Mathematical modeling and multi-criteria optimization of rotary electrical discharge machining process

U Shrinivas Balraj

Department of Mechanical Engineering,
Kakatiya Institute of Technology and Science, Warangal, India.

E-mail: u.shrinivas@gmail.com

Abstract. In this paper, mathematical modeling of three performance characteristics namely material removal rate, surface roughness and electrode wear rate in rotary electrical discharge machining RENE80 nickel super alloy is done using regression approach. The parameters considered are peak current, pulse on time, pulse off time and electrode rotational speed. The regression approach is very much effective in mathematical modeling when the performance characteristic is influenced by many variables. The modeling of these characteristics is helpful in predicting the performance under a given set of combination of input process parameters. The adequacy of developed models is tested by correlation coefficient and Analysis of Variance. It is observed that the developed models are adequate in establishing the relationship between input parameters and performance characteristics. Further, multi-criteria optimization of process parameter levels is carried using grey based Taguchi method. The experiments are planned based on Taguchi's L9 orthogonal array. The proposed method employs single grey relational grade as a performance index to obtain optimum levels of parameters. It is found that peak current and electrode rotational speed are influential on these characteristics.Confirmation experiments are conducted to validate optimal parameters and it reveals the improvements in material removal rate, surface roughness and electrode wear rate as 13.84%, 12.91% and 19.42% respectively.

1. Introduction

RENE80 nickel super alloy is extensively used in aerospace industry due to its high hardness, high strength and creep properties. This super alloy is difficult to machine due to high hardness, low thermal conductivity and high affinity to react with the tool materials at high temperature generated during machining [1]. Hence it requires a non-traditional machining such as electrical discharge machining (EDM). But it has very low metal removal rate (MRR) [2]. The solution to this problem is providing rotation to the electrode. It utilizes the rotary electrode that enhances the flushing of debris formed during machining due to forced circulation of dielectric in the machining zone. This results in better stability of the process. [3-6]. Consequently, rotary EDM (REDM) is adopted for machining RENE80 nickel super alloy in the present study.

Researchers in the past have focused their work on mathematical modeling and performance of EDM/REDM. Pradhan et al. [7] developed mathematical models relating to various significant machining parameters and different machining criteria such as MRR, TWR and overcut during micro-drilling of titanium super alloy by EDM. The models were tested at 95%, 90% and 75% confidence levels and found to be satisfactory. Mohan et al. [3] analyzed the effect of various

REDM parameters on responses like MRR, tool wear rate and SR while machining Al-SiC metal matrix composites. The authors reported that increasing the speed of the rotation of the electrode had positive effect on MRR, tool wear rate and SR than stationary electrode. Aliakbari and Baseri [4] performed REDM of mould steel and found that depending on the geometry of the electrode, the rotational speed of the electrode had different effects on MRR, SR and EWR. Wang et al [5] optimized the blind-hole drilling of $Al_2O_3/6061Al$ composite on REDM using Taguchi methodology. Lin et al [6] pointed out that electrical parameters have significant effect on all considered responses, whereas speed of electrode rotation had significant effect only on MRR. Kao et al. [8] dealt with multi-objective optimization of parameters for machining titanium alloys using GRA and the results showed considerable improvements in MRR, SR and EWR.

It is observed from the literature that limited work has been reported on mathematical modeling and multi-criteria optimization of REDM for RENE80 nickel super alloy. Hence this paper presents the development of mathematical modeling of MRR, SR and using regression approach and multi-criteria optimization of parameters using GRA based Taguchi method.

2. Methodology

2.1. Taguchi method

It is very popular and effective to deal with responses influenced by many parameters and is a systematic approach to determine the optimal process parameters. It mainly focuses on minimization of variation of the response of interest [9]. Further, it reduces the number of experiments drastically and saves time and cost of experimentation. In this method, signal to noise (S/N) ratio is used to measure the deviation of the response from the mean value. There are two types of S/N ratios i.e. Lower-the-better and Higher-the-better types given by Eqns.(1) and (2) respectively,

$$\eta = -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right], \tag{1}$$

$$\eta = -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n}y_i^2\right], \tag{2}$$

where η denotes S/N ratio of experimental values, y_i represents experimental value of the i^{th} experiment an n is the total number of experiments.

2.2. Regression approach

Regression analysis is widely used in manufacturing processes as a statistical tool for investigation of relationships between input parameters and output responses. Usually, the researchers are interested to find the causal effect of one input variable on output response for example, the effect of process parameter on machining performance. It is also important to evaluate the 'statistical significance' of the developed mathematical model i.e. the degree of confidence that the true existing relationship among variables is close to the estimated mathematical model. The regression model commonly used is given by,

$$Y = f(A, B, C \quad and \quad D). \tag{3}$$

here Y denotes the response variable like MRR, SR, TWR etc. f is the response function and A, B, C and D are process parameters. In the present study, A, B, C and D are peak current (I), pulse on time (T_{on}), pulse off time (T_{off}) and rotational speed of electrode (S). The general non-linear quadratic model consisting of only linear and quadratic effects is given by the following equation [Montgomery, 2001],

$$Y = \beta_0 + \beta_1 A + \beta_2 B + \beta_3 C + \beta_4 D + \beta_5 A^2 + \beta_6 B^2 + \beta_7 C^2 + \beta_8 D^2 + \epsilon \tag{4}$$

where $\beta_0, \beta_1 ... \beta_8$ are regression coefficients of process parameters and ϵ is the experimental error. β_0 is the free term of the regression model.$\beta_1, \beta_2, \beta_3$ and β_4 are linear coefficients and $\beta_5, \beta_6, \beta_7$ and β_8 are quadratic coefficients. The regression models and regression coefficients are established based on the experimental values of various performance characteristics.

2.3. Grey relational analysis

The grey system theory was proposed by Deng [10] and has evolved gradually to solve certain problems with complicated interrelationships among multiple responses of variety of machining processes. In this, linear normalization of data is usually required in the range 0-1. The optimum parameter levels for complicated multiple responses are converted into optimization of single GRG i.e. this grade is taken as the performance index for multi-criteria optimization [15]. For Larger-the-better type, normalization of the response i is given by Eq.(5),

$$Y_{ij}^* = \left[\frac{y_{ij} - min_j(y_{ij})}{max_j(y_{ij}) - min_j(y_{ij})} \right], \tag{5}$$

where, $Y_{ij}{}^*$ is the normalized value in the j^{th} experiment for $j = 1, 2, ..., q$
For Smaller-the-better type, normalization of the response i is given by Eq.(6),

$$X_{ij}^* = \left[\frac{max_j x_{ij} - x_{ij})}{max_j(x_{ij}) - min_j(x_{ij})} \right]. \tag{6}$$

Later, grey relational coefficient ξ_{ij} is found out by using Eq. (7)

$$\xi_{ij}^* = \left[\frac{min_i min_j(\Delta_{ij}) + \zeta max_i max_j(\Delta_{ij})}{\Delta_{ij} + \zeta max_i max_j(\Delta_{ij})} \right], \tag{7}$$

where, ζ is distinguishing coefficient and lie in the range 0 to 1. Generally, it is taken as 0.5[9].
$\Delta_{ij} = |X_{ij}^* - R|$ and $R = Max X_{ij}^*$
Now, find the GRG-γ_j for each j^{th} experiment using Eq.(8),

$$\gamma_j = \frac{1}{m} \left(\sum_{i=1}^{m} \sum_{j=1}^{q} w_i \xi_{ij} \right), \tag{8}$$

where, w_i is the weight for i^{th} response and m is the number of responses, $0 < \gamma_j < 1$ and $\sum_{i=1}^{m} w_i = 1$.

3. Experimental details

Pure electrolytic copper of diameter 14.3 mm is used as the electrode. The study material is RENE80 nickel super alloy with hardness 45HRC and density $8.2 g/cm^3$ Its chemical composition (wt%) is: Al 5-6; Cr 9.5-12; Ti 2.5-3.2; C 0.13-0.2; Mo 3.5-4.8; W 4.5-5.5; Co 4-4.5; B 0.02 max; Se 0.015 max; Si<0.4; Mn $<$ 0.4; Fe 0.5 max; Ni balance. The rotary setup is fabricated to impart the rotary motion to the electrode and fastened to the ram of die-sink EDM machine (Make -Askar Microns, Model V3525). The work piece dimensions are $70 \times 35 \times 4 mm^3$.The work piece and electrode are connected to the positive and negative polarities of power supply respectively. The machining time of each experiment is set for three minutes. The experiments

Table 1. L9 array and experimental results of MRR, SR and EWR.

	Level of parameters				Experimental results		
S.No	I (Amps)	T_{on} (μs)	T_{off} (μs)	S (rpm)	MRR (mg/min)	SR (um)	EWR (mg/min)
1	6	10	10	100	65.035	2.953	12.266
2	6	20	20	300	120.718	3.166	9.833
3	6	50	50	500	176.812	2.903	5.533
4	15	10	20	500	480.495	3.366	60.900
5	15	20	50	100	433.104	3.700	54.050
6	15	50	10	300	467.426	3.833	46.780
7	24	10	50	300	604.060	4.533	73.670
8	24	20	10	500	668.507	4.433	87.866
9	24	50	20	100	618.815	5.633	74.433

are repeated three times at constant gap voltage of 30V for each parametric combination to reduce experimental error and average values of responses are taken for analysis. MRR and EWR are measured by using Eq.9,

$$MRR/EWR = \frac{(W_1 - W_2)}{T}, \tag{9}$$

where W_1 = weight of the work piece/electrode before machining (mg), W_2 = weight of the work piece /electrode after machining (mg), T = machining time (minutes). The weights are measured by digital balance of accuracy of 1mg. SR is measured using surface roughness tester.

The majority of published literature indicates that parameters like peak current, pulse on time and pulse off time are dominant. Also, the comparative study between standard EDM and rotational EDM showed that rotational EDM is superior to standard EDM in machining performance. Hence rotational speed of electrode is taken as the fourth parameter.The selection of an orthogonal array depends on the number of degrees of freedom (DOF). The DOF are equal to number of parameter levels −1. A particular array is selected if the DOF are equal to or less than the DOF of that array. In this paper, four parameters with three levels are selected. Also the pilot experimentation indicates that the interaction among these parameters is negligible. Thus, total DOF are eight. Hence an orthogonal array L9 is selected as this array can has maximum DOF equal to eight. The values of the parameters are taken based on the experimental region and the machine specifications.

4. Results and discussion

The selected L9 orthogonal array based on the Taguchi method and the average experimental values of three trials of MRR, SR and EWR are given in Table 1.

4.1. Mathematical models

Mathematical models are developed for MRR, SR and EWR based on experimental results using statistical software MINITAB15 and are given in Table 2. The validation of developed models is done based on the value of correlation coefficient (R^2 value). It can be seen that R^2 values of these models are more than 95% and hence they are adequate. Another way of checking the adequacy of the model is to carry out ANOVA of these models. The summary of ANOVA results of all models including linear and quadratic terms are shown in Table 2. According to ANOVA, if the calculated F-ratio value is greater than the tabulated F-ratio value at considered

Table 2. Developed mathematical models and their ANOVA.

					F-value		
S.No	Response	Mathematical model	R^2	$R^2_{adj.}$	Mathematical model	Linear terms	Quadratic terms
1	MRR	- 291 + 59.7 A + 3.90 B+ 0.0282 D + - 1.05 A*A - 0.0493 B*B +0.000243 D*D	99.9	99.8	2179.9***	6327.23***	237.07***
2	SR	2.92 - 0.0093 A + 0.0172 B - 0.00113 D + 0.00375 A*A -0.000075 B*B - 0.000000 D*D	96.6	86.40	9.83*	28.56**	0.88
3	EWR	- 4.68 + 1.48 A + 0.763 B- 0.0744D + + 0.0541 A*A - 0.0199 B*B + 0.000169 D*D	99.7	98.7	104.43***	302.5***	10.86*

*** significant at 99% confidence level, **significant at 95% confidence level, *significant at 90% confidence level. $F_{0.01,6,2}$ = 99.3, $F_{0.05,6,2}$ = 19.3, $F_{0.10,6,2}$ = 9.33, $F_{0.05,3,2}$ = 19.2, $F_{0.10,3,2}$ = 9.16, $F_{0.01,3,2}$= 99.3

Table 3. Normalized values, Grey relational coefficients and GRG of MRR, SR and EWR.

	Normalized values			Grey relational coefficient			
S.No	*MRR*	*SR*	*EWR*	*MRR*	*SR*	*EWR*	*GRG*
1	0.000	0.982	0.918	0.333	0.965	0.859	0.719
2	0.092	0.904	0.948	0.355	0.838	0.905	0.700
3	0.185	1.000	1.000	0.380	1.000	1.000	0.793
4	0.688	0.830	0.328	0.616	0.747	0.426	0.596
5	0.610	0.708	0.411	0.562	0.631	0.459	0.551
6	0.667	0.659	0.499	0.600	0.595	0.500	0.565
7	0.893	0.403	0.172	0.824	0.456	0.377	0.552
8	1.000	0.440	0.000	1.000	0.472	0.333	0.602
9	0.918	0.000	0.163	0.859	0.333	0.374	0.522

confidence level, then the model is considered as statistically significant [7]. The mathematical models for MRR and EWR are strongly significant at 99% confidence level and that of SR is significant at 90% confidence level.

4.2. Multi-criteria optimization of process parameter levels using GRA

The normalized values of responses are measured by using Eqs. 5 and 6 and are shown in Table 3 along with the ideal sequence of value 1. In this study, all the responses are equally weighted i.e.0.33 as all three responses are considered with equal importance. Based on Eqns.7 and 8, GRG for each experiment is calculated. The results of grey relational coefficients, GRGs

Table 4. ANOVA of GRGs.

Parameter	Degrees of freedom	Sum of squares	Variance	F-ratio	Contribution(%)
I	2	0.0598	0.0299	$498.7^{a,b}$	86.73
$T_{on}{}^{\sharp}$	2	0.00012	0.00006		-0.17
T_{off}	2	0.0012	0.0006	10.01^{b}	1.75
S	2	0.00783	0.0039	$65.28^{a,b}$	11.35
$Error^{\sharp}$	2	0.00012	0.00006		
$Total$	8	0.0690			100

a significant at 95% confidence level, $F_{0.05,2,2} = 19$,
b significant at 90% confidence level, $F_{0.10,2,2} = 9$,
\# pooled parameter

Table 5. Results of confirmation experiments.

Initial data	Optimal process parameters			
Response	$I_1T_{on2}T_{off2}S_2$	Prediction $I_1T_{on1}T_{off3}S_3$	Experiment $I_1T_{on1}T_{off3}S_3$	% improvement
MRR	120.718	-	137.433	13.84%
SR	3.166	-	2.580	18.50%
EWR	9.833	-	7.923	19.42%
GRG	0.7097	0.807		13.71%
	Overall average $GRG = 0.6222$			

and their ranks are given in Table 3. The results show that experiment number 3 has the highest GRG. Hence, it can be expected that levels of each process parameter are superior to attain a better multiple responses. ANOVA results of GRGs are shown in Table 4. It depicts that the peak current and electrode rotational speed have percentage contributions as 86.73% and 11.65% respectively. The response graph of the GRGs is shown in Figure 1. From this graph, optimum combinational levels of parameters are identified as $I_1T_{on1}T_{off3}S_3$.Further the confirmation experiments are carried out at optimum parameter levels $I_1T_{on1}T_{off3}S_3$. The results are compared with one of the experiments in orthogonal array $I_1T_{on2}T_{off2}S_2$ [8]. The additive model is used to evaluate the predicted GRG. The corresponding results are given in Table 5. It can be observed that the GRG for multiple responses improved from 0.7097 to 0.807 i.e. 13.71%. MRR increased from 120.718 mg/min to 137.433 mg/min, SR decreased from 3.166μm to 2.580μm and EWR decreased from 9.833 mg/min to 7.923 mg/min. The percentage improvements in MRR, SR and EWR are 13.84%,12.91% and 19.42% respectively.

5. Conclusion

Mathematical models for MRR, SR and EWR are developed using regression approach and the validity of these models is checked by R2 value and ANOVA. Models are found statistically significant and adequate. Multi-criteria optimization of parameter levels is explored using GRA. It is observed that peak current and speed of electrode rotation are significant for multi-criteria optimization of MRR, SR and EWR simultaneously. The optimal levels of parameters are peak current-6A, pulse on time-10μs, pulse off time-50μs and speed of electrode rotation-500rpm. There is an improvement in GRG from 0.7097 to 0.807 i.e.13.71%. MRR increased from 120.718 mg/min to 137.433mg/min. SR decreased from 3.166μm to 2.580μm. The increase in MRR

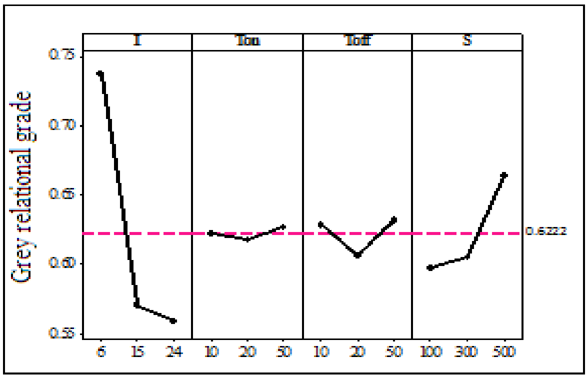

Figure 1. Response graph of GRG.

accompanied by the decrease in SR is attributed to the rotational speed.At higher speed, the debris is completely washed away from the machining zone due to centrifugal action of dielectric fluid. EWR decreased from 9.833 mg/min to 7.923 mg/min. This is due to the fact that at higher speed, the dielectric fluid is forced into machining zone which results in uniform distribution of the heat and cooling of the electrode. Hence EWR is decreased at optimal levels. The percentage improvements in MRR, SR and EWR are 13.84%, 12.91% and 19.42% respectively. This work is useful to the EDM industries dealing with nickel super alloys and provides the guidance to predict the performance and select optimal parameters for overall performance improvement.

References

[1] Choudhary I A and El-Baradie M A 1998 *Journal of Material Processing Technology* B**77** 278-284
[2] Ho K H and Newman S T 2003*International Journal of Machine Tools and Manufacture* B**43** 1287-1300
[3] Mohan B et.al 2002 *Journal of Material Processing Technology* B**124** 297-304
[4] Aliakbari E and Baseri H *International Journal of Advanced Manufacturing Technology* B**10.1007/s00170-011-3862-9**
[5] Wang C C and Yan B H 2002 *Journal of Material Processing Technology* B**102** 90-102
[6] Lin Y C et.al *International Journal of Advanced Manufacturing Technology* B**10.1007/s00170-011-3745-0**
[7] Pradhan B B et.al 2009 *International Journal of Advanced Manufacturing Technology* B**41** 1094-1106
[8] Kao Y C et.al 2010 *International Journal of Advanced Manufacturing Technology* B**47** 395-402
[9] Phadke M S 1989 *Prentice Hall New Jersy*
[10] Deng J 1989 *Journal Grey System* B**1(1)** 1-24