

# Selenocentric reference coordinates net in the dynamic system

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**Abstract.** In this work, the selenocentric dynamic reference net was developed for the first time in the field of selendesy in order to address problems with space navigation. Three tasks were addressed in this research: a) the analysis of the mathematical model of the orthogonal coordinate transformation accuracy; b) the identification of the basic dynamic reference system objects with ones that are contained in reducing catalogues; and c) the extension of the base points net of the basic dynamic reference system. The result was a dynamic coordinate system summary that contains 1 162 objects. The correlation analysis of this net was carried out and was found to coincide with modern dynamic coordinate systems that have been obtained. This selenocentric reference catalogue covers the full visible area of the Moon.

## 1. Introduction

Currently, Moon is a subject of research in many space experiments in the fields of astronomy and planetary science. The modern space missions “Clementine” [1], “Lunar Reconnaissance Orbiter” (LRO) [2] and “KAGUYA (SELENE)” [3] swiftly and radically changed attempts at the study of the moon [4]. Today, there are prospects in the industrial and robotic exploration of the moon for creating long-term manned lunar bases. Development of these space technologies imposes special requirements on the results of coordinate-time support. These requirements include the development of reference systems, establishing mutual orientation of dynamic and inertial coordinate systems and the investigation of the dynamics and geometry of the celestial bodies. This fully applies to the dynamic and geometric parameters of the moon that refer to its centre of mass. However, despite the outstanding results in this direction derived from observations of space missions “Apollo”, “Zond”, “LRO” and “Kaguya” the task of creating a global selenocentric network has still not been completely solved.

## 2. Formulation of the problem

Modern reference selenodetic networks are not equally accurate for the different coordinate axes, and they may even have an ellipsoidal distribution of errors. It should be noted that there are three main aspects to the problem [4].

First, Doyle et al. [5] carried out a detailed analysis of the internal precision of the Apollo system. Based on this analysis, we can formulate the following conclusions. To transform the topographical origin of Apollo three ALSEP stations were used. Since the mean square errors of the transformation were less than  $m$  and a measurement error was about  $m$ , we can assume that the points near and between the three ALSEP stations have errors of positions less than  $m$ . The offset from the location of the ALSEP increases the error up to  $\pm 400$  m, and this is a big part of the observed area. The errors of the point positions lying near the borders of the studied areas can reach  $\pm 400$  m and even exceed  $\pm 1000$  m. When the mission LRO have been received, very accurate altitude data but horizontal coordinates of the lunar objects have the dubious binding to the dynamical coordinate system too. Despite the fact that the spacecraft can scan the lunar surface with accuracy down to a centimeter (for example LRO, Kaguya, Clementine etc.), we obtain a very precise physical lunar surface but inaccurate dimensioning of the dynamic coordinate system.

The accurate satellite binding of lunar objects can only be carried out using one of two methods:

- 1) simultaneous scanning of the lunar surface with the exact definition of the coordinates of the laser impingement point and the binding of the orbiter to the stars, or
- 2) installation of lighthouses on the entire lunar surface. At the present time, the technical capabilities for both of these options are unavailable.



Second, at the present time, all data for the lunar selenography can be divided into two types. On one hand, the first data type obtained by the lunar surface laser scanning from onboard satellites describes the lunar relief well, but it does not give the coordinates for the reference objects on the moon. Other data types obtained on the basis of directly binding lunar objects to the stars gives the exact coordinates of reference objects, but they do not describe the lunar relief with sufficient accuracy. All of these systems have a different frame of reference and orientation of the axes. It is well known that data obtained from all of the space missions relate to a quasi-dynamic frame of reference in which the centre of origin is the lunar centre mass (LCM), but their axes do not coincide with the axes of inertia on the moon.

Third, most modern selenodetic catalogues have a quasi-dynamic system of coordinates. Either they have a centre of origin of coordinates that does not coincide with the LCM or the axes do not coincide with the axes of inertia on the moon. At the present time, it should also be noted that there is no reference dynamic selenocentric coordinate system covering enough area on the lunar surface based on satellite observations. In addition, despite the accuracy of the lunar physical relief obtained by space missions, the reference surface of this relief has a rather vague figure. Therefore in most articles describing satellite topographic data, much attention is paid to the high accuracy of the obtained physical relief and the question of the surface reference is glossed over. So far, there is no method for constructing the lunar photogrammetric topographic satellite map based on the integration of the thousands of separate pictures of the lunar surface to a single system on an absolute basis. Thus, we cannot say that satellite topographic data allows for the production of full-value topographic models with a complete reference surface.

### 3. The method of the making of the dynamical selenocentric reference coordinate system DSC

The dynamical selenocentric catalogue (DSC) on the surface of the Moon was based on large-scaled Moon and stars pictures taken by new unique method of separate plates. In contrast to the methods of taking pictures of the Moon without stars, in this method we have an absolute orientation definition, 0-point of coordinate system and its scale. Choosing the Moon craters included to the bearing net DSC we used following criterions. At first, we chose round shaped craters. Secondly, these craters have not very big size. In third, chosen craters must be well-observed. In the fourth, the net craters must be in others selenodesic catalogue objects lists and satisfy IAU recommendations. Required parameters are got from 2 amendment equation of this type:

$$\mathbf{A} \times \boldsymbol{\theta} + \boldsymbol{\varepsilon} = \mathbf{Z}, \quad (1)$$

$\mathbf{A}(A_{ij})$  – structured transition matrix,  $\boldsymbol{\theta}(\Delta\xi, \Delta\eta, \Delta\zeta)$  – column vector of required parameters,  $\boldsymbol{\varepsilon}$  – column vector of random observational errors,  $\mathbf{Z}$  – column vector of observations.

The determination of the required parameters  $\hat{\boldsymbol{\theta}}(\hat{\Delta\xi}, \hat{\Delta\eta}, \hat{\Delta\zeta})$  is:

$$\hat{\boldsymbol{\theta}} = (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} (\mathbf{A}^T \mathbf{P} \mathbf{Z}), \quad (2)$$

and their errors are determined by the covariance matrix

$$\mathbf{D}(\hat{\boldsymbol{\theta}}) = \frac{\mathbf{V}^T \mathbf{P} \mathbf{V}}{2m-3} (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1}, \quad (3)$$

$\mathbf{V}$  – vector of residual deviations.

The analysis of the DSC catalogue showed it fully satisfy following criterions: includes a sufficient number of control points for discovering the Moon figure and implementation of the exact binding to them; contains objects with coordinates referred to the Moon's ephemeris center of mass and covering quite big part of the Moon surface; presented points' accuracy reaches  $\pm 35$  meters in planning coordinates and in average  $\pm 60$  meters in heights.

The target of the research is to improve the accuracy of the condensation and expansion selenocentric control system based on optimal coordinate transformations. Elements of the matrix and the displacement vector can be obtained from common points for DSC and converted into a system in the particular directory.

The main problem with extending the DSC (system  $\mathbf{X}$ ) is a precise determination of the orientation matrix elements and displacement vector of the origin of coordinates during the transition from the  $\mathbf{Y}$  coordinate system to another system by common points:

$$\mathbf{X} = \mathbf{A}\mathbf{Y} + \mathbf{X}_0. \quad (4)$$

$\mathbf{A}$  – the transition matrix,  $\mathbf{X}_0$  – the origin of the coordinates displacement vector of the  $\mathbf{Y}$  to the zero point of the  $\mathbf{X}$  coordinate system. The exact solution of the problem becomes more important in the coordinates extrapolation. In our case, it is especially important as the objects of the reverse side of the moon are outside the set of control points.

Achieving the highest possible accuracy of transformation coordinates (TC) is provided by the following tasks:

a) Justifying the applicability of the adaptive regression modelling (ARM-approach), which is provided for the formation of the model (4);

- Assessing the quality of the model TC;
- Diagnosing compliance with the terms of the computational scheme MNE (in particular, Gauss-Markov conditions);
- A numerical adjustment to a material breach of the condition;

b) Methods of the ARM-approach application development, which includes:

- Criteria for the accuracy of the transition;
- A set of competing mathematical models of the TC;
- The corresponding set of methods for structural and parametric identification;
- The scenario data, providing an assessment of the prognostic properties of the model diagnostics of the conditions and adaptation in violation to provide the required optimal estimates of the properties (consistency, unbiasedness and efficiency).

In the ARM-approach [6], it is postulated that the matrix model structure of TC (5) is unknown for each pair of catalogues. In general form, for example, the first equation (4) can be rewritten in the matrix of the regression equation form:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad (5)$$

adding the error term  $\boldsymbol{\varepsilon}$  and considering that the first row of the matrix is the  $\boldsymbol{\beta}$  vector. Obviously, in structural identification, the parameters of the equation are simultaneously estimated, namely the vector  $\boldsymbol{\beta}$  elements for the simple case (5).

Because of the errors in the coordinates determination in both systems and the possible multicollinearity (interdependence) of the estimates, the matrix  $\mathbf{A}$  often does not satisfy the orthogonality conditions of the transition from  $\mathbf{Y}$  to  $\mathbf{X}$ , written as:

$$\mathbf{A}\mathbf{T}\mathbf{A} = \mathbf{E}, \det \mathbf{A} = 1 \quad (6)$$

In this regard, the basic deterministic transformation should be considered to be the equation (4), considered together with the conditions (6). This task, with accuracy up to the centres of the  $\mathbf{Y}$  and  $\mathbf{X}$  systems' divergence and the scale factor problem, is solved by a numerical method of optimization.

At this stage, as a model of approximation, the transformation was applied to model (4) without the condition (6). Later, in the concentration of the fundamental DSC, algebraic polynomials of the second and third degree, as well as the two-component description where the first component is a deterministic model (4) and (6), and the second includes marked polynomials to describe the residue after the first component, will be used. The optimal model will be derived from the ARM approach.

#### 4. Summary and conclusions

For the moon, there are several dynamical coordinate systems, and among them, the most informative is the Dynamical Selenocentric Catalogue (DSC), which was built at the Engelhardt Astronomical Observatory (EAO). Plane coordinates in the catalogue were obtained by direct binding of lunar objects to the stars, as well as altitude data that was adjusted based on the data LRO mission. Since its creation, lunar objects that are directly tied to the stars can be considered DSC-made in a dynamic coordinate system.

The analysis of the DSC catalogue showed that it fully satisfied the following criteria: the inclusion of a sufficient number of control points for discovering the moon's figure, the implementation of the exact binding to those points, containing objects with coordinates that referred to the moon's ephemeris centre of mass and covering a large part of the moon's surface, and presented points that have an accuracy that reaches metres in planning coordinates and on average meters in heights. In this work, direct binding to DSC was used to create the union lunar dynamic network on the lunar surface, covering the best lunar sphere and containing sufficient reference points for navigation tasks.

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