

Large amplitude free vibrations of Timoshenko beams at higher modes using coupled displacement field method

K.Krishna Bhaskar

Department of Mechanical Engineering, University College of Engineering, JNTUK, Kakinada, Andhra Pradesh, India.

E-mail: krisbhaskar22@yahoo.co.in

K.Meera Saheb

Department of Mechanical Engineering, University College of Engineering, JNTUK, Kakinada, Andhra Pradesh, India.

E-mail: meera.aec@gmail.com

Abstract. A simple but accurate continuum solution for the shear flexible beam problem using the energy method involves in assuming suitable single term admissible functions for the lateral displacement and total rotation. This leads to two non-linear temporal differential equations in terms of the lateral displacement and the total rotation and are difficult, if not impossible, to solve to obtain the large amplitude fundamental frequencies of beams as a function of the amplitude and slenderness ratios of the vibrating beam. This situation can be avoided if one uses the concept of coupled displacement field where in the fields for lateral displacement and the total rotation are coupled through the static equilibrium equation. In this paper the lateral displacement field is assumed and the field for the total rotation is evaluated through the coupling equation. This approach leads to only one undetermined coefficient which can easily be used in the principle of conservation of total energy of the vibrating beam at a given time, neglecting damping. Finally, through a number of algebraic manipulations, one gets a nonlinear equation of Duffing type which can be solved using any standard method. To demonstrate the simplicity of the method discussed above the problem of large amplitude free vibrations of a uniform shear flexible hinged beam at higher modes with ends immovable to move axially has been solved. The numerical results obtained from the present formulation are in very good agreement with those obtained through finite element and other continuum methods for the fundamental mode, thus demonstrating the efficacy of the proposed method. Also some interesting observations are made with variation of frequency Vs amplitude at different modes.

1. Introduction

The classic work of Woinowsky Krieger [1] on the large amplitude vibrations of slender beams inspired many a researcher to develop simple continuum formulations [2-4] and finite element formulations [5, 6] with some simplifying assumptions. However, large amplitude vibrations of shear flexible beams have not been received much attention till recently except in the work of Rao and Raju [7] who have used the simplified finite element formulation. Introduction of the effects of shear deformation and rotary inertia in the formulation yields coupled nonlinear differential



equations. Further the oscillations being non-harmonic with two independent variables, namely, the lateral deflection and total rotation, solution of such complicated equations by assuming both the spatial and temporal distribution is rather difficult. On the other hand, if the two variables appearing in the formulation are coupled to make the final equation a single non-linear equation, then the solution becomes much simpler. The reason being, in this system of single non-linear equation, assuming suitable admissible functions in space can eliminate the space variable and the resulting temporal equation can be solved using any standard method. Another interesting part of the present work is that unlike researchers arrived at finite element analogues based on the continuum theories, the authors are making an attempt to arrive at a continuum analogue of the recent coupled finite element for mutation [8, 9]. The purpose of the present paper is to demonstrate the efficacy of the proposed method by applying to the large amplitude free vibrations of a uniform, shear flexible hinged-hinged and clamped-clamped beams in conjunction with the coupled displacement theory and the principle of conservation of total energy. The results are obtained in closed form expressions of the non-linear to linear radian frequency ratios for simply supported and clamped beams at various modes for various slenderness and maximum amplitude ratios, and are obtained using the harmonic balance method [10]. Comparison of the present results with those existing in literature for the first mode for shear flexible beams shows that the present results obtained from the concept of coupled displacement field analysis are accurate.

2. Shear deformable beam theory for static equilibrium used for coupled displacement field

The kinematics of a shear flexible beam theory (Timoshenko beam theory) can be written as

$$\bar{u}(x, z) = z\theta, \quad (1)$$

$$\bar{w}(x, z) = w(x, z), \quad (2)$$

where \bar{u} and \bar{w} are the axial and transverse displacements at a generic point of the beam, z is the distance of the generic point from the neutral axis, w represents the transverse displacement and θ represents the total rotation anywhere on the beam axis and x, z are the independent spatial variables. The axial strain and shear strain are given by

$$\varepsilon_x = z \frac{\partial \theta}{\partial x}, \quad (3)$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \theta, \quad (4)$$

Now, the expressions for the strain energy U and potential energy of the externally applied loads W are given by

$$U = \frac{EI}{2} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx + \frac{kGA}{2} \int_0^L \left(\frac{\partial w}{\partial x} + \theta \right)^2 dx, \quad (5)$$

$$W = \int_a^b p(x)w(x)dx, \quad (6)$$

where EI is the flexural rigidity, GA is shear rigidity, k is the shear coefficient (taken as 5/6 in the present study), $p(x)$ is the static load per unit length on the beam and L is its length. Applying the principle of minimization of total potential energy, as

$$\delta(U - W) = 0, \quad (7)$$

the following static equilibrium equations can be obtained [11]

$$EI \frac{\partial^2 \theta}{\partial x^2} - kGA \left(\frac{\partial w}{\partial x} + \theta \right) = 0, \quad (8)$$

$$kGA \left(\frac{d^2 w}{dx^2} + \frac{d\theta}{dx} \right) + p = 0. \quad (9)$$

Equations (8) and (9) are coupled equations and can be solved for obtaining the solution for static analysis of shear deformable beams. A close observation of equation (8) shows that it is dependent on the load term p and equation (9) is independent of the load terms p . Hence equation (9) is used to couple the bending rotation θ and the transverse displacement w , so that the two independent variable problem become a single variable problem and the resulting large amplitude free vibration problem becomes much simpler.

3. Coupled displacement field theory

In this theory, we assume an admissible function θ for hinged-hinged beam which satisfies all the essential boundary conditions and symmetric condition, in the beam domain, and evaluate the lateral displacement w distribution using equation (9), θ distribution along the length of the beam is assumed as

$$\theta = a \frac{n\pi}{L} \cos \frac{n\pi x}{L}, \quad (10)$$

where a is the central lateral displacement of the beam and n is the mode number Equation (9) can be rewritten as

$$\frac{\partial w}{\partial x} = -\theta + \frac{EI}{kGA} \theta'', \quad (11)$$

$$\theta'' = -a \left(\frac{n\pi}{L} \right)^2 \cos \frac{n\pi x}{L}, \quad (12)$$

Substituting the function θ and θ'' in equation (11), we obtain the displacement field for w , after integration as

$$w = \lambda a \sin \frac{n\pi x}{L}, \quad (13)$$

$$\lambda = \frac{\frac{-n\pi}{L}}{\left(1 + \left(\frac{n\pi x}{L} \right)^2 \frac{EI}{kGA} \right)}. \quad (14)$$

It may be noted here that because of the coupled field theory w , the transverse displacement distribution, containing the same undetermined coefficient a as for θ distribution, w satisfies all the essential boundary and symmetric conditions and given by

$$w(0) = w(L) = \left(\frac{dw}{dx} \right)_{x=L/2} = 0. \quad (15)$$

4. Large amplitude vibrations

Large amplitude vibrations can be studied, once the coupled displacement field for the derived lateral displacement w , for an assumed θ distribution is evaluated using the principle of conservation of total energy at any instant of time, which states that

$$U + T + W = \text{constant}, \quad (16)$$

The expression for U , T and W are given by

$$U = \frac{EI}{2} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx + \frac{kGA}{2} \int_0^L \left(\frac{\partial w}{\partial x} + \theta \right)^2 dx, \quad (17)$$

$$T = \frac{\rho A}{2} \int_0^L \dot{w}^2 dx + \frac{\rho I}{2} \int_0^L \dot{\theta}^2 dx, \quad (18)$$

$$W = \frac{T_a}{2} \int_0^L \frac{I}{2} \left(\frac{\partial w}{\partial x} \right)^2 dx, \quad (19)$$

where U is the strain energy, T is the kinetic energy, W is the work done by the tension developed because of large amplitudes (deformations), ρ is the mass density, A is the area of cross - section, I is the moment of inertia, T_a is the tension developed in the beam because of large deformations, and $(\dot{\bullet})$ denotes differentiation with respect to time. T_a is obtained following Woinowsky Krieger [1] as

$$T_a = \frac{EI}{2Lr^2} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx, \quad (20)$$

where r is the radius of gyration. It is to be noted here that w in Equation (20) does not contain shear flexible terms, and T_a is evaluated in terms of $(q = a/r)$. Substituting the expressions for θ and w (obtained from the coupled field), the expression for U , T and W are obtained as

$$U = \frac{EI}{4L} a^2 (n\pi)^2 \left[1 + \frac{(n\pi)^2 EI}{L^2 kGA} \right], \quad (21)$$

$$T = \frac{\rho AL^3}{4(n\pi)^2} \dot{a}^2 \left\{ \left[1 + \frac{(n\pi)^2 EI}{L^2 kGA} \right]^2 + \frac{I}{A} \frac{(n\pi)^2}{L^2} \right\}, \quad (22)$$

$$W = \frac{EI}{32r^2} \left(\frac{n\pi}{L} \right)^2 L a^4 \left[1 + \frac{(n\pi)^2 EI}{L^2 kGA} \right]. \quad (23)$$

Substituting the expressions for U , T and W in equation (16) and simplifying, noting that $I = Ar^2$, we get

$$\dot{q}^2 + \alpha_1 q^2 + \alpha_2 q^4 = \text{constant} \quad (24)$$

,

$$\text{where } \alpha_1 = \frac{EI \left(\frac{n\pi}{L} \right)^4 \left[1 + \frac{(n\pi)^2 E}{kG\beta^2} \right]}{\rho A \left\{ \left[1 + \frac{(n\pi)^2 E}{kG\beta^2} \right]^2 + \frac{(n\pi)^2}{\beta^2} \right\}}, \quad (25)$$

$$\alpha_2 = \frac{EI \left(\frac{n\pi}{L} \right)^4 \left[1 + \frac{(n\pi)^2 E}{kG\beta^2} \right]^2}{\rho A \left\{ \left[1 + \frac{(n\pi)^2 E}{kG\beta^2} \right]^2 + \frac{(n\pi)^2}{\beta^2} \right\}}. \quad (26)$$

In equations (25) and (26) E and G can be eliminated by the standard relation,

$$G = \frac{E}{2(1 + \nu)}, \quad (27)$$

where ν is the Poisson ratio, and $\beta = L/r$, the slenderness ratio of the beam. Using the Harmonic balance method (HBM) [10] as discussed in the next section can solve equation (23).

5. Harmonic balance method

The direct numerical integration method (DNI) [3], proposed by the first author equation (24) can be solved to the desired degree of accuracy. However, for an elegant closed form solution one can advantageously use the HBM which is briefly discussed in this section. Differentiating equation (24), we get

$$\ddot{q} + \alpha_1 q + 2\alpha_2 q^3 = 0. \quad (28)$$

This is the famous Duffing equation and is solved by assuming

$$q = q_m \sin \omega_{NL} t. \quad (29)$$

Where ω_{NL} is the non-linear radian frequency and q_m is the maximum amplitude ratio a_m/r . From equations (29) and (27) and after simplification, and noting that, by neglecting the higher harmonics of ω_{NL} and using the approximate relation $\sin^3 \omega_{NL} t \approx \frac{3}{4} \sin \omega_{NL} t$ we get

$$\omega_{NL}^2 = \alpha_1 + \alpha_2 q_m^2. \quad (30)$$

From equation (29), If $q_m = 0$, i.e, for linear vibrations, linear radian frequency,

$$\omega_L = \omega_{NL}, \quad \omega_L^2 = \alpha_1. \quad (31)$$

From equations (25), (26) and (30), after simplification we obtained as,

$$\left(\frac{\omega_{NL}}{\omega_L} \right)^2 = 1 + \frac{3}{16} \left[1 + \frac{(n\pi)^2 E}{kG \beta^2} \right] \left(\frac{a_m}{r} \right)^2. \quad (32)$$

Equation (33) is an elegant form to calculate the ratios of ω_{NL}^2/ω_L^2 for various values of amplitude and slenderness ratios of the beam. In terms of the Poisson ratio, equation (33) can be written as

$$\left(\frac{\omega_{NL}}{\omega_L} \right)^2 = 1 + \frac{3}{16} \left[1 + \frac{3.12(n\pi)^2}{\beta^2} \right] \left(\frac{a_m}{r} \right)^2, \quad (33)$$

For very large β i.e. for slender beams, where shear deformation can be neglected, equation (34) becomes

$$\left(\frac{\omega_{NL}}{\omega_L} \right)^2 = 1 + \frac{3}{16} \left(\frac{a_m}{r} \right)^2, \quad (34)$$

which is a standard result [4]. In case of a clamped-clamped beam, a single term trigonometric admissible function for θ that satisfies the required boundary conditions is taken as

$$\theta = a \frac{2n\pi}{L} \cos \frac{2n\pi x}{L}, \quad (35)$$

and by following the similar procedure as discussed for hinged-hinged beam the ratios of the nonlinear to the linear radian frequencies can be obtained for the clamped-clamped beam, as

$$\left(\frac{\omega_{NL}}{\omega_L} \right)^2 = 1 + \frac{3}{64} \left[1 + \frac{12.48(n\pi)^2}{\beta^2} \right] \left(\frac{a_m}{r} \right)^2. \quad (36)$$

The relations for calculating the fundamental frequency parameter for both the cases was given in Table 1.

Table 1. Fundamental frequency parameter for hinged-hinged and clamped-clamped beams

	Fundamental frequency (λ) = $\frac{\rho A \omega_L^2 L^4}{EI}$
Hinged-hinged beam	$\frac{(n\pi)^4 \left[1 + \frac{3.12(n\pi)^2}{\beta^2} \right]}{\left\{ \left[1 + \frac{3.12(n\pi)^2}{\beta^2} \right]^2 + \frac{(n\pi)^2}{\beta^2} \right\}}$
Clamped-clamped beam	$\frac{16(n\pi)^4 \left[1 + \frac{12.48(n\pi)^2}{\beta^2} \right]}{3 \left\{ \left[1 + \frac{12.48(n\pi)^2}{\beta^2} \right]^2 + \frac{(2n\pi)^2}{3\beta^2} \right\}}$

6. Numerical results and discussion

Using the formulation described above, the large amplitude behavior of a uniform shear flexible hinged-hinged and clamped-clamped beams are obtained in terms of ω_{NL}/ω_L (ratio of non-linear radian frequency to the linear radian frequency) in terms of various a_m/r (maximum amplitude ratios) and L/r (slenderness ratios). As a demonstration of the proposed formulation, this beam is considered with axially immovable ends (Fig.1) for hinged-hinged beam. Table 1 shows expressions for linear frequency parameters for hinged-hinged and clamped-clamped beams. Numerical results in the form of linear frequency parameters for various slenderness ratios are given in Table 2 and Table 3 at different modes with Poisson ratio of 0.3 for both hinged-hinged and clamped-clamped beams. From these tables linear frequencies are increasing with mode number. The present results in terms of ω_{NL}/ω_L are presented in Table 4 and Table 5 respectively for hinged-hinged and clamped-clamped beams for fundamental mode. For the sake of comparison and validation of the proposed method, the same results obtained by the finite element method (FEM) [7] and by using the direct numerical integration method (DNI) [3] are also included in Table 4 and Table 5 for hinged-hinged case for Poisson ratio 0.3 at mode 1. Similar results are presented in Table 6 and Table 7 for hinged-hinged and clamped-clamped beams at various modes. The present results for the extreme cases of $L/r = 10$ and $a_m/r = 5$ match very well with the results of FEM and DNI. Further, the present results match excellently with those of Ref. [4] for slender beams, showing the efficacy of the proposed method and the accuracy of results over a range of $L/r = 10$ to $L/r = 100$. Then both shear deformable beam and slender beams can be solved using the present formulation. Further, no shear locking phenomenon [12] exists in the present formulation.

7. Conclusions

A coupled displacement field formulation presented in this paper, to study the variation of large amplitude vibrations of a uniform shear flexible hinged-hinged and clamped-clamped beams at higher modes with axially immovable ends. This classical problem is chosen as a demonstration problem, to show the efficacy of the formulation. The increase of frequency ratio is considerable at higher modes and this is more considerable in hinged-hinged beams than clamped-clamped beams. The results obtained using the present formulation are found to be in good agreement with those results obtained by FEM and DNI for a wide range of L/r ratios, including the slender beams, and a_m/r ratios. The preliminary studies on the shear flexible beams with other boundary conditions, with axially immovable ends, give promising results, which indicate the generality of the present formulation. Further the present method proposed does not exhibit shear locking phenomenon commonly encountered in the formulations of shear flexible beams.

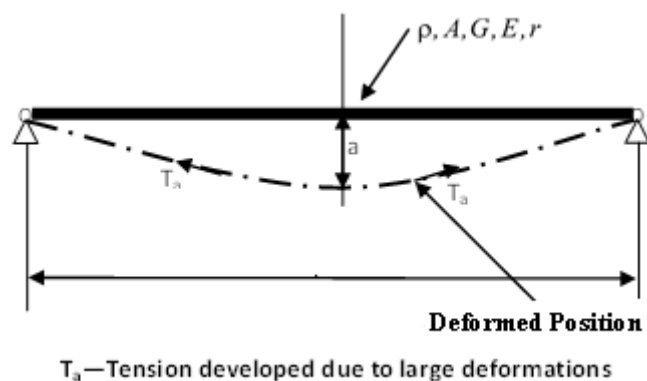


Figure 1. A shear flexible hinged-hinged beam undergoing large deformations

Table 2. Values of non-dimensional linear frequency parameter ($\sqrt{\lambda}$) with shear for hinged-hinged beam

β	Mode					
	1	DNI [13]	2	3	4	5
10	8.3971	—	25.4525	44.398	63.4843	82.367
25	9.5742	9.567	35.3385	71.5827	113.7518	159.0779
50	9.7984	9.79	38.2968	83.1529	141.3539	209.9281
100	9.8575	9.849	39.1934	87.3235	153.1874	235.4498

Table 3. Values of non-dimensional linear frequency parameter ($\sqrt{\lambda}$) with shear for clamped-clamped beam

β	Mode					
	1	DNI [13]	2	3	4	5
10	15.0685	—	37.1896	58.7957	79.9787	100.9363
25	20.6962	20.61	67.3388	121.7498	177.3149	232.4349
50	22.2157	22.17	82.7847	168.9496	269.3554	376.7119
100	22.6574	22.63	88.8629	193.8516	331.1387	493.8748

Table 4. ω_{NL}/ω_L for a uniform shear flexible hinged-hinged beam at mode 1

a_m/r	β									
	10		25		50			100		
	Present Study	Present Study	FEM [7]	DNI [3]	Present Study	FEM [7]	DNI [3]	Present Study	FEM [7]	DNI [3]
0.0	1	1	1	1	1	1	1	1	1	1
0.2	1.0049	1.0039	-	1.0039	1.0037	-	1.0038	1.0037	-	1.0038
0.4	1.0194	1.0156	1.0156	1.0156	1.0150	1.0150	1.0151	1.0149	1.0149	1.0149
0.6	1.0432	1.0348	-	1.0347	1.0336	-	1.0335	1.0333	-	1.0332
0.8	1.0756	1.0610	1.0605	1.0608	1.0589	1.0585	1.0588	1.0584	1.0585	1.0582
1.0	1.1159	1.0939	1.0925	1.0933	1.0907	1.0897	1.0902	1.0899	1.089	1.8940
2.0	1.4075	1.3367	1.3213	1.3313	1.3263	1.3142	1.3212	1.3237	1.3125	1.3186
3.0	1.7910	1.6645	1.6146	1.6501	1.6456	1.6052	1.6318	1.6409	1.6257	1.6272
4.0	2.2191	2.0366	-	2.0116	2.0092	-	1.9850	2.0023	-	1.9783
5.0	2.6706	2.4328	-	2.3968	2.3969	-	2.3619	2.3878	-	2.3531

Table 5. ω_{NL}/ω_L for a uniform shear flexible clamped-clamped beam at mode 1

a_m/r	β									
	10		25		50			100		
	Present Study	Present Study	FEM [7]	DNI [3]	Present Study	FEM [7]	DNI [3]	Present Study	FEM [7]	DNI [3]
0	1	1	1	1	1	1	1	1	1	1
0.2	1.0021	1.0011	-	1.0011	1.001	-	1.001	1.0009	-	1.0009
0.4	1.0083	1.0045	1.039	1.0045	1.0039	1.0039	1.0039	1.0038	1.0035	1.0037
0.6	1.0187	1.0101	-	1.01	1.0088	-	1.0088	1.0085	-	1.0084
0.8	1.0329	1.0178	1.0153	1.0178	1.0156	1.0142	1.0156	1.0151	1.014	1.0149
1	1.051	1.0277	1.0238	1.0276	1.0243	1.0228	1.0242	1.0235	1.0218	1.0231
2	1.1911	1.1066	1.0895	1.1058	1.094	1.08	1.0933	1.0908	1.0834	1.0892
3	1.3935	1.2268	1.1855	1.2239	1.2011	1.1771	1.1987	1.1946	1.1759	1.1902
4	1.6354	1.3777	-	1.3711	1.3368	-	1.3313	1.3264	-	1.3178
5	1.9017	1.5502	-	1.5389	1.4932	-	1.4836	1.4786	-	1.4647

Table 6. ω_{NL}/ω_L for a uniform shear flexible hinged-hinged beam for modes 2,3,4 and 5

a_m/r	n=2				n=3			
	β				β			
	10	25	50	100	10	25	50	100
0	1	1	1	1	1	1	1	1
0.2	1.0083	1.0045	1.0039	1.0038	1.0141	1.0054	1.0042	1.0038
0.6	1.0727	1.0396	1.0348	1.0336	1.1201	1.0476	1.0368	1.0341
0.8	1.126	1.0694	1.0611	1.059	1.2053	1.0832	1.0646	1.0599
1	1.1911	1.1066	1.094	1.0908	1.3067	1.1273	1.0992	1.0921
2	1.6354	1.3777	1.3368	1.3264	1.9571	1.4432	1.354	1.3307
3	2.1835	1.7379	1.6645	1.6457	2.7144	1.8537	1.6955	1.6536
4	2.7746	2.1428	2.0366	2.0092	3.5101	2.309	2.0815	2.0207
5	3.3861	2.5714	2.4328	2.3969	4.3231	2.7871	2.4915	2.412

a_m/r	n=4				n=5			
	β				β			
	10	25	50	100	10	25	50	100
0	1	1	1	1	1	1	1	1
0.2	1.022	1.0067	1.0045	1.0039	1.0321	1.0083	1.0049	1.004
0.6	1.1834	1.0587	1.0396	1.0348	1.26	1.0727	1.0432	1.0357
0.8	1.3083	1.1021	1.0694	1.0611	1.4299	1.126	1.0756	1.0627
1	1.4533	1.1556	1.1066	1.094	1.6224	1.1911	1.1159	1.0963
2	2.3341	1.5303	1.3777	1.3368	2.7438	1.6354	1.4075	1.3445
3	3.3179	2.0047	1.7379	1.6645	3.9609	2.1835	1.791	1.6785
4	4.335	2.5233	2.1428	2.0366	5.2071	2.7746	2.2191	2.057
5	5.3667	3.0636	2.5714	2.4328	6.4655	3.3861	2.6706	2.4594

Table 7. ω_{NL}/ω_L for a uniform shear flexible clamped-clamped beam for modes 2,3,4 and 5

a_m/r	n=2				n=3			
	β				β			
	10	25	50	100	10	25	50	100
0	1	1	1	1	1	1	1	1
0.2	1.0055	1.0017	1.0011	1.0010	1.0113	1.0026	1.0014	1.0010
0.4	1.0220	1.0067	1.0045	1.0039	1.0444	1.0104	1.0054	1.0042
0.6	1.0488	1.0150	1.0101	1.0088	1.0973	1.0231	1.0121	1.0093
0.8	1.0853	1.0265	1.0178	1.0156	1.1674	1.0408	1.0214	1.0165
1	1.1305	1.0411	1.0277	1.0243	1.2518	1.0631	1.0333	1.0257
2	1.4533	1.1556	1.1066	1.0940	1.8077	1.2330	1.1273	1.0992
3	1.8714	1.3247	1.2268	1.2011	2.4703	1.4733	1.2685	1.2119
4	2.3341	1.5303	1.3777	1.3368	3.1735	1.7554	1.4432	1.3540
5	2.8196	1.7597	1.5502	1.4932	3.8953	2.0621	1.6407	1.5172

a_m/r	n=4				n=5			
	β				β			
	10	25	50	100	10	25	50	100
0	1	1	1	1	1	1	1	1
0.2	1.0192	1.0039	1.0017	1.0011	1.0294	1.0055	1.0021	1.0012
0.4	1.0749	1.0155	1.0067	1.0045	1.1129	1.0220	1.0083	1.0049
0.6	1.1618	1.0345	1.0150	1.0101	1.2397	1.0488	1.0187	1.0110
0.8	1.2735	1.0605	1.0265	1.0178	1.3980	1.0853	1.0329	1.0194
1	1.4041	1.0931	1.0411	1.0277	1.5784	1.1305	1.0510	1.0302
2	2.2104	1.3339	1.1556	1.1066	2.6393	1.4533	1.1911	1.1159
3	3.1213	1.6593	1.3247	1.2268	3.7978	1.8714	1.3935	1.2457
4	4.0673	2.0290	1.5303	1.3777	4.9863	2.3341	1.6354	1.4075
5	5.0285	2.4228	1.7597	1.5502	6.1876	2.8196	1.9017	1.5915

Acknowledgments

The authors are highly thankful to authorities of University College of Engineering(A), JNTUK-Kakinada for extending the necessary support for publishing the paper.

References

- [1] Woinowsky and Krieger S 1950 The effect of an axial force on the vibration of hinged bars *Journal of Applied Mechanics* **17** 35-36
- [2] Srinivasan A V 1965 Large amplitude free oscillations of beams and plates *AIAA J* **3** 1951-1953
- [3] Rao G V and Raju K K 2002 A direct numerical integration method to study the large amplitude vibration of slender beams with immovable ends *Journal of Institute of Engineers (I)* **83** 42-44
- [4] Rao G V and Raju K K 2003 Large amplitude free vibrations of beams an energy approach *ZAMM* **83** 493-498
- [5] Chuh Mei 1972 Non-linear vibrations of beams by matrix displacement method *AIAA J* **10** 355-357
- [6] Rao G V, Raju I S and Raju K K 1976 Finite element formulation for the large amplitude free vibrations of beams and orthotropic circular plates *Computers and Structures* **6** 169-172
- [7] Rao G V, Raju I S and Raju K K 1976 Non-linear vibrations of beams considering shear deformation and rotary inertia *AIAA J* **14** 685-687
- [8] Singh G and Rao G V 2000 Shear flexible finite elements retrospect and prospects *Journal of Institution of Engineers (I)* **81** 12-19
- [9] Raveendranath P, Singh G and Rao G V 2001 A three noded shear flexible curved beam element based on coupled displacement field interpolation *International Journal of Num. Meth. Engg* **51** 85-102
- [10] Azrar L and white R G 1999 A semi - analytical approach to the nonlinear dynamic response problem of s-s and c-c beams at large vibration amplitudes. Part1: General theory and application to the single mode approach to free and forced vibration analysis *Journal of Sound and Vibration* **224** 183-207
- [11] Reddy J N 1993 *An introduction to the finite element method* (McGraw-Hill Inc, New York)
- [12] Prathap G and Bhashyam G R 1982 Reduced integration and the shear flexible beam element *International Journal of Num. Meth. Engg* **18** 195-210
- [13] Rao G V, Meera sahib K and RangaJanardhana G 2006 Concept of coupled displacement field for large amplitude free vibrations of shear flexible beams *Journal of Vibration and Acoustics* **128**