

# Modified generalized Chaplygin gas model in Bianchi type-V space-time geometry with dynamical $G$ and $\Lambda$

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**Abstract.** In this paper a new set of exact solutions of Einsteins field equations have been obtained for Bianchi type V space-time geometry. The first part of the paper deals with perfect fluid cosmological models in the presence of variable gravitational and cosmological constants. The second part of the paper is devoted to study the modified generalized Chaplygin gas model. The physical and dynamical behaviors of the models have been discussed with the help of graphical representation.

## 1. Introduction

It has been well established in the literature [1–5] that the universe is currently accelerating. However, final satisfactory explanation about physical mechanism and driving force of accelerated expansion of the universe is yet to achieve. To understand the accelerating behavior of the universe, role of varying cosmological constant is also considered as very important. The current interest in stems mainly from observations of type Ia high red shift supernovae which indicate that the universe is accelerating fueled perhaps by a small cosmological - term [6]. Cosmological constant problem and consequences on cosmology with a time varying cosmological constant are investigated by some researchers [7–11].

A variation of  $G$  has many interesting consequences both in geology and astrophysics. Canuto and Narlikar [12] have shown that the  $G$  varying cosmology is consistent with what so ever cosmological observations presently available. Singh and Kotambkar [13, 14] have discussed cosmological models with  $G$  and  $\Lambda$  in higher dimensional space-time. Vishwakarma [15] has investigated a Bianchi type I model with variable  $G$  and  $\Lambda$ . Singh et. al. [16] have studied a new class of cosmological models with variable  $G$  and  $\Lambda$ . Singh and Kale [17] have discussed anisotropic bulk viscous cosmological models with variable  $G$  and  $\Lambda$ .

The observed universe is homogeneous and isotropic, hence space- time is usually described by Fridman- Lemaitra-Robertson-Walkar (FLRW) cosmology. But it is widely believed that FLRW model does not give a correct matter description in the early stage of universe. The inconsistencies found in the cosmic microwaves background (CMB) and the large structure observations are stimulations to grow interest in the study of anisotropic cosmological model of the universe. Recent experimental data support the existence of an anisotropic phase, which turns in to an isotropic one during the evolution of the universe. A number of authors have investigated Bianchi type V cosmological model in general relativity



in different context [18–21]. Recently Yadav et. al. [22], Yadav and Sharma [23] have discussed about transit universe in Bianchi type V space-time with variable  $G$  and  $\Lambda$ .

There is mounting evidence from supernovae [24], WMAP [25–29], BAO oscillation data that the universe at present is dominated by a smooth component with negative pressure, the so called dark energy, leading to accelerated expansion. To avoid the problems associated with cosmological models with and quintessence models, recently it has been shown that Chaplygin gas (CG) may be useful for describing dark energy because of its positive energy density and negative pressure. Due to effectiveness of CG in explaining the evolution of the universe, several generalization of Chaplygin gas have been proposed in the literature [30,32]. The generalized Chaplygin gas (GCG), described by equation of state

$$p = \frac{-B}{\rho^\alpha}$$

is considered, where constants  $B$  and  $\alpha$  satisfy respectively  $B > 0$  and  $0 < \alpha \leq 1$  [33, 34]. State finder diagnostic for modified Chaplygin gas in Bianchi type V universe has been discussed by Adhav [35]. Number of researchers investigated cosmological models with modified generalized Chaplygin gas [36–39]. Observational constraints on modified Chaplygin gas in Horava-Lifschitz gravity with dark radiation have been studied by Paul et. al. [40].

## 2. Field equations

The Einstein field equation with time dependent  $G$  and  $\Lambda$  may be written as

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi GT_i^j + \Lambda g_i^j. \quad (1)$$

The energy momentum tensor of cosmic fluid is given by

$$T_i^j = (\rho + p)u_i u^j - p g_i^j, \quad (2)$$

where  $\rho$  is the energy density,  $p$  represents equilibrium pressure and  $u^i$  is the fluid four velocity of the fluid satisfying the condition  $u_i u^i = 1$ .

The spatially homogeneous and anisotropic Bianchi type V space-time metric is given by

$$ds^2 = dt^2 - R_1^2(t)dx^2 - R_2^2(t)e^{2kx}dy^2 - R_3^2(t)e^{2kx}dz^2. \quad (3)$$

The Einstein field equation (1) and the energy momentum tensor (2) for the space-time metric (3) yield the following equations:

$$\frac{\dot{R}_1}{R_1} \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_2}{R_2} \frac{\dot{R}_3}{R_3} + \frac{\dot{R}_3}{R_3} \frac{\dot{R}_1}{R_1} - \frac{3k^2}{R_1^2} = 8\pi G\rho + \Lambda. \quad (4)$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1}{R_1} \frac{\dot{R}_2}{R_2} - \frac{k^2}{R_1^2} = -8\pi G\rho + \Lambda. \quad (5)$$

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2}{R_2} \frac{\dot{R}_3}{R_3} - \frac{k^2}{R_1^2} = -8\pi G\rho + \Lambda. \quad (6)$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1} \frac{\dot{R}_3}{R_3} - \frac{k^2}{R_1^2} = -8\pi G\rho + \Lambda. \quad (7)$$

$$2\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = 0. \quad (8)$$

By combining equations (4)-(7) one can easily obtain the continuity equation as

$$\dot{\rho} + (\rho + p) \left( \frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) + \rho \frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0. \quad (9)$$

The energy momentum conservation equation ( $T_{ij}^{ij}$ ) suggests

$$\dot{\rho} + 3(\rho + p)H = 0, \text{ where Hubble parameter } H = \frac{1}{3} \left( \frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right). \quad (10)$$

Equations (9) and (10) implies

$$\rho \frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0. \quad (11)$$

The physical quantities of observational interest viz. the expansion scalar  $\Theta$ , shear scalar  $\sigma^2$ , the relative anisotropy and deceleration parameter  $q$  are defined by

$$\Theta = 3H = \frac{\dot{V}}{V}, \text{ (Volume } V = R_1 R_2 R_3), \quad (12)$$

$$\sigma^2 = \frac{1}{2} \left[ \left( \frac{\dot{R}_1}{R_1} \right)^2 + \left( \frac{\dot{R}_2}{R_2} \right)^2 + \left( \frac{\dot{R}_3}{R_3} \right)^2 \right] - \frac{\Theta^2}{6}, \quad (13)$$

$$\text{Relative anisotropy} = \frac{\sigma^2}{\rho}, \quad (14)$$

$$q = -1 - \frac{\dot{H}}{H^2}. \quad (15)$$

### 3. Solutions of field equations

It can be easily seen that the system have five equations from equation (4)-(8) with seven unknowns  $R_1$ ,  $R_2$ ,  $R_3$ ,  $\rho$ ,  $p$ ,  $G$  and  $\Lambda$ . Hence in order to obtain complete set of exact solutions, two additional physically plausible relations among these variables are required. Equation (5) and equation (7) suggest a relation between scale factors  $R_2$  and  $R_3$  as

$$\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = \frac{k}{(R_2 R_3)^{\frac{3}{2}}}. \quad (16)$$

We consider the ansatz

$$R_1 = t^\alpha e^{t^\beta}. \quad (17)$$

Here  $\alpha$  and  $\beta$  are constants can be determined on the basis of observational limits on cosmological parameters.

Under this condition equation (8) may be written as

$$\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = 2 \left( \frac{\alpha}{t} + \beta t^{\beta-1} \right). \quad (18)$$

With help of equations (8) and (16) and (17) takes the form

$$\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = \frac{k}{(t^\alpha e^{t^\beta})^3}. \quad (19)$$

From equations (18) and (19), one can easily obtain

$$R_2 = C_1 t^\alpha e^{t^\beta} \exp\left(\int \frac{k}{2(t^\alpha e^{t^\beta})^3} dt\right) \quad (20)$$

$$R_3 = C_2 t^\alpha e^{t^\beta} \exp\left(\int \frac{-k}{2(t^\alpha e^{t^\beta})^3} dt\right) \quad (21)$$

By use of the aforesaid equations, we have computed expressions for physical parameters viz. spatial volume  $V$ , Hubble parameter  $H$ , expansion scalar  $\Theta$ , shear scalar  $\sigma^2$  and deceleration parameter  $q$

$$V = C t^{3\alpha} e^{3t^\beta}, \quad H = \frac{\alpha}{t} + \beta t^{\beta-1}, \quad \Theta = 3\left(\frac{\alpha}{t} + \beta t^{\beta-1}\right), \quad \sigma^2 = \frac{k^2}{4(t^\alpha e^{t^\beta})^6}, \quad q = -1 - \frac{[-\alpha + \beta(\beta-1)t^\beta]}{(\alpha + \beta t^\beta)^2}, \quad (22)$$

where  $C = C_1 C_2$ . The model has a singularity at the origin of the universe. The expression for expansion scalar  $\Theta$  clearly shows that for all  $\beta < 1$ , expansion ceases and shear dies out for when cosmic time  $t$  takes very large value. If  $\beta > 1$  the universe expands forever and for all  $\alpha > 1, \beta > \frac{1}{2}$  cosmological model shows accelerated expansion of the universe.

### 3.1. Case I

In this case we consider the equation of state connecting the pressure and the energy density as

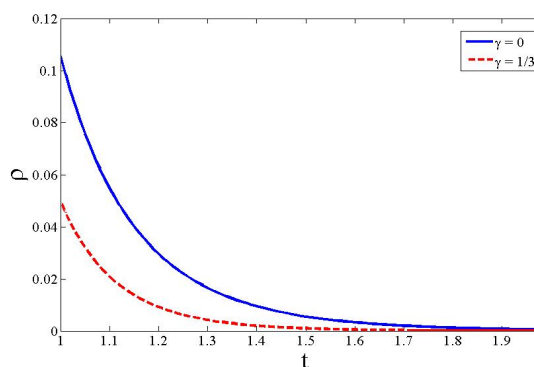
$$p = \gamma \rho, \quad 0 \leq \gamma \leq 1, \quad (23)$$

$\gamma = 0, \gamma = \frac{1}{3}$  and  $\gamma = 1$  represents dust model, radiation dominated model and matter dominated model respectively. Using equations (10), (22) and (23), we can obtain the expression for energy density and by use of equations (4)-(5), (17), (21)-(22) we can obtain expression for gravitational constant as

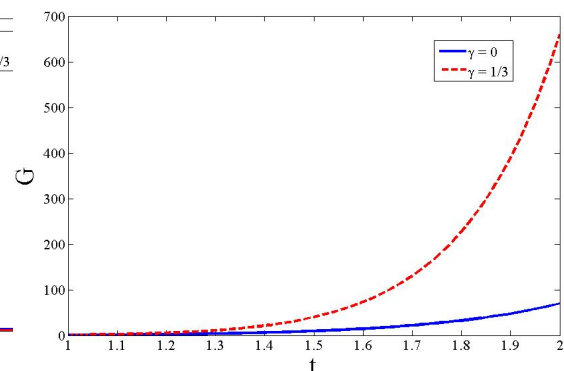
$$\rho = \frac{D}{(t^\alpha e^{t^\beta})^{3(1+\gamma)}}, \quad (24)$$

where  $D$  is a constant of integration, and

$$G = \frac{-1}{8\pi D(1+\gamma)} \left[ \frac{k^2}{2} (t^\alpha e^{t^\beta})^{3(\gamma-1)} + 2\beta(\beta-1)t^{\beta-2} (t^\alpha e^{t^\beta})^{3(1+\gamma)} + 2k^2 (t^\alpha e^{t^\beta})^{3\gamma+1} - \frac{2\alpha}{t^2} (t^\alpha e^{t^\beta})^{3(1+\gamma)} \right]. \quad (25)$$



**Figure 1.** Variation of energy density with cosmic time  $t$ .



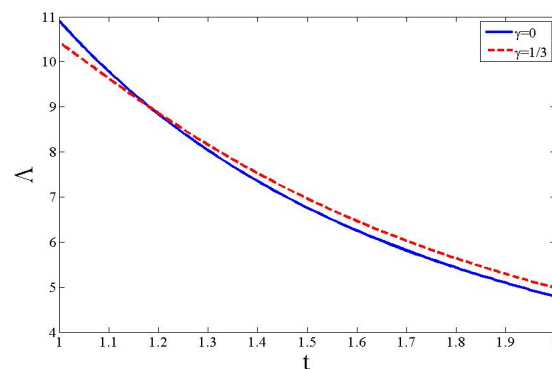
**Figure 2.** Variation of gravitational constant with cosmic time  $t$ .

By use of equations (22) and (24) expression (14) for relative anisotropy takes the form

$$\text{Relative anisotropy} = \frac{k^2}{4D(t^\alpha e^{t^\beta})^{3(1-\gamma)}}. \quad (26)$$

Using equations (5), (23) and (25), we get the expression for cosmological constant as

$$\Lambda = 3 \left( \frac{\alpha}{t} + \beta t^{\beta-1} \right)^2 + \frac{1}{1+\gamma} \left[ 2\beta(\beta-1)t^{\beta-2} - \frac{2\alpha}{t^2} + \frac{k^2}{4}(1-\gamma)(t^\alpha e^{t^\beta})^{-6} - k^2(3\gamma+1)(t^\alpha e^{t^\beta})^{-2} \right]. \quad (27)$$



**Figure 3.** Variation of Cosmological “constant” with cosmic time  $t$ .

Anisotropy parameter of expansion is given by

$$\Delta \equiv \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2 = D_1 (t^\alpha e^{t^\beta})^{-6} \left( \frac{\alpha}{t} + \beta t^{\beta-1} \right)^{-2}, \text{ where } D_1 = \frac{k^2}{6}.$$

The state finder parameters for this model takes the form

$$r = \frac{\ddot{R}}{RH^3} = \frac{(\alpha^3 - 3\alpha^3 + 2\alpha) + [3\alpha\beta(\alpha + \beta - 2) + \beta(\beta^2 - 3\beta + 2)]t^\beta + 3\beta^2(\alpha + \beta - 1)t^{2\beta} + \beta^3 t^{3\beta}}{(\alpha + \beta t^\beta)^3}, \quad (28)$$

$$s = \frac{r-1}{3(q-\frac{1}{2})} = \frac{2\alpha \left[ (\alpha^3 - 3\alpha^3 + 2\alpha) + [3\alpha\beta(\alpha + \beta - 2) + \beta(\beta^2 - 3\beta + 2)]t^\beta + 3\beta^2(\alpha + \beta - 1)t^{2\beta} + \beta^3 t^{3\beta} - (\alpha + \beta t^\beta)^3 \right]}{(6-9\alpha)(\alpha + \beta t^\beta)^3}. \quad (29)$$

The critical energy density and the vacuum energy density are respectively given by

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{-3D(1+\gamma)(\alpha + \beta t^\beta)^2}{(t^\alpha e^{t^\beta})^{3(1+\alpha)} \left[ -2\alpha + 2k^2 t^2 (t^\alpha e^{t^\beta})^{-2} + \frac{k^2 t^2}{2} (t^\alpha e^{t^\beta})^{-6} + 2\beta(\beta-1)t^\beta \right]}. \quad (30)$$

$$\rho_v = \frac{3(\alpha + \beta t^\beta)^2 + \frac{1}{1+\gamma} \left( -2\alpha + 2\beta(\beta-1)t^\beta + \frac{k^2 t^2 (1-\gamma)}{4} (t^\alpha e^{t^\beta})^{-6} + k^2(3\gamma+1)t^2 (t^\alpha e^{t^\beta})^{-2} \right)}{\frac{-1}{D(\gamma+1)} (t^\alpha e^{t^\beta})^{3(1+\gamma)} \left[ -2\alpha + 2k^2 t^2 (t^\alpha e^{t^\beta})^{-2} + \frac{k^2 t^2}{2} (t^\alpha e^{t^\beta})^{-6} + 2\beta(\beta-1)t^\beta \right]}. \quad (31)$$

Mass density parameter and density parameter of the vacuum are given by

$$\Omega_M = \frac{\rho}{\rho_c} = \frac{-1}{3(1+\gamma)(\alpha + \beta t^\beta)^2 \left[ \frac{k^2 t^2}{2} (t^\alpha e^{t^\beta})^{-6} + 2k^2 t^2 (t^\alpha e^{t^\beta})^{-2} - 2\alpha + 2\beta(\beta-1)t^\beta \right]}. \quad (32)$$

$$\Omega_\Lambda = \frac{\rho_v}{\rho_c} = 1 + \frac{-1}{3(\alpha + \beta t^\beta)^2} \left[ \frac{1}{1+\gamma} \left( \frac{k^2(\gamma-1)t^2}{4} (t^\alpha e^{t^\beta})^{-6} - k^2(3\gamma+1)t^2 (t^\alpha e^{t^\beta})^{-2} + 2\beta(\beta-1)t^\beta - 2\alpha \right) \right]. \quad (33)$$

From equation (30) to (33) one can see that the critical energy density, critical vacuum energy density, mass density parameter and density parameter of vacuum are decreasing with evolution of the universe.

### 3.2. Case II

It is known that ordinary matter fields available from standard model of particle physics in general relativity, fails to account the present observation. Therefore modifications of the matter sectors of the Einstein-Hilbert action with exotic matter is considered in the literature, Chaplygin gas is considered to be one of the such candidates for dark energy. The generalized Chaplygin gas equation is given by  $p = -\frac{B}{\rho^a}$ ,  $0 < a \leq 1$ . As we can see from equation of GCG is corresponding to almost dust at high density which is not agreeing completely with present universe. Therefore in this case we consider modified Chaplygin gas (MCG) with the following equation of state

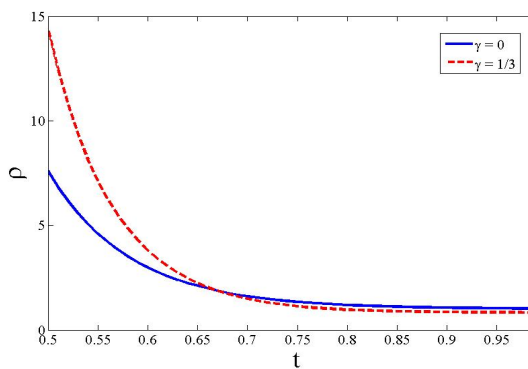
$$p = \gamma\rho - \frac{B}{\rho^a}, \quad (34)$$

where  $\gamma$  is positive constant. To have constant negative pressure at low energy density and high pressure at high energy density, MCG model is appropriate choice. By use of equations (10), (17) and (34) one can obtain energy density of MCG and by use of equations (4)-(5), (17), (20)-(21), (34) one can obtain gravitational constant of MCG as,

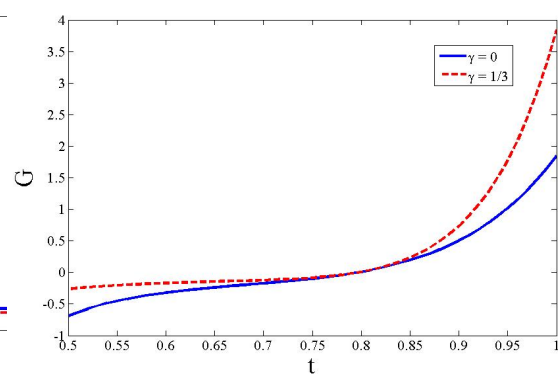
$$\rho = \left[ \rho_0 + C(t^\alpha e^{t^\beta})^{-3n} \right]^{\frac{1}{a+1}}, \quad (35)$$

where  $\rho_0 = \frac{B}{\gamma+1}$ ,  $n = (\gamma+1)(a+1)$  and  $C$  is a constant of integration.

$$G = \frac{(t^\alpha e^{t^\beta})^{3n}}{8\pi C(\gamma+1)} \left[ \rho_0 + C(t^\alpha e^{t^\beta})^{-3n} \right]^{\frac{a}{a+1}} \left[ \frac{2\alpha}{t^2} - 2\beta(\beta-1)t^{\beta-2} - \frac{k^2}{2} (t^\alpha e^{t^\beta})^{-6} - 2k^2 (t^\alpha e^{t^\beta})^{-2} \right]. \quad (36)$$



**Figure 4.** Variation of energy density with cosmic time  $t$ .

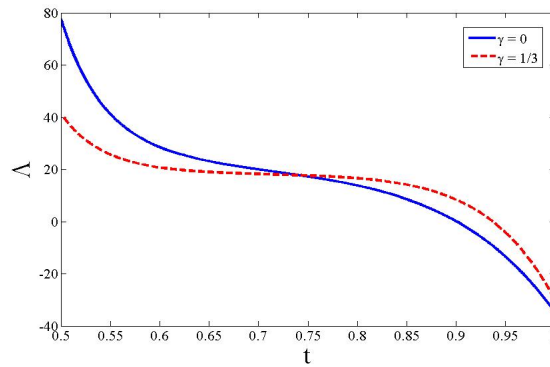


**Figure 5.** Variation of gravitational constant with cosmic time  $t$ .

Using equations (5), (35) and (36) we get the expression for cosmological constant as

$$\Lambda = 3 \left( \frac{\alpha}{t} + \beta t^{\beta-1} \right)^2 + \frac{2M_1}{t^2} (t^\alpha e^{t^\beta})^{3n} \left\{ \alpha - \beta(\beta-1)t^\beta - \frac{k^2 t^2}{4} (t^\alpha e^{t^\beta})^{-6} - k^2 t^2 (t^\alpha e^{t^\beta})^{-2} \right\} \\ + \frac{2}{(\gamma+1)t^2} \left\{ -\alpha + \beta(\beta-1)t^\beta + \frac{k^2 t^2 (1-\gamma)}{8} (t^\alpha e^{t^\beta})^{-6} - \frac{k^2 t^2 (3\gamma+1)}{2} (t^\alpha e^{t^\beta})^{-2} \right\}, \quad (37)$$

where  $M_1 = \frac{\gamma \rho_0 - B}{C(\gamma+1)}$ .



**Figure 6.** Variation of Cosmological “constant” with cosmic time  $t$ .

The critical energy density and the vacuum energy density  $\rho_c$  and  $\rho_v$  in this case are given by

$$\rho_c = \frac{3C(\gamma+1)(\alpha + \beta t^\beta)^2}{(t^\alpha e^{t^\beta})^{3n} [\rho_0 + C(t^\alpha e^{t^\beta})^{-3n}]^{\frac{a}{a+1}} [2\alpha - 2\beta(\beta-1)t^\beta - k^2 t^2 (t^\alpha e^{t^\beta})^{-6} - 2k^2 t^2 (t^\alpha e^{t^\beta})^{-2}]}, \quad (38)$$

$$\rho_v = \frac{3(\alpha + \beta t^\beta)^2 + 2M_1(t^\alpha e^{t^\beta})^{3n} \left\{ \alpha - \beta(\beta-1)t^\beta - \frac{k^2 t^2}{4} (t^\alpha e^{t^\beta})^{-6} - k^2 t^2 (t^\alpha e^{t^\beta})^{-2} \right\} \\ + \frac{2}{\gamma+1} \left\{ -\alpha + \beta(\beta-1)t^\beta + \frac{k^2 t^2 (1-\gamma)}{8} (t^\alpha e^{t^\beta})^{-6} - \frac{k^2 t^2 (3\gamma+1)}{2} (t^\alpha e^{t^\beta})^{-2} \right\}}{\frac{1}{C(\gamma+1)} (t^\alpha e^{t^\beta})^{3n} [\rho_0 + C(t^\alpha e^{t^\beta})^{-3n}]^{\frac{a}{a+1}} [2\alpha - 2\beta(\beta-1)t^\beta - \frac{k^2 t^2}{2} (t^\alpha e^{t^\beta})^{-6} - 2k^2 t^2 (t^\alpha e^{t^\beta})^{-2}]}, \quad (39)$$

whereas mass density parameter and the density parameter of the vacuum are given by

$$\Omega_M = \frac{1}{3C(\gamma+1)(\alpha + \beta t^\beta)^2} [\rho_0 (t^\alpha e^{t^\beta})^{3n} + C] \left[ 2\alpha - 2\beta(\beta-1)t^\beta - \frac{k^2 t^2}{2} (t^\alpha e^{t^\beta})^{-6} - 2k^2 t^2 (t^\alpha e^{t^\beta})^{-2} \right], \quad (40)$$

$$\Omega_\Lambda = 1 + \frac{1}{3(\alpha + \beta t^\beta)^2} \left[ 2M_1(t^\alpha e^{t^\beta})^{3n} \left\{ \alpha - \beta(\beta-1)t^\beta - \frac{k^2 t^2}{4} (t^\alpha e^{t^\beta})^{-6} - k^2 t^2 (t^\alpha e^{t^\beta})^{-2} \right\} \right. \\ \left. + \frac{2}{\gamma+1} \left\{ -\alpha + \beta(\beta-1)t^\beta + \frac{k^2 t^2 (1-\gamma)}{8} (t^\alpha e^{t^\beta})^{-6} - \frac{k^2 t^2 (3\gamma+1)}{2} (t^\alpha e^{t^\beta})^{-2} \right\} \right]. \quad (41)$$

From equation (38) to (41) one can see that the critical energy density, vacuum energy density, mass density parameter and density parameter of vacuum are decreasing with time.

#### 4. Discussion

In this paper we have discussed Bianchi Type V space-time model with equilibrium pressure and modified generalized Chaplygin gas. The energy density  $\rho$  and cosmological constant  $\Lambda$  are decreasing with evolution of the universe whereas  $G$  becomes infinite as  $t \rightarrow \infty$  in all the models. If  $\beta > 1$  the universe expands forever and for  $\frac{1}{2} < \beta < 1$  the universe expansion vanishes when  $t \rightarrow \infty$ . Shear dies out with evolution of the universe for large value of  $t$ . For all  $\alpha > 1, \beta > \frac{1}{2}$  equation (30) shows that present cosmological model suggests accelerated expansion of the universe. Considering present day limit for deceleration parameter  $q = -0.53^{+0.17}_{-0.13}$ , equation (30) suggests  $0.6097 < \beta < 0.7462$ . In order to have clear idea of variation in behavior of cosmological parameters relevant graphs have been plotted. In first case for  $\gamma = 0$  energy density is rapidly decreasing than for  $\gamma = \frac{1}{3}$  whereas  $G$  is increasing slowly for  $\gamma = 0$  than for  $\gamma = \frac{1}{3}$  which is in fair agreement with observations. In both the cases cosmological constant is decreasing fastly for  $\gamma = 0$  than for  $\gamma = \frac{1}{3}$  which goes with observations.

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