

Study of SH-type wave propagating in an anisotropic layer sandwiched between an orthotropic medium and an in-homogeneous half-space

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Abstract. The present paper investigates the effect of initial stress and the existence of SH-type wave in an initially stressed porous stratum sandwiched between an initially stressed orthotropic medium and an in-homogeneous half-space. Exponential variations of rigidity and density have been taken in the lower half-space. The dispersion equation has been derived in closed form. It is observed that the propagation of SH type wave is influenced by the medium characteristics such as porosity, initial stress, anisotropy and in-homogeneity parameter.

1. Introduction

The study of seismic wave propagation has long been a prime interest to the seismologist as it is responsible for constituting and layering the earth's interior. Earth's structure is very complex in nature containing various types of layers including porous layer contribute amazing characteristics such as anisotropy, heterogeneity and stress parameter also. Generally these pores can contain oil or gas or water and rock will be saturated with one of these three. The studies of SH-type wave propagation in a fluid saturated anisotropic media have received considerable attention in the field of Earthquake engineering, Geophysical problem and soil dynamics. Scattering of SH-type wave in multi-layered media with irregular interfaces was developed by Ding and Darvinski [1]. A model for spherical SH-wave propagation in self-reinforced linearly elastic media was formulated by Chattopadhyay and Michel [2]. By the virtue of Biot's dynamic theory [3, 4] on fluid saturated porous media Deresiewicz [5] was the first who studied the propagation of Love waves in a porous layer resting on an elastic homogeneous isotropic semi-infinite space. Propagation of SH-type wave in a fluid saturated elastic plate was discussed by Pradhan et al. [6]. Sharma and Gogna [7] studied the propagation of Love waves in an initially stressed medium consisting of a slow elastic layer lying over a liquid-saturated porous solid half-space. Possibility of Love wave propagation in a porous layer under the effect of linearly varying directional rigidities was introduced by Gupta et al. [8]. The present paper deals with the propagation of SH-type wave in an anisotropic layer sandwiched between orthotropic layer and an in-homogeneous half-space. SH-type waves propagate in the layer and attenuate along the thickness of the substrate so that its particle of motion and direction of propagation are in a same horizontal line. Since the composition of the Earth is a heterogeneous medium



under high initial stresses so the medium in-homogeneous, orthotropic, anisotropic and initial stresses contribute a significant effect in the propagation of seismic waves.

2. Formulation of the problem

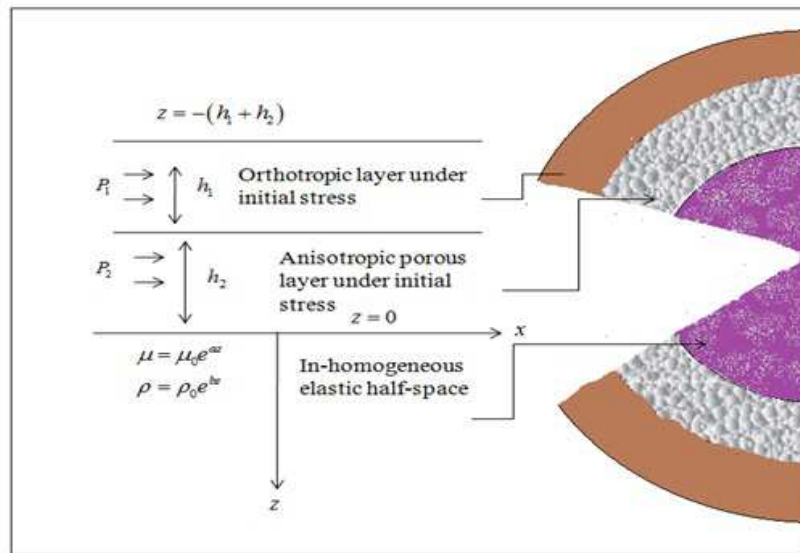


Figure 1. Schematic diagram of the physical problem.

3. Basic preliminaries

3.1. Solution for the upper layer

The upper layer is considered as the orthotropic medium with finite thickness h_2 under initial stress P_1 along x axis. Let u_1, v_1, w_1 are the displacement components, ω_x, ω_y and ω_z are the rotational component along x, y and z axis respectively. Now the dynamical equations of motion for this medium in the absence of body forces are (Biot [9]):

$$\left. \begin{aligned} \frac{\partial S_{11}}{\partial x} + \frac{\partial S_{12}}{\partial y} + \frac{\partial S_{13}}{\partial z} - P_1 \left(\frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) &= \rho_1 \frac{\partial^2 u_1}{\partial t^2} \\ \frac{\partial S_{21}}{\partial x} + \frac{\partial S_{22}}{\partial y} + \frac{\partial S_{23}}{\partial z} - P_1 \frac{\partial \omega_z}{\partial x} &= \rho_1 \frac{\partial^2 v_1}{\partial t^2} \\ \frac{\partial S_{31}}{\partial x} + \frac{\partial S_{32}}{\partial y} + \frac{\partial S_{33}}{\partial z} - P_1 \frac{\partial \omega_y}{\partial x} &= \rho_1 \frac{\partial^2 w_1}{\partial t^2} \end{aligned} \right\} \quad (1)$$

where $S_{ij} (i, j = 1, 2, 3)$ are the incremental stress components for the half-space and ρ_1 is the density of the material in this medium. Using the relations between the strain and incremental stress components with SH-type surface wave condition $u_1 = 0, w_1 = 0$ and $v_1 = v_1(x, z, t)$, the aforementioned equation of motion from (1) for the upper orthotropic medium can be written as

$$\frac{\partial}{\partial x} \left(R_3 \frac{\partial v_1}{\partial x} \right) + \frac{\partial}{\partial z} \left(R_1 \frac{\partial v_1}{\partial z} \right) - P_1 \frac{\partial}{\partial x} \left(\frac{1}{2} \frac{\partial v_1}{\partial x} \right) = \rho_1 \frac{\partial^2 v_1}{\partial t^2}. \quad (2)$$

The above equation can be restated as

$$\left(R_3 - \frac{P_1}{2} \right) \frac{\partial^2 v_1}{\partial x^2} + R_1 \frac{\partial^2 v_1}{\partial z^2} = \rho_1 \frac{\partial^2 v_1}{\partial t^2}. \quad (3)$$

The only two non-zero components are $S_{12} = 2Q_3 e_{12}$ and $S_{23} = 2Q_1 e_{23}$. Here e_{ij} is the strain components defined by

$$e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

For a wave propagating along x direction, it may be assumed that $v_1 = V_1(z)e^{ik(x-ct)}$ then the equation (3) can be converted to

$$\frac{d^2 V_3}{dz^2} - m^2 V = 0, \quad (4)$$

where

$$m^2 = \frac{k^2}{R_1} \left[\left(R_3 - \frac{P_1}{2} \right) - c^2 \rho_1 \right].$$

So, the solution for the pre-stressed upper semi-infinite orthotropic medium can be taken as

$$v_1 = (A_1 e^{mz} + A_2 e^{-mz}) e^{ik(x-ct)}, \quad (5)$$

where A_1, A_2 are arbitrary constants and c is the phase velocity.

3.2. Solution for the intermediate anisotropic medium

The effective equation of motion in an anisotropic porous layer under the effect of initial stress, ignoring the viscosity of fluid and body forces are given by (Biot [10])

$$\left. \begin{aligned} \frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} - P_2 \frac{\partial \omega'_x}{\partial y} + P_2 \frac{\partial \omega'_y}{\partial z} &= \frac{\partial^2}{\partial t^2} (\rho_{11} u_2 + \rho_{12} U_x) \\ \frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} - P_2 \frac{\partial \omega'_z}{\partial x} &= \frac{\partial^2}{\partial t^2} (\rho_{11} v_2 + \rho_{12} V_y) \\ \frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{32}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} - P_2 \frac{\partial \omega'_y}{\partial x} &= \frac{\partial^2}{\partial t^2} (\rho_{11} w_2 + \rho_{12} W_z) \\ \frac{\partial S}{\partial x} = \frac{\partial^2}{\partial t^2} (\rho_{12} u_2 + \rho_{22} U_x), \quad \frac{\partial S}{\partial y} = \frac{\partial^2}{\partial t^2} (\rho_{12} v_2 + \rho_{22} V_y), \quad \frac{\partial S}{\partial z} = \frac{\partial^2}{\partial t^2} (\rho_{12} w_2 + \rho_{22} W_z) \end{aligned} \right\} \quad (6)$$

where $\sigma_{ij} (i, j = 1, 2, 3)$ are the incremental stress components, (u_2, v_2, w_2) and (u_x, v_y, w_z) are the components of the displacement vector for solid and liquid respectively along x, y and z direction respectively, S is the stress vector due to the liquid part of porous which is related to the fluid pressure p by the relation $-S = fp$.

The angular component ω'_x, ω'_y and ω'_z are defined as

$$\omega'_x = \frac{1}{2} \left(\frac{\partial w_2}{\partial y} - \frac{\partial v_2}{\partial z} \right), \quad \omega'_y = \frac{1}{2} \left(\frac{\partial u_2}{\partial z} - \frac{\partial w_2}{\partial x} \right), \quad \omega'_z = \frac{1}{2} \left(\frac{\partial v_2}{\partial x} - \frac{\partial u_2}{\partial y} \right).$$

The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} are related to the densities ρ', ρ_s and ρ_w of the layer, solid and liquid respectively by

$$\rho_{11} + \rho_{12} = (1 - f) \rho_s, \quad \rho_{12} + \rho_{22} = f \rho_w,$$

so that the mass densities of the bulk material is

$$\rho' = \rho_{11} + 2\rho_{12} + \rho_{22} = \rho_s + f(\rho_w - \rho_s).$$

These mass co-efficient also obey the following inequalities

$$\rho_{11} > 0, \quad \rho_{22} > 0, \quad \rho_{12} < 0, \quad \rho_{11}\rho_{22} - \rho_{12}^2 > 0.$$

For the SH-type wave propagating along x direction, having the displacement of particle along the y direction as the displacement along the other direction vanishes, we have

$$\left. \begin{aligned} u_2 &= 0, & w_2 &= 0, & v_2 &= v_2(x, z, t) \\ U_x &= 0, & W_z &= 0, & V_y &= V(x, z, t) \end{aligned} \right\}. \quad (7)$$

Substituting relation(7)into dynamical system of equation (6), the equation of motion which are not identically satisfied are

$$\left. \begin{aligned} \frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{23}}{\partial z} - P_2 \frac{\partial \omega_z'}{\partial x} &= \frac{\partial^2}{\partial t^2} (\rho_{11} v_2 + \rho_{12} V_2) \\ \frac{\partial S}{\partial y} = 0 &= \frac{\partial^2}{\partial t^2} (\rho_{12} v_2 + \rho_{22} V_2) \end{aligned} \right\}. \quad (8)$$

With the help of stress-strain relation $\sigma_{23} = 2Le_{yz}$ and $\sigma_{12} = 2Ne_{xy}$ the above equation transformed to

$$\left. \begin{aligned} \left(N - \frac{P_2}{2}\right) \frac{\partial^2 v_2}{\partial x^2} + L \frac{\partial^2 v_2}{\partial z^2} &= \frac{\partial^2}{\partial t^2} (\rho_{11} v_2 + \rho_{12} V) \\ \frac{\partial^2}{\partial t^2} (\rho_{12} v_2 + \rho_{22} V) &= 0 \end{aligned} \right\}. \quad (9)$$

From $\frac{\partial^2}{\partial t^2} (\rho_{12} v_2 + \rho_{22} V) = 0$ and $\rho_{12} v_2 + \rho_{22} V = d''$, so that $V = \frac{d'' - \rho_{12}}{\rho_{22}}$.

Now $\frac{\partial^2}{\partial t^2} (\rho_{11} v_2 + \rho_{12} V) = \rho_2 \frac{\partial^2 v_2}{\partial t^2}$ where $\rho_2 = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}$.

Hence equation (9) can be rewritten as

$$\left(N - \frac{P_2}{2}\right) \frac{\partial^2 v_2}{\partial x^2} + L \frac{\partial^2 v_2}{\partial z^2} = \rho_2 \frac{\partial^2 v_2}{\partial t^2}. \quad (10)$$

From equation (9), we found that the velocity of shear wave along the x direction is $\sqrt{\frac{N - \frac{P_2}{2}}{\rho_2}}$ and along the z direction is $\sqrt{\frac{L}{\rho_2}}$. The shear wave velocity in the porous medium along the x direction can be taken as

$$\beta' = \sqrt{\frac{N - \frac{P_2}{2}}{\rho_2}} = \beta_a \sqrt{\frac{1 - \xi_2}{d}},$$

where $d = \gamma_{11} - \frac{\gamma_{12}^2}{\gamma_{22}}$, $\beta_a = \sqrt{\frac{N}{\rho'}}$ is the velocity of shear wave in the corresponding initial stress free, non-porous, anisotropic, elastic medium along the x direction. $\xi_2 = \frac{P_2}{2N}$ is the non-dimensional parameter due to the initial stress P_2 and $\gamma_{11} = \frac{\rho_{11}}{\rho'}$, $\gamma_{12} = \frac{\rho_{12}}{\rho'}$, $\gamma_{22} = \frac{\rho_{22}}{\rho'}$ are the non-dimensional parameters for the material of the porous layer as obtained by Biot[11].

Thus for the SH-type wave propagating along the x direction the solution of equation(10) can be taken as

$$v_2(x, z, t) = V_2(z) e^{ik(x-ct)}. \quad (11)$$

Applying equation (11) in equation (10) we have

$$\frac{d^2 V_2}{dz^2} + q'^2 k^2 V_2 = 0, \quad (12)$$

where $q' = \sqrt{\frac{1}{L} \left(c^2 \rho_2 - N + \frac{P_2}{2}\right)}$.

Therefore the solution of equation (12) takes the form as

$$v_2 = [A_3 \cos(qz) + A_4 \sin(qz)] e^{ik(x-ct)}.$$

3.3. Solution for the lower half-space

The lower semi-infinite medium is considered as heterogeneous elastic half-space. The displacement components of this medium are u_3, v_3 and w_3 along x, y, z direction respectively. Using SH-type wave conditions $u_3 = w_3 = 0$ and $v_3 = v_3(x, z, t)$, the equation of motion corresponding to the displacement due to SH-type wave can be written as

$$\frac{\partial t_{21}}{\partial x} + \frac{\partial t_{23}}{\partial z} = \frac{\partial^2}{\partial t^2} (\rho_3 v_3), \quad (13)$$

where t_{ij} are the incremental stress components in the half-space and ρ_3 is the density of the material of the half-space. Using the stress-strain relation $t_{21} = 2\mu e_{xy}$, $t_{23} = 2\mu e_{yz}$ and the in-homogeneity conditions the equation (13) can be written as

$$\mu_0 e^{az} \frac{\partial^2 v_3}{\partial x^2} + a\mu_0 e^{az} \frac{\partial v_3}{\partial z} + \mu_0 e^{az} \frac{\partial^2 v_3}{\partial z^2} = \rho_0 e^{bz} \frac{\partial^2 v_3}{\partial t^2}, \quad (14)$$

where $e_{xy} = \frac{1}{2} \left(\frac{\partial v_3}{\partial x} + \frac{\partial u_3}{\partial y} \right)$ and $e_{yz} = \frac{1}{2} \left(\frac{\partial w_3}{\partial y} + \frac{\partial v_3}{\partial z} \right)$.

Let $v_3(x, z, t) = V_3(z) e^{ik(x-ct)}$ be the solution of the above equation. Then equation (14) can take the form as

$$\frac{d^2 V_3(z)}{dz^2} + a \frac{dV_3(z)}{dz} - k^2 \left(1 - \frac{c^2 e^{bz}}{c_0^2 e^{az}} \right) V_3(z) = 0, \quad (15)$$

where $c_0 = \sqrt{\frac{\mu_0}{\rho_0}}$ is the shear wave velocity. Substituting $V_3(z) = \phi(z) e^{-\frac{az}{2}}$ in equation (15) to eliminate the term $\frac{dV_3(z)}{dz}$ we obtain

$$\phi''(z) + \left[k^2 \frac{c^2 e^{bz}}{c_0^2 e^{az}} - k^2 \left(1 - \frac{a^2}{4k^2} \right) \right] \phi(z) = 0.$$

Using non-dimensional parameters $\gamma = \left(1 + \frac{a^2}{4k^2} - \frac{b}{a} \frac{c^2}{c_0^2} \right)^{\frac{1}{2}}$, $\eta = \frac{2\gamma k(1+az)}{a}$ and $\omega = kc$, the above equation reduces to

$$\phi''(\eta) + \left(\frac{R}{\eta} - \frac{1}{4} \right) \phi(\eta) = 0, R = \frac{c^2(a-b)k}{2c_0^2 a^2 \gamma}. \quad (16)$$

The Solution of Whittaker's equation (16) is given by

$$\phi(\eta) = D_1 W_{R, \frac{1}{2}}(\eta) + D_2 W_{R, \frac{1}{2}}(-\eta), \quad (17)$$

where D_1, D_2 are arbitrary constants and $W_{R, \frac{1}{2}}(\eta)$ is the Whittaker function. The solution of equation (17) may be given by Whittaker and Watson [12], satisfying the condition $\lim_{z \rightarrow \infty} V_3(z) \rightarrow 0$ i.e. $\lim_{\eta \rightarrow \infty} \phi(\eta) \rightarrow 0$ may be taken as $\phi(\eta) = D_1 W_{R, \frac{1}{2}}(\eta)$. Hence the displacement component $v_3(z)$ in the in-homogeneous half-space is given by

$$v_3(x, z, t) = D_1 W_{R, \frac{1}{2}} \left[\frac{2\gamma k}{a} (1+az) \right] e^{-\frac{az}{2}} e^{ik(x-ct)}.$$

4. Boundary conditions

The suitable boundary conditions according to the formulation of the problem are as follows

(i) Continuity of the displacement components $v_1 = v_2$ at $z = -h_1$ which gives

$$A_1 e^{-mh_1} + A_2 e^{mh_1} = A_3 \cos(qh_1) - A_4 \sin(qh_1). \quad (18)$$

(ii) Stress components are continuous at $z = -h_1$ that is

$$m \left(A_1 e^{-mh_1} - A_2 e^{mh_1} \right) = \frac{R_1}{L} q \left[A_3 \sin(qh_1) + A_4 \cos(qh_1) \right]. \quad (19)$$

(iii) At $z = 0$ the velocity components of layer and half-space are continuous that is

$$A_3 = D_1 \left(\frac{2\gamma k}{a} \right) e^{-\frac{\gamma k}{a}} \left[1 + (1 - R) \frac{\gamma k}{a} \right]. \quad (20)$$

(iv) At $z = 0$, the stress components of layer and half-space are continuous that implies

$$A_4 q = \frac{\mu_3}{\mu_2} D_1 \left(\frac{2\gamma k}{a} \right) e^{-\frac{\gamma k}{a}} \left[1 + (1 - R) \frac{\gamma k}{a} \right] \left[\frac{(1 - R) \gamma k}{1 + (1 - R) \frac{\gamma k}{a}} - \left(\gamma k + \frac{a}{2} \right) + a \right]. \quad (21)$$

(v) At $z = -(h_1 + h_2)$ stress component for the upper layer vanishes i.e

$$A_1 e^{-m(h_1+h_2)} + A_2 e^{m(h_1+h_2)} = 0. \quad (22)$$

Eliminating A_1, A_2, A_3, A_4 and D_1 from equations (18),(19),(20),(21) and (22) we have

$$\begin{vmatrix} e^{-mh_1} & e^{mh_2} & -\cos(qh_1) & \sin(qh_1) & 0 \\ me^{-mh_1} & -me^{mh_2} & -\frac{L}{R_1} q \sin(qh_1) & -\frac{L}{R_1} q \cos(qh_1) & 0 \\ 0 & 0 & 0 & q & -\frac{\mu_3}{L} \beta \\ 0 & 0 & 1 & 0 & -\alpha \\ e^{-m(h_1+h_2)} & e^{m(h_1+h_2)} & 0 & 0 & 0 \end{vmatrix} = 0,$$

where $\alpha = \left(\frac{2\gamma k}{a} \right) e^{-\frac{\gamma k}{a}} \left[1 + (1 - R) \frac{\gamma k}{a} \right]$ and

$\beta = \left(\frac{2\gamma k}{a} \right) e^{-\frac{\gamma k}{a}} \left[1 + (1 - R) \frac{\gamma k}{a} \right] \left[\frac{(1-R)\gamma k}{1+(1-R)\frac{\gamma k}{a}} - \left(\gamma k + \frac{a}{2} \right) + a \right]$. This gives that

$$\tan(qh_1) = \frac{mq\alpha - \frac{\mu_3}{R_1} \beta q \tanh(mh_2)}{\frac{\mu_3}{L} \beta m + \alpha \frac{L}{R_1} q^2 \tanh(mh_2)}. \quad (23)$$

This is the dispersion equation of SH-type wave in an anisotropic porous layer sandwiched between an orthotropic medium and an in-homogeneous half-space.

5. Particular cases

5.1. Case 1

If we neglect the upper orthotropic layer then equation (24) can be introduced as

$$\tan(qh_1) = \frac{-\mu_3 \beta q}{\mu_2 \alpha q^2}. \quad (24)$$

5.2. Case 2

when the lower half-space be homogeneous that is $a \rightarrow 0, b \rightarrow 0$ this implies that $\gamma = \sqrt{1 - \frac{c^2}{c_0^2}}$ and therefore equation (24) reduces to

$$\tan \left[kH \left(\frac{N}{L} \right) d \left\{ \frac{c^2}{\beta_p^2} - \frac{1 - \xi_2}{d} \right\} \right]^{\frac{1}{2}} = \frac{\mu_3}{L} \frac{\sqrt{1 - \frac{c^2}{c_0^2}}}{\sqrt{\left(\frac{N}{L} \right) d \left\{ \frac{c^2}{\beta_p^2} - \frac{1 - \xi_2}{d} \right\}}}. \quad (25)$$

5.3. Case 3

If the intermediate layer is initially stress free, isotropic, non-porous elastic layer (i.e. $\xi_2 = 0$, $N = L = \mu_2$, $d = 1$) then the above equation reduces to

$$\tan \left[kH \left\{ \frac{c^2}{\beta_p^2} - 1 \right\} \right]^{\frac{1}{2}} = \frac{\mu_3}{\mu_2} \frac{\sqrt{1 - \frac{c^2}{c_0^2}}}{\sqrt{\frac{c^2}{\beta_p^2} - 1}}. \quad (26)$$

This is the general equation of Love wave established by Love [13] and hence validating the solutions of the problem discussed in the current paper.

6. Numerical computation and discussion

In order to show the effect of porosity, anisotropy, in-homogeneity and initial stresses on the propagation of SH-type wave, we take the following numerical values from Gubbins [14] and Kar and Kalyani [15]):

(1) For the upper isotropic homogeneous medium

$$\mu_1 = 6.54 \times 10^{10} \text{ N/m}^2, \rho_1 = 1409 \text{ kg/m}^3$$

(2) For the intermediate anisotropic porous layer

$$N = 5.00 \times 10^{10} \text{ N/m}^2, \rho_2 = 9890 \text{ kg/m}^3$$

(3) For the lower semi-infinite orthotropic half-space

$$Q_1 = 5.82 \times 10^{10} \text{ N/m}^2, Q_3 = 3.99 \times 10^{10} \text{ N/m}^2.$$

In figures 2-5, different curves are plotted from the dispersion equation to show the effect of

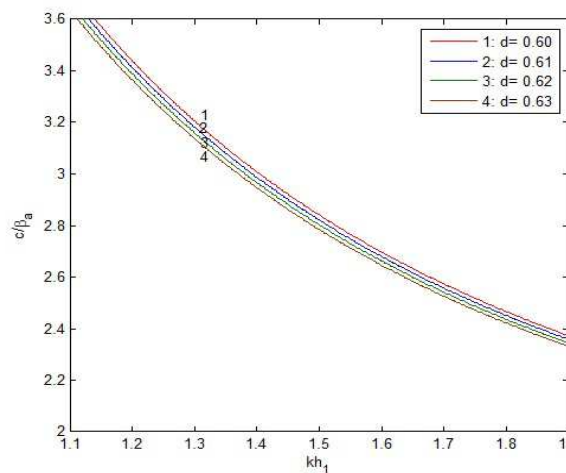


Figure 2. Dimensionless phase velocity against dimensionless wave number under the effect of porosity parameter (d).

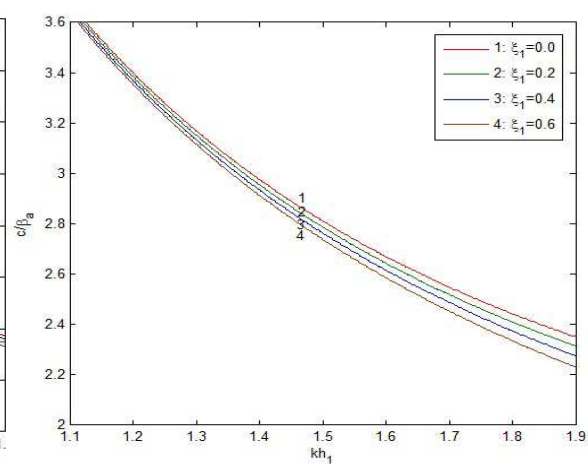


Figure 3. Effect of initial stress ξ_1 in the orthotropic layer on the propagation of SH-type waves.

various parameter. All the figures have been plotted with vertical axis as non-dimensional phase velocity $\frac{c}{\beta_a}$ against horizontal axis as dimensional wave number kh_1 . Figure 2 depicts the effect of porosity on the propagation of SH-type wave. We observed that as the porosity increases i.e. as the value of d decreases, the velocity of SH-type wave increases. Figure 3 demonstrates the effect of initial stress (ξ_1) present in the upper orthotropic layer while the intermediate porous layer (i.e. $\xi_2 = 0$) is initially stress free. It has been observed that an increase in Compressive initial

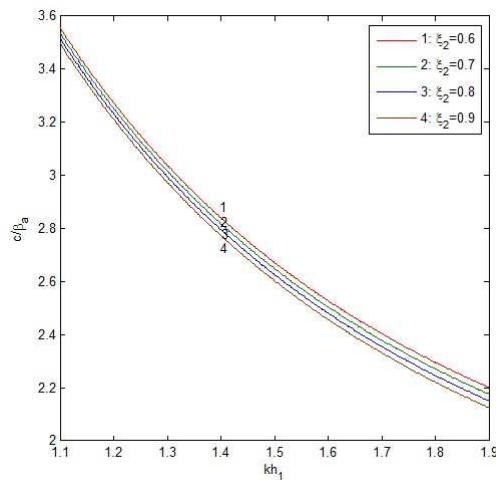


Figure 4. Effect of initial stress ξ_2 in the porous layer on the propagation of SH-type waves for $d=0.6154$.

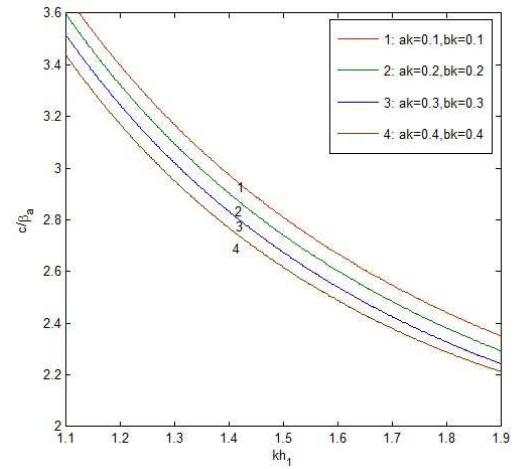


Figure 5. Effect of in-homogeneity parameter $\frac{a}{k}, \frac{b}{k}$, in the lower half-space on the propagation of SH-type wave for $d=0.6154$.

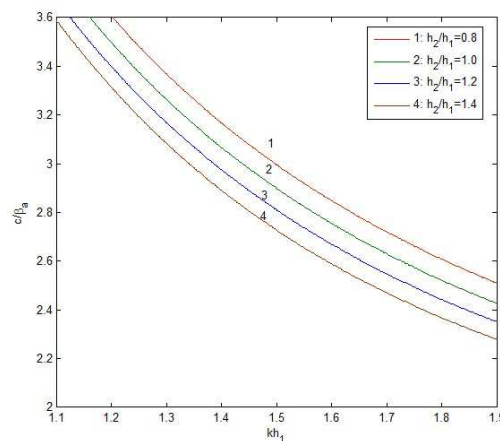


Figure 6. Dimensionless phase velocity against dimensionless wave number for different value of $\frac{h_2}{h_1}$ at a constant value of $d=0.6154$.

stress in the orthotropic layer decreases the velocity of SH-type wave. Figure 4 reflects the effect of initial stress in the porous layer when the stress for the upper orthotropic layer vanishes. It has been found that the phase velocity of SH-type wave decreases, as the value of initial stress parameter increases. Figure 5 shows the effect of in-homogeneity parameter $\frac{a}{k}$ and $\frac{b}{k}$ on the propagation of SH-type wave in the absence of initial stress parameters. The curves of this figure represented that the phase velocity decreases with the increase of in-homogeneity parameter. Figure 6 represents the effect of the ratio of depth parameter $\frac{h_2}{h_1}$ on the propagation of SH-type wave. It can be added that as the width ratio increases the phase velocity decreases.

7. Conclusion

The present paper deals with the propagation of SH-type wave in distinct vertically anisotropic double layers overlying an in-homogeneous half-space. Analytical solutions for the displacement in both the layer and half-spaces have been obtained separately in closed form. The dispersion relation has been obtained in terms of Whittakers functions which are expanded asymptotically and the terms up to second degree are retained. It is observed from the dispersion equation that the porosity, anisotropy and initial stresses have a significant effect on the propagation of SH-type wave.

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