

Unsteady MHD free convection and mass transfer flow past a porous vertical plate in presence of viscous dissipation

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Abstract. This study examines the influence of unsteady magneto hydrodynamic free convection and mass transfer flow past a vertical porous plate in presence of viscous dissipation and heat source. The governing non linear partial differential equations are transformed into a system of coupled non linear partial differential equations and solved using a Finite element technique. The effects of various physical parameters on the dimensionless velocity, temperature and concentration profiles are depicted graphically and analyzed in detail.

1. Introduction

The phenomenon of hydromagnetic flow with heat and mass transfer in an electrically conducting fluid past a porous plate embedded in a porous medium has attracted the attention of a good number of investigators because of its varied applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and in the boundary layer control in the field of aerodynamics. Heat transfer in laminar flow is important in problems dealing with chemical reactions and in dissociating fluids. In view of its wide applications, Hasimoto [1] initiated the boundary layer growth on a flat plate with suction or injection. Yamamoto and Iwamura [2] explained the flow of a viscous fluid with convective acceleration through a porous medium. Mansutti et al. [3] have discussed the steady flow of a non Newtonian fluid past a porous plate with suction or injection. Jha [4] analyzed the effect of applied magnetic field on transient free convective flow in a vertical channel. Chandran et al. [5] have discussed the unsteady free convection flow of an electrically conducting fluid with heat flux and accelerated boundary layer motion in presence of a transverse magnetic field. Acharya et al. [6] have reported the problem of heat and mass transfer over an accelerating surface with heat source in presence of suction and blowing. The unsteady free convective MHD flow with heat transfer past a semi infinite vertical porous moving plate with variable suction has been studied by Kim [7]. Singh and Thakur [8] have given an exact solution of a plane unsteady MHD flow of a non Newtonian fluid. Sharma and Pareek [9] explained the behavior of steady free convective MHD flow past a vertical porous moving surface. Das et al. [10] estimated the mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction employing finite difference analysis. Das et al. [11] investigated numerically the unsteady free convective MHD flow past an accelerated vertical plate with suction and heat flux. Das and Mitra [12] discussed the unsteady mixed convective MHD flow and mass transfer past an accelerated infinite vertical



plate with suction. Recently, Das et al. [13] analyzed the effect of mass transfer on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. More recently, Das et al. [14] investigated the hydro magnetic convective flow past a vertical porous plate through a porous medium with suction and heat source. The study of stellar structure on solar surface is connected with mass transfer phenomena. Its origin is attributed to difference in temperature caused by the non homogeneous production of heat which in many cases can rest not only in the formation of convective currents but also in violent explosions. Mass transfer certainly occurs within the mantle and cores of planets of the size of or larger than the earth.

In the present study we therefore, propose to analyze the effect of mass transfer on unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with viscous dissipation and heat source. The present investigations can be utilized as a basis for studying more complex systems that arise in engineering and industrial applications.

2. Mathematical formulation

Consider the unsteady free convective mass transfer flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in presence of constant suction and heat source and transverse magnetic field. We made the following assumptions. In Cartesian coordinate system, let x' - axis is taken to be along the plate and the y' - axis normal to the plate. Since the plate is considered infinite in x' - direction, hence all physical quantities will be independent of x' - direction. The wall is maintained at constant temperature T'_w and concentration C'_w higher than the ambient temperature T'_∞ and concentration C'_∞ respectively. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. The homogeneous chemical reaction is of first order with rate constant K' between the diffusing species and the fluid is neglected. It is assumed that there is no applied voltage which implies the absence of an electric field. The fluid has constant kinematic viscosity and constant thermal conductivity and the boussinesq's approximation have been adopted for the flow. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present and hence sores and dufour effects are negligible. The magneto hydrodynamic unsteady mixed convective boundary layer equations under the boussinesq's approximations are:

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v'_0(\text{constant}), \quad (1)$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + v \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{v}{k'} u'. \quad (2)$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + \frac{v}{C_p} \left(\frac{\partial u'}{\partial t'} \right)^2 + S'(T' - T'_\infty). \quad (3)$$

Concentration Equation:

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2}. \quad (4)$$

The boundary conditions of the problem are :

$$u' = 0, v' = -v'_0, T' = T'_w + \epsilon(T'_w - T'_\infty)e^{i\omega' t'}, C' = C'_w + \epsilon(C'_w - C'_\infty)e^{i\omega' t'} \text{ at } y' = 0, \quad (5)$$

$$u' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty. \quad (6)$$

Introducing the following non dimensional variables and parameters,

$$y = \frac{y' v'_0}{v}, t = \frac{t' v'^2_0}{4v}, \omega = \frac{4v\omega'}{v'^2_0}, u = \frac{u'}{v'}, v = \frac{\eta_0}{\rho}, \quad (7)$$

$$M = \left(\frac{\sigma B_0^2}{\rho} \right) \frac{v}{v'^2_0}, K = \frac{v'^2_0 K'}{v^2}, \theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, C = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)}, \quad (8)$$

$$Pr = \frac{\nu}{k}, Gr = \frac{vg\beta(T'_w - T'_\infty)}{v'^3_0}, Gc = \frac{vg\beta^*(C'_w - C'_\infty)}{v'^3_0}, Sc = \frac{v}{D}, S = \frac{4vS'}{v'^2_0}, Ec = \frac{v'^2_0}{Cp(T'_w - T'_\infty)}, \quad (9)$$

$$Sc = \frac{v}{D}, S = \frac{4vS'}{v'^2_0}, Ec = \frac{v'^2_0}{Cp(T'_w - T'_\infty)}. \quad (10)$$

Substituting (7), (8), (9) and (10) in equations (2), (3) and (4) under boundary conditions (5) and (6), we get:

$$\frac{1}{4} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (Gr)\theta + (Gc)C + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K} \right) u, \quad (11)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \frac{1}{4} S\theta + (Ec) \left(\frac{\partial u}{\partial y} \right)^2, \quad (12)$$

$$\frac{1}{4} \frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2}. \quad (13)$$

The corresponding boundary conditions are:

$$u = 0, \theta = 1 + \epsilon e^{i\omega t}, C = 1 + \epsilon e^{i\omega t} \text{ at } y = 0, \quad (14)$$

$$u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty, \quad (15)$$

where B_0 , magnetic field component along y' -axis, C_p , specific heat at constant pressure, G_r , grashof number, G_c , modified trashof number, g , acceleration of gravity, K' , the permeability of medium, K , the permeability parameter, M , hartmann number, Pr , prandtl number, Sc , schmidt number, S , heat source parameter, Ec , eckert number, D , chemical molecular diffusivity, T' , temperature of fluid near the plate, T'_w , temperature of the fluid far away of the fluid from the plate, T'_∞ , temperature of the fluid at infinity, C , concentration of the fluid, C' , concentration of fluid near the plate, C'_w , concentration of the fluid far away of the fluid from the plate, C'_∞ , concentration of the fluid at infinity, t' , time in x', y' - coordinate system, t , time in dimensionless coordinates, u' , velocity component in x' - direction, u , dimensionless velocity component in x' - direction, v' , velocity component in y' - direction, v'_0 , dimensionless velocity component in y' - direction, x', y' , coordinate system, x, y , dimensionless coordinates, β , coefficient of volume expansion for heat transfer, β^* , coefficient of volume expansion for mass transfer, κ , thermal conductivity of the fluid, τ , electrical conductivity of the fluid, ν , kinematic viscosity, θ , non dimensional temperature, ρ , density of the fluid, ω , angular frequency, ωt , phase angle.

3. Method of solution

The systems of coupled nonlinear equations (11), (12) and (13) with the relevant boundary conditions (14) and (15) are solved numerically for the velocity, temperature and concentration distributions. The finite element method is used to obtain an accurate and efficient solution to the boundary value problem under consideration. The fundamental steps comprising the method are as follows:

3.1. Step 1: Discretization of the domain into elements:

The whole domain is divided into finite number of sub-domains, a process known as discretization of the domain. Each sub-domain is termed a finite element. The collection of elements is designated the finite element mesh.

3.2. Step 2: Derivation of the element equations:

The derivation of finite element equations i.e. algebraic equations among the unknown parameters of the finite element approximation, involves the following three steps: a) Construct the variational formulation of the differential equation. b) Assume the form of the approximate solution over a typical finite element. c) Derive the finite element equations by substituting the approximate solution into variational formulation.

3.3. Step 3: Assembly of element equations:

The algebraic equations so obtained are assembled by imposing the inter-element continuity conditions. This yields a large number of algebraic equations, constituting the global finite element model, which governs the whole flow domain.

3.4. Step 4: Impositions of boundary conditions:

The physical boundary conditions defined in equation (14) and (15) are imposed on the assembled equations.

3.5. Step 5: Solution of the assembled equations:

The final matrix equation can be solved by a direct or indirect (iterative) method. Numerical solutions for these equations are obtained by MATLAB. In order to prove the convergence and stability of finite element method, the same MATLAB was run with slightly changed values of the previous and no significant change was observed in the values of u , θ and C . This process is repeated until the desired accuracy of 0.0005 is obtained. Hence, the finite element method is stable and convergent.

4. Results and discussion

In order to get physical insight into the problem, we have carried out numerical calculations for nondimensional velocity field, temperature field, species concentration field and by assigning some specific values to the parameters entering into the problem and the effects of these values on the above fields are demonstrated graphically. In the present study we adopted the following default parameter values of finite element computations: $Gr = 5.0$, $Gc = 5.0$, $M = 1.0$, $K = 1.0$, $Pr = 0.71$, $E_c = 0.001$, $Sc = 0.6$, $\epsilon = 0.002$, $\omega = 1.0$, $t = 1.0$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

Figure 1 shows the typical velocity profiles in the boundary layer for various values of the magnetic parameter M . Due to the damping effect of the magnetic field, increase in the values of M have a tendency to slow the motion of the fluid and make it warmer as it moves along the vertical plate causing the tangential velocity to decrease. Figures 2 and 4 show the effects of the heat source parameter S on the velocity and temperature profiles respectively. It is clear that

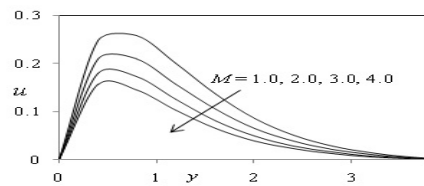


Figure 1. Effect of hartmann number on velocity profiles u .

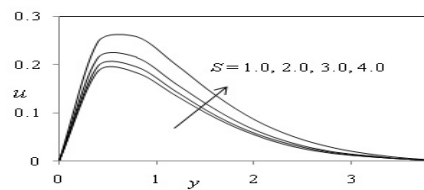


Figure 2. Effect of heat source parameter on velocity profiles u .

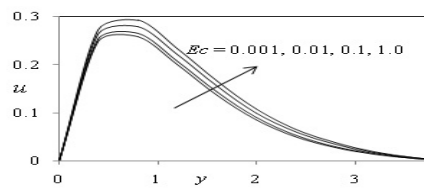


Figure 3. Effect of eckert number on primary velocity profiles u .

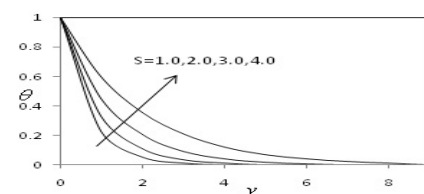


Figure 4. Effect of heat source parameter on temperature profiles θ .

as heat source parameter increases, an increase in the fluid temperature occurs and through the thermal buoyancy effect, the velocity increases. The tangential velocity and temperature profiles for various values of eckert numbers are shown in figure 3 and figure 4. From figure 3 we may conclude that the presence of viscous dissipation and joule heating increase in the tangential velocity and the temperature. This rise in temperature is due to the heat created by viscous dissipation and compression work ($E_c \neq 0$).

Figure 6 shows the effect of the schmidt number Sc on the concentration profile. As the schmidt number Sc increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity and also show that the reductions the concentration profiles are accompanied by the simultaneous reductions in the concentration boundary layers.

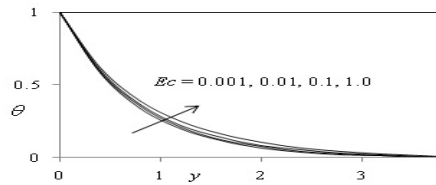


Figure 5. Effect of eckert number on temperature profiles θ .

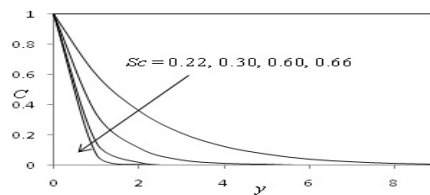


Figure 6. Concentration profiles for different values of Sc .

5. Conclusion

In this study, the governing non-linear partial differential equations are transformed into system of algebraic non-linear equations by using finite element method. The numerical results conclude that as magnetic parameter increases, the value of the velocity decreases. This conclusion meets the logic of magnetic field exerting a retarding force on the free convection flow. It is notice that there is a rise in the temperature and the velocity due to the heat created by the viscous dissipation, free convection and heat source. The effect of increasing schmidt number is to reduce the concentration boundary layer thickness of the flow field at all points.

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