

# Model of dynamical structures in synchrotron images of pulsar wind nebulae

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**Abstract.** The relativistic pulsar winds are forming the pulsar wind nebulae (PWN) — the unique extended sources of non-thermal radiation detected in all bands of the electromagnetic spectrum. High angular resolution imaging of the PWN with modern orbital and ground-based telescopes makes possible to study the highly non-equilibrium processes in the pulsar wind plasma. Physical interpretation of the observed dynamic structures in the nebulae requires modeling of the relativistic pulsar wind. The main mechanism of emission of the magnetized relativistic pair plasma in the PWN is the synchrotron radiation, and the observed dynamical structures may be related with a propagation of perturbations of the magnetic field. A kinetic approach to highly non-equilibrium relativistic pair plasma allows us to evaluate the structure of the perturbation of the magnetic field propagating transverse to the mean quasi-stationary magnetic field. We present synchrotron images with the dynamic structures in the PWN simulated in the relativistic pair plasma with the strong scattering of pairs by the stochastic magnetic field fluctuation.

## 1. Introduction

The pulsar wind nebulae (PWN), formed by the relativistic pulsar winds, present one of the most interesting classes of objects for the high energy astrophysics. The emission of these nebulae is detected across the electromagnetic spectrum. The conversion mechanism, responsible for the highly efficient transformation of the energy of a rotating neutron star into the electromagnetic energy of a PWN is one of the actual problems. High angular resolution observations of the PWN, possible thanks to modern orbital and ground-based telescopes, allow studying highly non-equilibrium processes in the pulsar wind plasma. Such observations revealed the presence of dynamical structures, like jets and wisps, in the Crab nebula[1,2]. The origin of the high energy gamma-ray flares, discovered in 2011 by Fermi and AGILE telescopes, is one of the last hot issues[2–4].

The study of the origin of the observed dynamical structures can help us to answer the question about their possible relationship with the gamma-ray flares. It also may be useful for understanding the physics of the processes, involved in the energy conversion mechanism in the relativistic pair plasma of the PWN. To interpret these structures, the modeling of the relativistic pulsar wind is required.



The main mechanism of emission of the magnetized relativistic pair plasma in the PWN is the synchrotron radiation. Thus, a propagation of perturbations of the magnetic field may be responsible for the observed dynamical structures.

Applying a kinetic approach to highly non-equilibrium relativistic pair plasma, we can evaluate the structure of the perturbation of the magnetic field propagating transverse to the mean quasi-stationary magnetic field. We describe the scheme of obtaining the evolution equation for the magnetic perturbation propagating in the relativistic pair plasma with the strong scattering of pairs by the stochastic magnetic field fluctuation. We discuss some solutions of this equation and present the synchrotron images of their evolution.

## 2. The evolution equation

### 2.1. The kinetic model

We consider a local weakly-nonlinear perturbation of the magnetic field, propagating transverse to the mean quasi-stationary magnetic field  $B_0$  in highly non-equilibrium relativistic pair plasma. Due to locality, one-dimensional analysis is applicable. We assume that  $\mathbf{B}_0 \parallel Oz$ , and that the perturbation, considered as a linearly polarized wave with electric field  $\mathbf{E} \parallel y$  and magnetic field  $\mathbf{b} \parallel z$ , propagates along the  $Ox$  axis. The 'collisional' regime of the strong scattering of pairs by the stochastic magnetic field fluctuation is supposed. We introduce the typical frequency of such 'collisions' (scatterings)  $\nu$  and the gyrofrequency  $\Omega = eB_0/mc\gamma$ . Here  $e$  is the positron charge,  $m$  – the mass of a particle,  $c$  – the light velocity,  $\gamma$  – the Lorentz-factor of a particle. The typical frequencies of the processes under consideration  $\omega$  are supposed to be small in comparison with  $\nu$ . The frequency of scatterings by the magnetic field fluctuation may not exceed the gyrofrequency. We write for the frequencies:

$$\omega \ll \nu \leq \Omega, \quad \nu = a\Omega \quad (1)$$

We use the kinetic approach and seek the distribution functions of positrons and electrons in the form of  $f_\alpha = f_0 + f_\alpha$ , where  $f_0$  is the isotropic part of the distribution function,  $\alpha = p, e$ . The subscripts 'p' and 'e' refer to positrons and electrons, respectively. The collision operator is taken in the form of the relaxation time approximation

$$-\nu(f - f_0) = -\nu\tilde{f}.$$

The system of kinetic equations for the components of pair plasma takes the form:

$$\partial_t f_p + v_x \partial_x f_p - \Omega \cdot \hat{\mathbf{O}} f_p + e \left( E_y - \frac{1}{c} v_x b \right) \partial_{p_y} f_p + \frac{e}{c} v_y b \partial_{p_x} f_p = -\nu_1 \tilde{f}_p + \nu_2 \tilde{f}_e \quad (2)$$

$$\partial_t f_e + v_x \partial_x f_e + \Omega \cdot \hat{\mathbf{O}} f_e - e \left( E_y - \frac{1}{c} v_x b \right) \partial_{p_y} f_p - \frac{e}{c} v_y b \partial_{p_x} f_p = -\nu_2 \tilde{f}_e + \nu_1 \tilde{f}_p. \quad (3)$$

Here  $\hat{\mathbf{O}} = [\mathbf{p} \times \partial_{\mathbf{p}}]$ . The collisional operators are taken in the symmetric with respect to the sorts of the particles form.

### 2.2. The derivation of the evolution equation

In this subsection we give a brief review of the derivation of the evolution equation, performed in [5].

The solution of the kinetic system (2)–(3) is obtained in two stages. At the first stage, the equations are linearized (the terms with nonlinear combinations of fields are neglected) and solved using the Fourier transforms. This allows to obtain the dispersion equation and the terms  $\delta f_{p,e}^1, \delta f_{p,e}^2, \delta f_{p,e}^3$  of the linear variation of the distribution function.

At the second stage, we solve the system using the successive approximation method, taking into account the nonlinear terms. In the first approximation again only linear terms are left. The equations are solved by building the Green functions. In the second approximation the nonlinear terms are considered and the same procedure is performed. This allows to obtain the nonlinear variation of the distribution function  $\delta f_{p,e}^4$ .

The current density, defined by the variations of the distribution functions, could be derived as  $j_y = \sum_{i,\alpha} e_\alpha \int v_y \delta f_\alpha^i d^3p$ . This result should be substituted to the Maxwell equation for  $rot \mathbf{B}$ .

Thus, we get the evolution equation, which takes the form of the Korteweg—de Vries—Burgers equation:

$$\partial_\tau h + \partial_\xi h + \partial_\xi^3 h + \lambda h \partial_\xi h = \chi \partial_\xi^2 h \quad (4)$$

Here  $h = b/B_0$ ;  $\tau, \xi$  — the dimensionless temporal and spatial coordinates. The coefficients of this equation are expressed via the integrals of the form of  $\sum_\alpha \int v_y \delta f_\alpha^i d^3p$ .

### 2.3. The initial value problem

The equation (4) has an analytical solution only in important special cases. One of such cases is realized when  $\nu \ll \Omega$ : in this limit one can neglect the term  $\sim \partial_\xi^2 h$  describing the dissipation. Thus, the equation will take the Korteweg—de Vries form. One of the Korteweg—de Vries equation solutions is a soliton — a long lived solitary wave, propagating without changing of its shape:

$$h(\xi, \tau) = \frac{h_0}{\cosh^2[w^{-1}(\xi - (1 + 4w^{-2})\tau)]}. \quad (5)$$

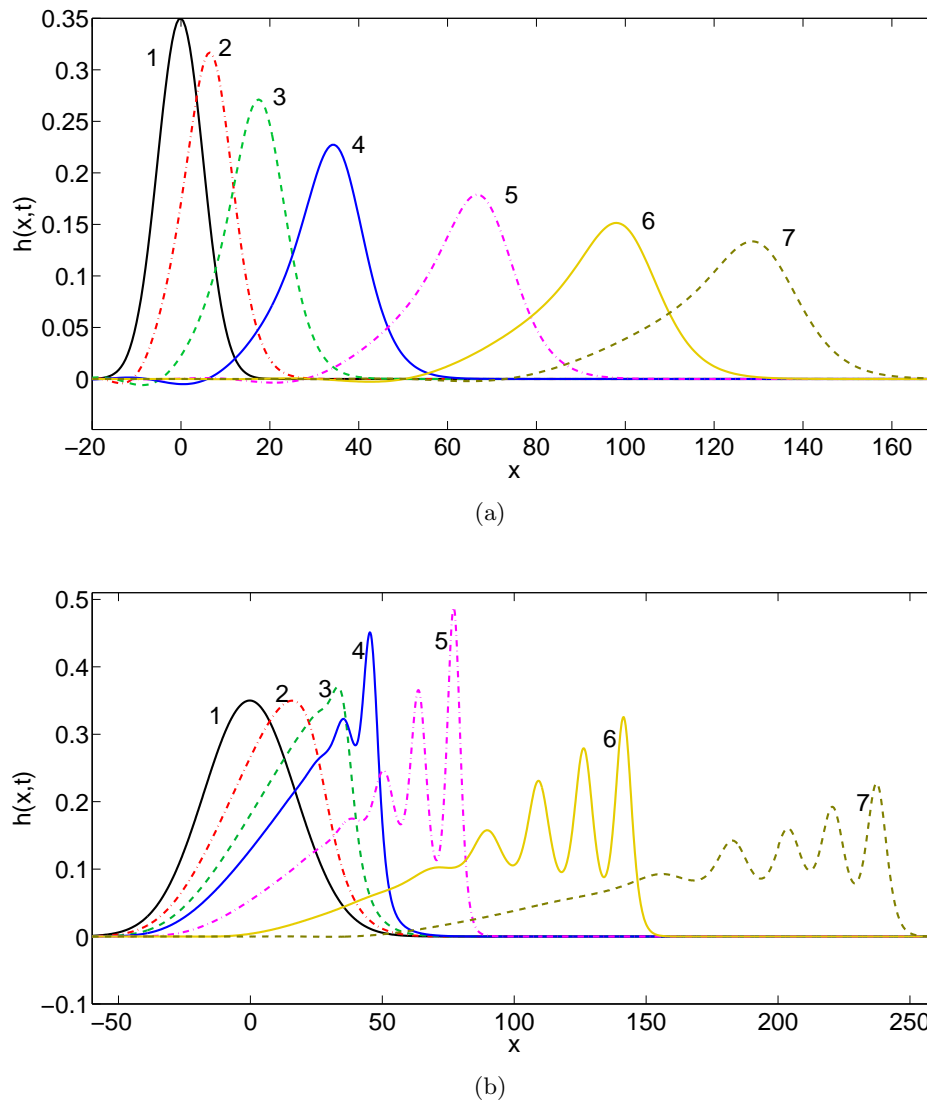
This solution has an important parameter — the soliton width:

$$w = \sqrt{12/\lambda h_0} \quad (6)$$

We emphasize that the derivation of the evolution equation is performed for an arbitrary isotropic part of the distribution function  $f_0$ . In the further modeling of the synchrotron images of the local magnetic field structures in the relativistic pair plasma of the Crab nebula we take  $f_0$  in the form of a broken power-law. We choose the parameters of  $f_0$  resembling the values which could be locally estimated from the spectral energy distribution of the average emission of the Crab nebula, presented in Figure 2 of [2]. We take the breaking Lorentz-factor  $\gamma_b = 10^6$ , and the spectral indices  $p_1 = 1.35$  and  $p_2 = 2.15$  at low and high energies, respectively. The induction of the mean quasi-stationary magnetic field is taken, according to observations, to be equal to 200  $\mu\text{G}$ . The concentration of particles is estimated from the assumption  $P \approx B_0^2/8\pi$ .

In Figure 1 we present the results of the numerical solution of the initial value problem for the equation (4) for the case of significant damping. The amplitude and the spatial scale of the initial perturbation of the magnetic field are matching the solitary peaks emerging in the simulation of the cyclotron instability by Spitkovsky & Arons [6] (see their Figure 2). We use the Gaussian form for the initial perturbation, and take the amplitude  $b/B_0 = 0.35$  and the standard deviation to be equal to  $1.4 \cdot 10^{16}$  cm. We suppose it to propagate with the velocity  $u = c/3$ .

In Figure 1(a) we present the results for the limit  $\nu = \Omega$ . One can see that the amplitude of the propagating perturbation decreases, and its width increases. In addition, the perturbation decelerates. This behaviour is similar to the specific relations between amplitudes, widths and velocities of the soliton solutions. The decreasing of the amplitude is quite slow:  $\sim 10\%$  from the initial value on the timescale of a month. The perturbation remains to be a relatively narrow peak (width of the order of a few units of  $10^{16}$  cm) for tens of days. On longer timescales a 'tail' emerges behind the peak likely due to damping.



**Figure 1.** The evolution of the initial perturbation (Gaussian with the amplitude  $b/B_0 = 0.35$  and the standard deviation equal to  $1.4 \cdot 10^{16}$  cm) in case of (a):  $\nu = \Omega$ ; (b):  $\nu = 0.1\Omega$ . The perturbation at various moments of time is presented. The list of the corresponding moments of time is shown in Table 1. The variable  $x = \xi - U\tau$ , where  $\tau, \xi$  — the dimensionless time and coordinate,  $U$  — the dimensionless velocity of the initial perturbation. The unit length is equal to  $2.8 \cdot 10^{15}$  cm in (a) and  $8.1 \cdot 10^{14}$  cm in (b).

The results shown in Figure 1(b) correspond to the frequency  $\nu = 0.1\Omega$ . Here the dispersion term  $\sim \partial_\xi^3 h$  dominates over the damping term, and one can see the formation of narrow peaks. The spatial scale of these peaks is of the order of a few units of  $10^{15}$  cm. This process is similar to the decay of the initial perturbation into solitons, which emerges in the collisionless case. In the same time, one can see clear manifestations of the damping.

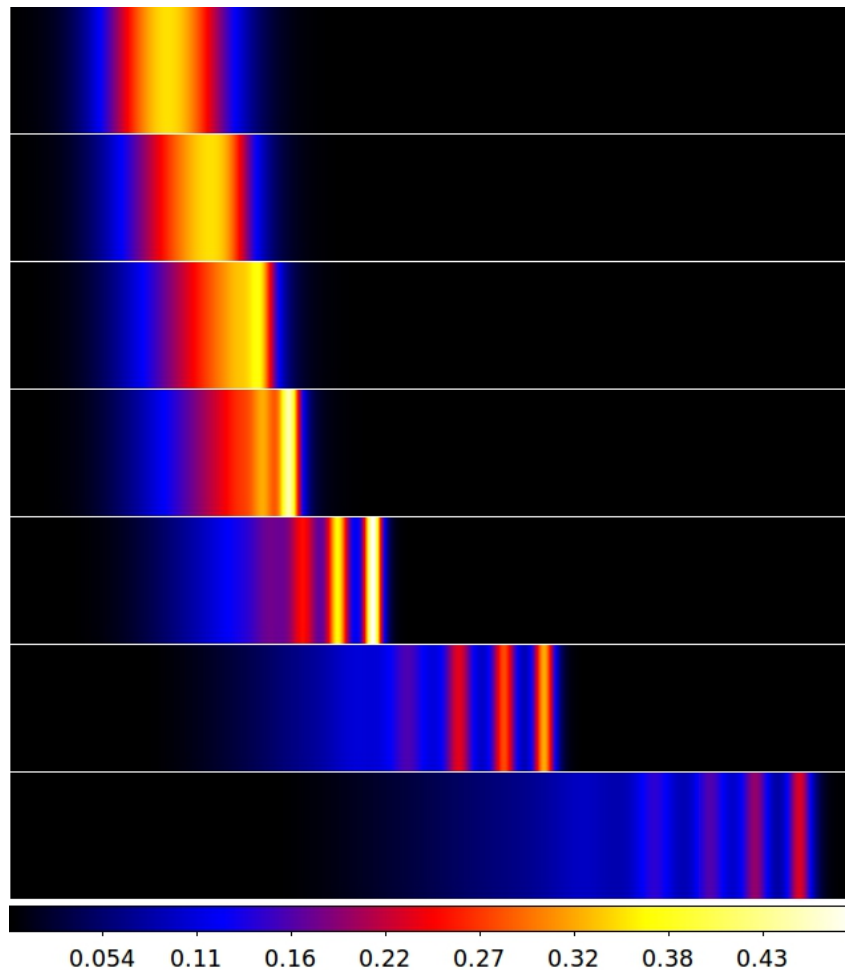
The angular resolution of the Hubble Space Telescope  $\sim 0.1''$  allows us to resolve an object of a size larger than  $3 \cdot 10^{15}$  cm in the Crab nebula. The localized perturbations of the magnetic field result in synchrotron features, which could be detected as bright wisps and filaments.

**Table 1.** The time moments of the evolution, shown in Figure 1, in units of  $10^7$  sec

<i>Figure</i>	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$
1(a)	0.0	0.37	1.03	2.06	4.11	6.17	8.23
1(b)	0.0	0.091	0.181	0.272	0.543	1.21	2.42

### 3. The synchrotron images

The main mechanism of emission of the magnetized relativistic pair plasma of the pulsar wind nebula is the synchrotron radiation. Thus, the propagating perturbations of the magnetic field could be responsible for the observed dynamical structures in the nebulae.



**Figure 2.** The synchrotron images of the evolution of the initial perturbation presented in Figure 1(b): the spatial distribution of the intensity (normalized to the background value). The images correspond to the moments of time in the bottom row of Table 1. The size of the simulated box is  $2.6 \cdot 10^{17}$  cm. The frame moves with the velocity of the initial perturbation. The wavelength of emission  $\lambda = 555$  nm (corresponding photon energy  $E = 2.23$  eV).

The solution of the initial value problem for the evolution equation (4) is a spatial distribution of the magnetic field induction. One has to evaluate the Stokes parameters for this distribution

to get the coordinate distribution of the intensity of emission and the parameters describing the polarization.

The synchrotron images of the evolution of the initial perturbation shown in Figure 1(b) are presented in Figure 2. The simulated box moves with the velocity of the initial perturbation.

One can see the formation of thin bright stripes, which correspond to narrow peaks, shown in Figure 1(b). These stripes have widths  $\sim 10^{15}$  cm. For the leading stripes the significant enhancement of the calculated emission intensity over the background level holds on a timescale of tens of days.

#### 4. Conclusions

In this paper we discuss the modeling of the dynamical structures in the highly-nonequilibrium relativistic pair plasma of the pulsar wind nebulae. We describe a kinetic model of a propagation of a weakly-nonlinear perturbation of the magnetic field transverse to the mean quasi-stationary magnetic field. The regime of the strong scattering of pairs by the stochastic magnetic field fluctuation is considered. The locality of the perturbation is supposed, thus, the one-dimensional analysis is performed. We review the procedure of derivation of the evolution equation from the kinetic equations for the pair plasma components. The obtained evolution equation has the form of the Korteweg—de Vries—Burgers (KdVB) equation.

We solve the initial value problem for the KdVB equation numerically, taking a localized perturbation of the magnetic field as the initial condition. We show that for the limit  $\nu = \Omega$  the perturbation retains the character of a single peak for a long time, but its amplitude decreases due to dissipation. The perturbation decelerates and broadens, and a long 'tail' emerges behind it likely due to damping. For the case of  $\nu = 0.1\Omega$ , when the dispersion dominates over the dissipation, the changes in the shape are more substantial: the groups of solitary peaks are formed. They have the spatial scale of the order of a few units of  $10^{15}$  cm. For this case we evaluate the synchrotron emission intensity at the obtained magnetic field distribution and build the images, representing the coordinate distribution of the intensity. In these images one can see bright thin stripes, corresponding to the peaks mentioned above. The widths of these stripes are of the order of the scale which could be resolved by the Hubble Space Telescope in the Crab nebula ( $3 \cdot 10^{15}$  cm). For the leading stripes the significant enhancement of the calculated synchrotron emission intensity over the background level holds on a timescale of tens of days.

Thus, the perturbations described by the evolution equation (4) can be responsible for the observed dynamical structures, like wisps in the Crab nebula. Also, it can be shown that the characteristic spatial scales of the derived structures are sensitive to the pressure of the pair plasma. This can open up a prospective of an observational constraining of the distribution function parameters and studying of the relativistic pairs gas properties.

#### References

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