^{12}C+^{12}C fusion in a multichannel folding model

M Assunção[1] and P Descouvemont[2]

[1]Departamento de Ciências Exatas e da Terra, Universidade Federal de São Paulo, 09972-270, Campus Diadema, São Paulo, Brazil
[2]Physique Nucléaire Théorique et Physique Mathématique, C.P. 229, Université Libre de Bruxelles (ULB), B 1050 Brussels, Belgium

Abstract.
The ^{12}C+^{12}C fusion reaction is investigated using a folding potential in a multichannel approach involving the ^{12}C($0^+_1, 2^+, 0^+_2, 3^-$) states. The ^{12}C densities (including transition densities) are taken from the RGM calculation of Kamimura. For the nucleon-nucleon interaction, we use the DDM3Y density-dependent interaction. Owing to the explicit presence of inelastic channels, the imaginary part of the optical potential only contains a short-range fusion contribution. The S-factor is then virtually insensitive to the precise value, and the model is free of any fitting parameter. From the coupled-channel system, we determine the elastic and fusion cross sections simultaneously. As elastic data are available around the Coulomb barrier, this simultaneous treatment offers a good test for the reliability of the model. In the fusion cross section, the role of the inelastic channels and, in particular of the ^{12}C(0^+_1)+^{12}C(0^+_2) channel involving the Hoyle state, is discussed.

1. Introduction

The ^{12}C+^{12}C reaction plays an important role in stellar nucleosynthesis [1], and in particular in the evolution of massive stars [2]. The extrapolation of the S-factor at low energies is made difficult by the presence of resonances whose interpretation is not clear (see a recent review in [2]). A recent work [3] suggests that excitations of ^{12}C are important for a reliable determination of the S factor. Most fusion calculations to date are performed in a single-channel model, i.e. involving the ^{12}C ground-state only, while the absorption is simulated by a phenomenological imaginary potential [4]. In light systems, however, it is known that inelastic channels may be important and require to be explicitly included in the calculation. In Ref. [3], the authors suggest that mutual excitations play an important role even at low energies, where excited channels are closed. At first sight, this effect may seem surprising since only a single channel is open. It is explained by distortion effects in the wave functions: the cross section is mostly sensitive to the inner part of the wave functions, where closed channels may have a significant amplitude.

The aim of our calculation is to investigate the ^{12}C+^{12}C fusion in a multichannel folding method [5]. We include the ^{12}C($0^+_1, 2^+, 0^+_2, 3^-$) states and also the corresponding mutual excitations. A folding method is performed using the density-dependent M3Y (DDM3Y) interaction [7] to describe the nucleon-nucleon (NN) interaction, and ^{12}C densities are taken from the RGM values of Kamimura [6]. These densities (elastic and inelastic) are obtained from a microscopic triple-alpha model, and are known to provide a precise description of many scattering data. In particular, the 0^+_2 state of ^{12}C is well described by the three-α microscopic calculation of Kamimura, and is expected to play a significant role in the ^{12}C+^{12}C system [8].

Another aspect that makes this work even more interesting is that the calculation is free of parameter, except for a weak dependence on the absorption potential. It provides a simultaneous description of elastic scattering and of fusion.

2. Theoretical framework

In a coupled-channel formalism, the ^{12}C+^{12}C potentials are defined as

$$V_{\alpha_1\alpha_2,\alpha_1'\alpha_2'}(\boldsymbol{r}) = \int\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\, v_{NN}(\boldsymbol{r}-\boldsymbol{r}_1+\boldsymbol{r}_2)\,\rho_1^{\alpha_1\alpha_1'}(\boldsymbol{r}_1)\,\rho_2^{\alpha_2\alpha_2'}(\boldsymbol{r}_2), \tag{1}$$

where $v_{NN}(\boldsymbol{r})$ represents the nucleon-nucleon nuclear interaction, $\boldsymbol{r} = (r, \Omega_r)$ is the relative coordinate, $\rho_k^{\alpha\alpha'}(\boldsymbol{r}_k)$ are the ^{12}C nuclear densities, and labels α_k refer to different ^{12}C states. The same formalism is applied to the Coulomb interaction. In the present work, we include $^{12}\mathrm{C}(0_1^+, 2^+, 0_2^+, 3^-)$ states, which means that ten ^{12}C+^{12}C channels are introduced in the coupled-channel system.

In practice, the evaluation of the double integral (1) is performed by using Fourier transforms for the nuclear as well as for the Coulomb interactions [5]. The densities $\rho_k^{\alpha\alpha'}(\boldsymbol{r})$ are taken from the 3α microscopic calculation of Kamimura [6]. These densities, elastic ($\alpha = \alpha'$) as well as inelastic ($\alpha \neq \alpha'$), are known to reproduce many experimental data.

As usual, the densities $\rho_k^{\alpha\alpha'}(\boldsymbol{r})$ are expanded in multipoles [6], and the radial wave functions $g_c^{J\pi}(r)$ are obtained from the coupled-channel system

$$-\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right] g_c^{J\pi}(r) + \sum_{c'} V_{cc'}^{J\pi}(r) g_{c'}^{J\pi}(r) = (E - E_1^c - E_2^c) g_c^{J\pi}(r), \tag{2}$$

where μ is the reduced mass of the system, and E_i^c are the ^{12}C energies. In this equation, label c stands for $c = (\alpha_1, \alpha_2, I, \ell)$, where I is the channel spin and ℓ the relative orbital momentum.

In this approach, the absorption to other channels is simulated by adding an imaginary component to the potentials as (see Ref.[12] for details)

$$V_{cc'}^{J\pi}(r) \longrightarrow V_{cc'}^{J\pi}(r) + iW_{cc'}^{J\pi}(r). \tag{3}$$

In the present calculation, the coupled-channel system (2) explicitly includes inelastic channels in a wide energy range. To define the fusion component of the potential, a short-range absorption potential [11] is included as

$$W_{cc'}(r) = -\frac{W_0}{1+\exp((r-R_0)/a)}\delta_{cc'}. \tag{4}$$

The range R_0 is chosen smaller than the barrier radius, and this potential acts at short distances only. In this multichannel calculation, the values were taken as $W_0 = 10$ MeV, $R_0 = 3$ fm, and $a = 0.1$ fm, and the same conditions were employed to investigate the elastic-scattering and fusion processes. The calculation are stable within $1-2\%$ when these parameters are modified. An important consequence is that the model is free of parameters, and that all cross sections are obtained without any adjustment.

The coupled-channel system (2) is solved with the R-matrix method [9]. In the internal region ($r \leq a$), the radial functions $g_c^{J\pi}(r)$ are expanded over a Lagrange basis [10]. In the external region, they are given by linear combinations of Coulomb functions. The matching provides the collision matrix $\boldsymbol{U}^{J\pi}$. Notice that, at low energies, most of ^{12}C+^{12}C excited channels are closed (the first open channel is $^{12}\mathrm{C}(0_1^+)$+$^{12}\mathrm{C}(2^+)$ which opens at 4.44 MeV).

The elastic cross sections are computed from the collision matrices by using standard formulas [4]. The fusion cross section is defined as [13]

$$\sigma_F(E) = \frac{2\pi}{k^2} \sum_{J \text{ even}} (2J+1) P_J(E), \tag{5}$$

where k is the wave number, and where the fusion probability $P_J(E)$ is obtained from

$$P_J(E) = -\frac{2}{\hbar v} \sum_c \int |g_c^{J\pi}(r)|^2 \, W_{cc}(r) \, dr, \tag{6}$$

where v is the relative velocity [4].

At low energies, the fusion and reaction cross sections are identical, and $P_J(E)$ can be expressed as

$$P_J(E) = 1 - |U_{11}^J|^2, \tag{7}$$

where U_{11}^J is the elastic element of the collision matrix, associated with the ^{12}C+^{12}C ground-state channel.

For the ^{12}C+^{12}C reaction, the fusion cross section is traditionally converted in a modified S factor as

$$\tilde{S}(E) = \sigma_F(E) E \exp(2\pi\eta + 0.46\, E), \tag{8}$$

where η is the Sommerfeld parameter. The linear term in the exponential accounts for an additional energy dependence (E is expressed in MeV).

3. Results and Discussions

The present folding model was first applied to ^{12}C+^{12}C elastic scattering at energies around the Coulomb barrier, where experimental data are available [14]. Thus, the idea was to assess the accuracy of the model, which is well known experimentally at energies close to the Coulomb barrier [14], and then apply it to the calculation of fusion cross sections. The procedure was started from a single-channel approximation, and we progressively included additional channels.

The comparison between theory and experiment is presented in Fig.1. When the energy increases, and in particular at $E = 10$ MeV, inelastic channels significantly improve the theoretical cross section. At 6 MeV, the physics of the problem is essentially determined by the Coulomb interaction, and the role of the inelastic channels is hardly visible. The most sensitive angular range is beyond $\theta = 70°$, where the single-channel approximation provides a poor fit of the data. Including the 2^+ state improves the overall agreement, but adding further the 0_2^+ Hoyle state provides an excellent agreement with the data. Note that good fits can be obtained even in the single-channel approximation [15], but after fitting the imaginary potential to optimize the agreement with experiment. The same behavior is observed at higher energies, i. e., the inclusion of the Hoyle state in the calculation significantly improves the agreement with experiment (see Figure in Ref. [16]). Most likely, breakup channels start playing a role at these energies, and the imaginary potential should be adapted.

The modified S factor is displayed in Fig.2, where the experimental data have been corrected as suggested by Aguilera *et al.* [25]. Above the Coulomb barrier (≈ 6.5 MeV) the data are well reproduced by the calculation, and the role of inelastic channels is minor. When the energy decreases, the sensitivity with respect to the number of excited channels is more and more important, in agreement with Ref.[3]. At $E = 1$ MeV, the multichannel calculation provides an enhancement of about a factor of three, in comparison with the single-channel approach.

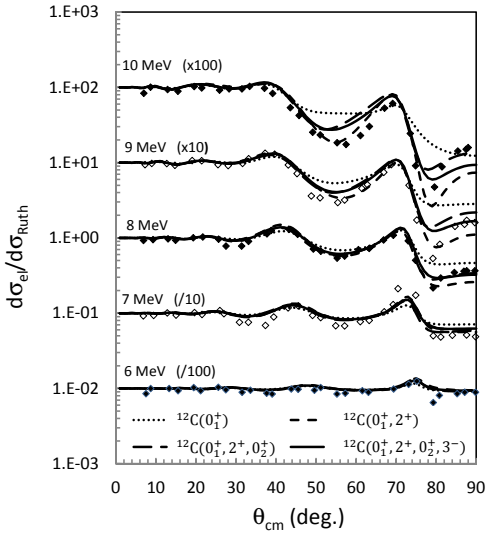

Figure 1. Ratios of the elastic and Rutherford cross sections around the Coulomb barrier, for increasing numbers of ^{12}C+^{12}C inelastic channels. Labels correspond to c. m. energies. Experimental data are from Ref.[14].

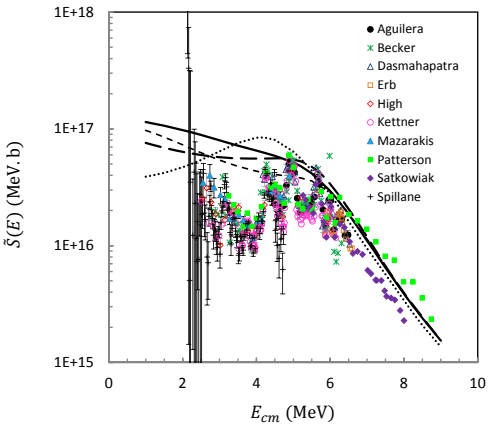

Figure 2. (Color online). Modified S factor (8) for increasing numbers of ^{12}C+^{12}C inelastic channels (the curves are as in Fig.1). Experimental data are taken from Refs.[17–26].

Of course, fluctuations are absent from the present theory. Although molecular resonances are predicted by the calculation with a real potential, they are strongly hindered by the absorption part of the potential.

The interpretation of the theoretical S factor can be seen in Fig.3 through a decomposition in angular momenta J (upper panel) and in the various channels (lower panel). The fusion cross section is essentially given by the contribution of $J = 0^+$ and $J = 2^+$; $J = 4^+$ provides less than 10 %, and other partial waves are negligible. The contributions of the different channels confirm that the fusion cross sections are strongly affected by inelastic channels. These channels are closed at low energies, but the corresponding wave functions $g_c^{J\pi}(r)$ have a significant amplitude in the inner region. Even if they tend to zero at large distances, the short-range potential $W(r)$ makes integrals (6) sensitive to the inner part of the wave function only. Consequently the contribution of inelastic channels to the fusion cross section (5) may be important, and even larger than the ground-state contribution. The role of the Hoyle state is supported by the

importance of the $^{12}C(0_1^+)+^{12}C(0_2^+)$ channel, which is even dominant above 3.5 MeV.

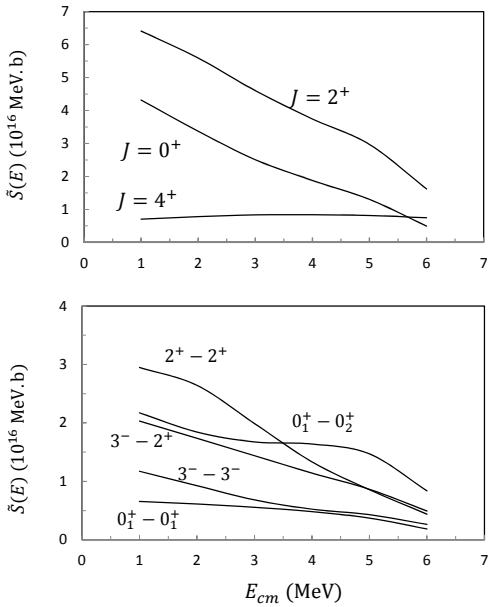

Figure 3. Decompositions of the modified S factor (8) in partial waves (upper panel), and in the main channel contributions (lower panel).

4. Conclusion

In the present work, the $^{12}C+^{12}C$ fusion process was investigated in a multichannel model. The coupling potentials are generated from ^{12}C densities obtained in a microscopic cluster model. The calculation does not contain any fitting parameter, and provides simultaneously the fusion and elastic cross sections. Around the Coulomb barrier the elastic data are well reproduced by the model provided that all inelastic channels, and in particular those involving the 0_2^+ state, are included. The results confirm the conclusion of Ref.[3], i.e. that inelastic channels play an important role, and must be taken into account for a precise description of the fusion cross section.

Acknowledgments

We are grateful to Prof. M.S. Hussein for useful discussions, and to Profs. M. Kamimura and F. Strieder for providing us with the ^{12}C densities, and with the data of Ref.[26]. We acknowledge FAPESP for financial support. This text presents research results of the IAP programme P7/12 initiated by the Belgian State Federal Services for Scientific, Technical and Cultural Affairs.

References

[1] Wiescher M, Käppeler F, Langanke K 2012 *Annu. Rev. Astron. Astrophys* **50** 165
[2] Pignatari M, *et al.* 2013 *The Astrophysical Journal* **762** 31
[3] Esbensen H, Tang X and Jiang C L 2011 *Phys. Rev. C* **84** 064613
[4] Canto F and M. S. Hussein M S 2013 *Scattering Theory of Molecules, Atoms and Nuclei*, *World Scientific Publishing*
[5] Satchler G R and Love W G 1979 *Phys. Rep.* **55C** 183
[6] Kamimura M 1981 *Nucl. Phys. A* **351** 456
[7] Kobos A, *et al.* 1982 *Nucl. Phys. A* **384** 65
[8] Ito M, Y.Sakuragi Y, Hirabayashi Y 2001 *Phys. Rev. C* **63** 064303
[9] Descouvemont P and Baye D 2010 *Rep. Prog. Phys.* **73** 036301

[10] Baye D 2006 *Phys. Stat. Sol. (b)* **243** 1095
[11] Rhoades-Brown M, Braun-Munzinger P 1984 *Phys. Lett. B.* **136** 19
[12] T. Khoa D T and Satchler G 2000 *Nucl. Phys. A* **668** 3
[13] F. Canto L F,*et al.* 2006 *Phys. Rep.* **424** 1
[14] Treu W, *et al.* 1980 *Phys. Rev. C* **22** 2462
[15] Gasques L R, *et al.* 2005 *Phys. Rev. C* **72** 025806
[16] Assunção M and Descouvemont P 2013 *Phys. Lett. B* **723** 355
[17] Patterson J R and Zaidinis S 1969 *Astrophys. J.* **157** 367
[18] Mazarakis M G and W. E. Stephens W E 1973 *Phys. Rev. C* **7** 1280
[19] High M and Cujec B 1977 *Nucl. Phys. A* **282** 181
[20] Erb K A, *et al.* 1980 *Phys. Rev. C* **22** 507
[21] Kettner K, Lorenz-Wirzba H and Rolfs 1980 *Z. Phys. A* **298** 65
[22] Becker H, *et al.* 1981 *Z. Phys. A* **303** 305
[23] Dasmahapatra B, Cujec B and Lahlou F 1982 *Nucl. Phys. A* **384** 257
[24] Satkowiak L J, *et al.* 1982 *Phys. Rev. C* **26** 2027
[25] Aguilera E F, *et al.* 2006 *Phys. Rev. C* **73** 064601
[26] Spillane T, *et al.* 2007 *Phys. Rev. Lett.* **98** 122501