity) but for a range of values where α differs from the standard value. We then calculate by how much its central temperature (in fact the ratio $K_B T_c / (m_p c^2)$, where T_c is the central temperature) must differ from that of our Sun, by solving equation (6) to determine the value of n .

We have fixed specific values of $\Delta\alpha/\alpha$ and $S = 0$ and let R vary freely. Since we are only considering the PP chains the main sensitivity of the polytropic model will be to changes in α_p (as well as α itself), and therefore we can only hope to constrain a linear combination of R and S . We may then assume $S = 0$ and get a constraint for R , which from equation (1) will in fact be a constraint on $R + S/4$. We have also assumed $R_{\min} \approx -R_{\max}$ in the range we have considered; $10^{-8} < |\Delta\alpha/\alpha| < 10^{-2}$.

This is an excellent approximation except at the top limit where, although $R_{\max}/R_{\min} \approx -0.7$, it is still adequate in the sense that $R_{\max} + R_{\min} \approx 0$. This is not critical because variations of this order are not expected in the light of current experimental constraints.

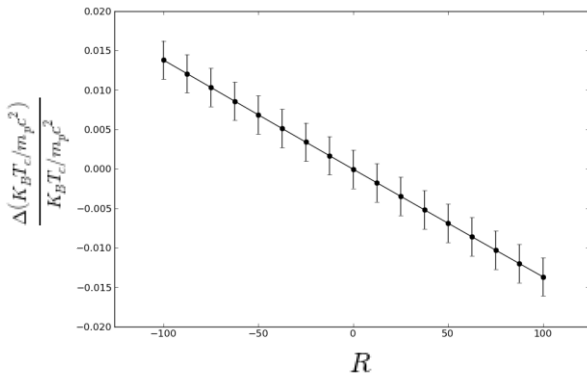


FIG. 1: The relative change of the Sun's central temperature, for $\Delta\alpha/\alpha = -10^{-4}$, as a function of the unification parameter R (with the choice $S=0$). Allowing a maximal variation of one percent yields the limit $|R| < 90$. For a description on how the error bars have been estimated, please see the main text.

In Fig. 1 we show an example for $\Delta\alpha/\alpha = -10^{-4}$, in which case $R_{\max} \approx 90$. The error bars are estimates of the upper limit for the uncertainty in our numerical calculation. It is related to both the error incurred when integrating the Lane-Emden equation in search of ξ_s and θ_s' and the uncertainty associated with finding n . The latter is dominant due to the much greater precision that is possible when locating n numerically the surface of the polytrope.

By repeating the above analysis for different values of α it is possible to identify the region in the (R, S, α) parameter-space which is consistent with the currently estimated experimental uncertainty for the central temperature of our Sun. This is summarized in Fig. 2, and yields our final bound

$$|4R + S| \leq 10^{-1.44} \left| \frac{\Delta\alpha}{\alpha} \right|^{-1} \quad (7)$$

As a simple illustration, if we take the typical values suggested in [2] of $R \approx 30$ and $S \approx 160$, we find $|\Delta\alpha/\alpha| < 1.3 \times 10^{-4}$. This constraint is slightly weaker than the one obtained in [4] for Population III stars, which is not surprising given the limitations of the polytropic model. Moreover, both our constraint and that of [4] are weaker than those obtained spectroscopically along the line of sight of quasars. However, we emphasise that such measurements are obtained in very low density absorption systems, whereas stars are much denser environments. The two types of constraints therefore complement each other for the purpose of constraining models where nature's fundamental couplings are environment-dependent.

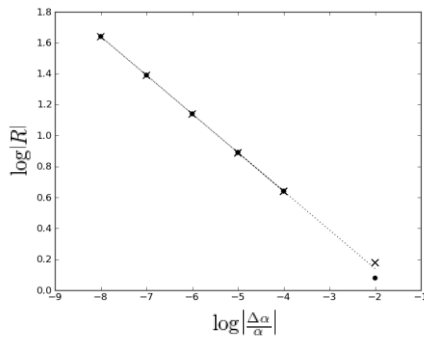


FIG. 2: Bounds on the (α, R) parameter space, for the case $S = 0$. The region above and to the right of the dashed line is excluded by our analysis. The dots (crosses) show the modulus of the estimated values of the maximum (minimum) allowed R for particular choices of α .

References

- [1] J. P. P. Vieira, C. J. A. P. Martins, and M. J. P. F. Monteiro, Phys. Rev. D, 86, 043003_1-043003_6 (2012)
- [2] A. Coc, N. J. Nunes, K. A. Olive, J.-P. Uzan, and E. Vangioni, Phys.Rev. D76, 023511 (2007), astro-ph/0610733
- [3] F. C. Adams, JCAP 0808, 010 (2008), 0807.3697
- [4] S. Ekstrom, A. Coc, P. Descouvemont, G. Meynet, K. A. Olive, et al., A. & A. 514, A62 (2010), 0911.2420