# Design and manufacture of perpendicular bi-stable cantilever for vibrational energy harvesting on the basis of stochastic resonance

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**Abstract.** This paper describes extremely simple configuration of novel vibrational energy harvester, which can harness low frequency (less than 5 Hz, such as various environmental vibrations) over a broad frequency band for the first time. A design that utilizes a phenomenon called stochastic resonance can give significantly enhanced vibration mode for increasing efficiency, and simple bi-stable cantilever with tip mass installed a basement vertically fulfils the requirements for stochastic resonance. We fabricated bi-stable cantilever with tip mass and validated whether the cantilever could be used as an effective low frequency vibration energy harvester. In the experiment, when a 1 Hz periodic force and environmental noise vibration were applied, stochastic resonance occurred. The amplitude of the energy harvester increased over tenfold (over 30 mm).

## 1. Introduction

Recently, the idea called "Internet of Things [1]" is obtaining a great deal of attention and the number of wireless sensors has been growing exponentially. In this situation, there will be too many sensors to charge or to exchange in our society, and the field of energy harvesting is gathering attention as a solution of sensors' power source problem. Energy harvesting is a method for scavenging power from environmental energy sources like solar energy, thermal energy and vibrational energy instead of batteries. However, the solar power is unavailable in dark environment and thermal energy is unavailable in cold environment. By contrast, vibrational energy is more robust for condition changes. Therefore, more and more researcher are focusing on vibrational energy harvesting.

To convert vibrational energy to electrical energy, there are three types of vibrational energy harvesting. First type is electromagnetic induction energy harvesting. The electromagnetic induction type requires large magnets, so its structure is likely to become bulky. Second type is electrostatic energy harvesting. This type has difficulty in controlling gaps between electrodes in order that the gaps are small and electrodes does not touch each other. Third type is piezo energy harvesting. Energy density of this type is three times larger as compared to the two counterparts [2].

In previous studies, cantilevers with piezo films are utilized for piezo vibrational energy harvesting frequently. However, there are two major problems in using piezo films on cantilevers [3]. First, ambient vibration frequency is much lower than the resonant frequency of standard energy harvesters imposed by harvester's stiffness. Second, electro-mechanical conversion efficiency decreases promptly when environmental vibration frequencies deviates from resonant frequencies of energy harvesters due to their narrow frequency band operation design.

The objective of this work is to design and manufacture a new piezo energy harvester that can convert low frequency vibrations to electrical energy over a wide frequency band.

## 2. Proposed energy harvester

To fulfil the research objective, we proposed to use a remarkable phenomenon called stochastic resonance. When stochastic resonance occurs, amplitude of bi-stable oscillators increases [4]. To make stochastic resonance occur, three conditions need to be fulfilled: double-well potential, a periodic force and environmental noise vibration. A cycle of stochastic resonance is shown in figure 1. In this cycle, first, a periodic force reduces the potential barrier height. Second, oscillator state jumps to the other well, being triggered by environmental noise vibration. When this cycle periodically repeats, the oscillator fluctuates between two wells at the resonant point.

Various methods have been proposed to build double-well potential systems, such as the use of permanent magnets [5]. When permanent magnets are utilized to build double-well potential system, fabrication process becomes complex and miniaturization of devices becomes difficult. In this work, we proposed to use vertical bi-stable cantilevers because its structure is simple to fabricate and the miniaturization is possible. Figure 2 shows the structure of perpendicular bi-stable cantilever where $v$ and $u$ are respectively the horizontal and vertical displacement of the tip mass, $s$ is the distance of the beam from the basement, $v_{\mathrm{p}}$ and $u_{\mathrm{p}}$ are respectively the horizontal and vertical displacement at the point where the distance of the beam is s, $x$ and $y$ are respectively the horizontal and vertical displacement of the basement and $M_{\mathrm{t}}$ is the tip mass. In this study, vertical vibration $\ddot{y}$ is taken into account for the first time in addition to horizontal vibration [6]. By considering vertical vibration and horizontal vibration, vibrations in all directions could be simulated.

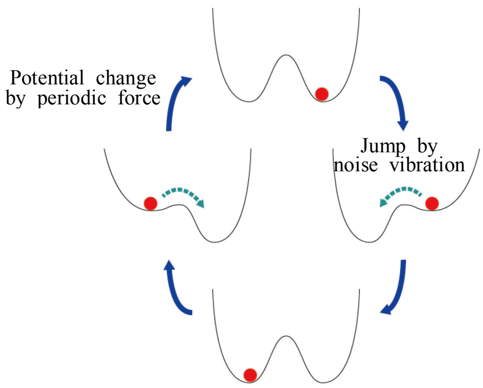



**Figure 1.** The cycle of stochastic resonance.

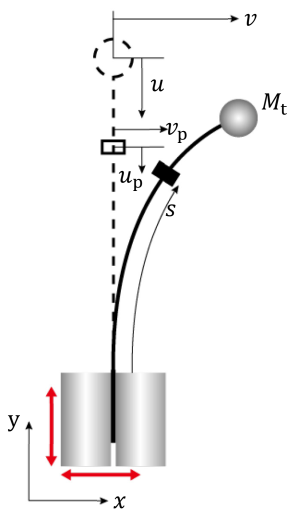



**Figure 2.** Mechanical system of cantilever.

The kinetic energy of this cantilever's system $T$ is defined by

$$T = \frac{1}{2}\rho A \int_0^L \left[\left(\dot{v}_{\mathrm{p}}(s,t) + \dot{x}\right)^2 + \left(\dot{u}_{\mathrm{p}}(s,t) - \dot{y}\right)^2\right] \mathrm{d}s + \frac{1}{2}M_{\mathrm{t}}[(\dot{v} + \dot{x})^2 + (\dot{u} - \dot{y})^2] + \frac{1}{2}I_{\mathrm{t}}\dot{\phi}^2 \quad (1)$$

where $\rho$ is density of the beam, $A$ is the cross section area of the beam, $L$ is the length of the beam, $x$ is the horizontal displacement of the basement, $w$ is the vertical displacement of the basement, $I_{\mathrm{t}}$ is moment of inertia of the end mass, $\phi$ is the slope of the tip mass and the dot means time-derivative.

The position potential energy of the system $\Pi$ is defined by.

$$\Pi = \frac{1}{2}EI\int_0^L\left(\kappa(s,t)\right)^2\mathrm{d}s - \rho Ag\int_0^L\left(u_\mathrm{p}(s,t)-y\right)\mathrm{d}s - M_\mathrm{t}g(u-y) \tag{2}$$

where $E$ is Young Modulus of the beam, $\kappa$ is the beam's curvature and $g$ is acceleration of gravity.

Simplifying Lagrange's equation, equation of motion of the system is obtained as in equation (3).

$$\begin{aligned}&[M_\mathrm{t} + \rho AN_1 + I_\mathrm{t}N_5^2 + (\rho AN_3 + M_\mathrm{t}N_4^2 + I_\mathrm{t}N_5^4)v^2]\ddot{v} + (M_\mathrm{t}N_4^2 + \rho AN_3 + I_\mathrm{t}N_5^4)v\dot{v}^2\\ &\quad + (EIN_6 - N_9\rho Ag - N_4M_\mathrm{t}g + 2EIN_7v^2)v\\ &\quad = -(\rho AN_2 + M_\mathrm{t})\ddot{x} + (\rho AN_9 + M_\mathrm{t}N_4)v\ddot{y} + N(t)\end{aligned} \tag{3}$$

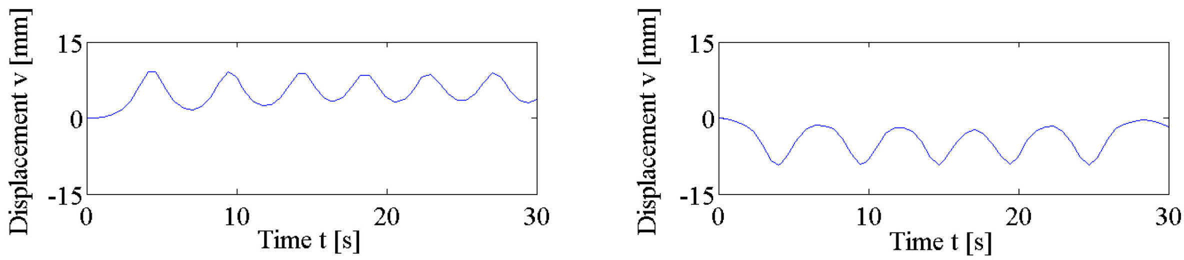
where $N_1$-$N_9$ are constants and $N(t)$ is environmental noise vibration.

## 3. Results

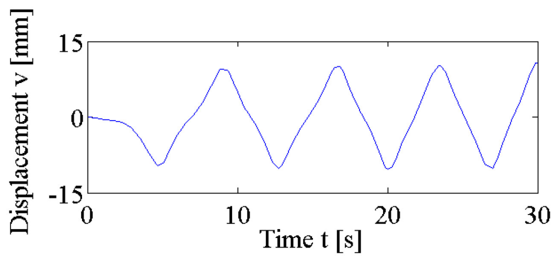
In this section, the simulation and experiment are described. The simulation and experiment results are discussed at the end of this section.

### *3.1. Simulation*

Optimization of the parameters is conducted under the following conditions. First, tip mass must be over the mass where beam starts buckling. Second, the range of equilibrium position is set between 1 mm and 50 mm. Based on these conditions, tip mass and beam thickness were determined as 7.2 g and 100 µm, respectively. Then, three numerical simulations were implemented as shown in figure 3 (a), (b) and (c) and parameter setting in simulation is described in table 1. As figure 3 (a) and (b) shows, when the energy harvester is excited only by ambient noise or periodic force, tip mass of the beam oscillates around only one equilibrium position. By contrast, when periodic force and environmental noise vibration oscillate the vertical bi-stable cantilever, the beam's tip mass is able to oscillate between two equilibrium positions, increasing the beam oscillation considerably. Therefore, the amplitude of the tip mass increases more than twofold (more than 20 mm).




(a) (b)




(c)

**Figure 3.** Simulation results. Three types of simulations are conducted: (a) excited by only environmental noise vibration, (b) excited by only a periodic force and (c) excited by both of them.

**Table 1.** Parameter setting in simulation.

| Length of the beam (mm) | Width of the beam (mm) | Tip mass (g) | Thickness of the beam (µm) | Young Modulus ($GNm^{-2}$) | Density of the beam ($kgm^{-3}$) |
|---|---|---|---|---|---|
| 50 | 4 | 7.2 | 100 | 197 | 7930 |

### *3.2. Experiment*
In this subsection, experimental setting and results are described.

*3.2.1. Experimental setting.* An energy harvester is designed to utilize stochastic resonance and fabricated. Figure 4 shows fabricated energy harvester. Specification of the energy harvester is showed in table 2. This energy harvester is the second version because the first version of the energy harvester was fabricated without consideration of the vertical vibration mode of actual vibration generator. Furthermore, the second version of the tip mass is triangular prism type because the first version of the tip mass which is quadrangular type was likely to rotate in addition to vertical vibration. The beam and the tip mass were made of stainless steel (SUS304) and carbon steel (S45C), respectively. The energy harvester and a laser sensor (IL-100, Keyence, Japan) are installed on a vibration generator (m060, IMV, Japan). Amplitude of the beam was measured at 20 mm above the basement. A periodic force's frequency is 1 Hz, respectively. Directions of the periodic force and environmental noise vibration are vertical directions.

**Table 2.** Specifications of energy harvester.

| Length (mm) | Width (mm) | Tip mass (g) | Thickness of the beam (μm) |
|---|---|---|---|
| 50 | 4 | 7.2 | 100 |

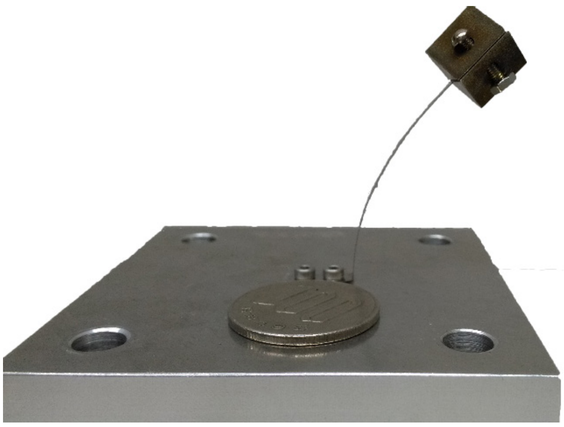


**Figure 4.** Fabricated energy harvester. The length and width of the energy harvester is 50mm and 4mm, respectively. The tip mass is 7.2 g.

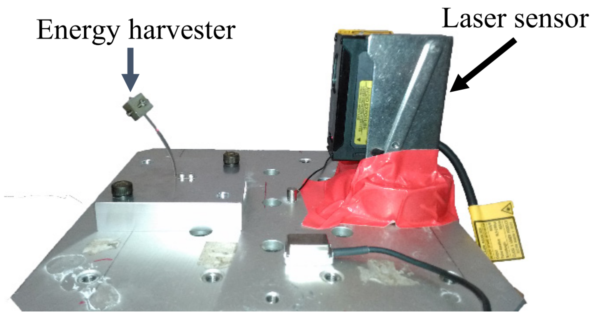
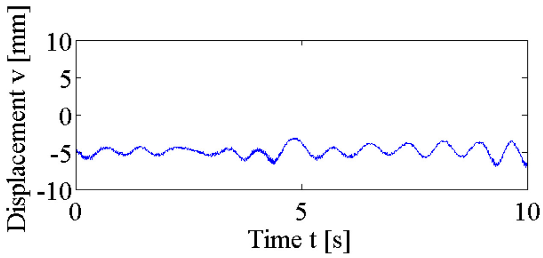
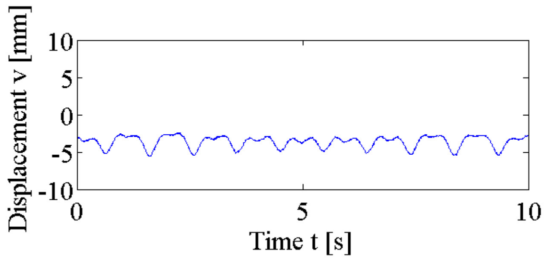



**Figure 5.** Experimental setting. Energy harvester and laser sensor are installed on vibration generator.

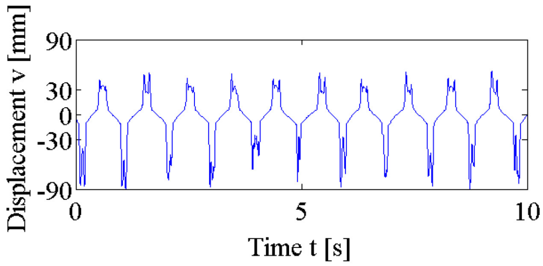
*3.2.2. Experimental results.* Figure 6 (a) shows the result when only the periodic force was applied and Figure 6 (b) shows the result when only the environmental noise vibration is applied. In both cases, the oscillator vibrates around only one equilibrium as seen in simulation, so the amplitude is about 3 mm. On the other hand, Figure 6 (c) shows the result when both the periodic force and the ambient noise vibration are applied, and the oscillator can move between two equilibrium positions, so stochastic resonance occurred and the amplitude of the tip mass increased over tenfold (over 30 mm).



(a)



(b)



(c)

**Figure 6.** Experimental results. Three types of experiments were conducted: (a) excited by only environmental noise vibration, (b) excited by only a periodic force and (c) excited by both of them.

### *3.3. Discussion*
The disturbance of wave showed in figure 6 (c) is due to tip mass's blocking laser sensor light. To eliminate this disturbance, the point that will be measured by the laser sensor should be lowered to less than at 20 mm above the basement. In the simulation and experiment, only when the periodic force and environmental noise vibration excited the energy harvester, the tip mass could vibrate between two equilibrium positions. These results verified stochastic resonance's occurrence.

## 4. Conclusion
In conclusion, this paper designed and demonstrated a new energy harvester that can be used for low frequency vibration energy harvesting over a wide frequency band. Our current effort is to attach a piezo film to the energy harvester in order to generate electrical power. For future work, we will downsize the energy harvester and the energy harvester will be tested in real environment such as on bridges and cars.

## References
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