

# Band widening of piezoelectric vibration energy harvesters by utilizing mechanical stoppers and magnets

T. Maeguchi<sup>1</sup>, A. Masuda<sup>1</sup>, H. Katsumura<sup>2</sup>, H. Kagata<sup>2</sup> and H. Okumura<sup>2</sup>

<sup>1</sup> Kyoto Institute of Technology, Matsugasaki, Sakyo-ku, Kyoto 606-8585, Japan

<sup>2</sup> Panasonic Corporation, Kadoma 571-8506, Japan

E-mail: m4623044@edu.kit.ac.jp

**Abstract.** This paper presents a design of a piezoelectric hardening-type nonlinear vibration energy harvester which has widened resonance band while maintaining the same peak performance at the resonance frequency as that of the reference linear harvester. To this end, a pair of mechanical stoppers and a pair of repulsive magnets are introduced in this study. An experimental prototype device is designed by using a stainless steel-based piezoelectric cantilever, and numerical simulations and experiments are conducted to examine the validity of the presented design strategy. It is concluded that using the magnets to shift the resonance peak toward the lower frequency and using stoppers to expanding the resonance band toward the higher frequency can broaden the resonance band effectively maintaining the peak response. The damping due to the contact of the tip mass with the stopper is one of the key parameters which should be as small as possible to enhance the band widening effect.

## 1. Introduction

There has been increasing interests in vibration energy harvesting technologies that aim to retrieve discarded kinetic energy from environment as the power source of wireless sensors. Many researchers have proposed resonance-type devices with a linear oscillator, in which the natural frequency of the oscillator is matched with the dominant frequency of the vibration source. In this type of energy harvesters, the mechanical Q factor of the oscillator is designed as large as possible to maximize the energy harvesting performance. The larger Q factor, however, bounds resonance in a narrower frequency band, therefore, the harvested power decreases significantly if the dominant frequency of the source moves out of this band. This trade-off is recognized as one of the most significant issues in resonance-type vibration energy harvesters [1] because the actual vibration sources present in the environment are likely to have fluctuation in their frequency components. A number of efforts to pursue wider-band vibration energy harvesters have been reported in the literature [1, 2]. Among them, the concept of introducing a nonlinear oscillator to widen the resonance frequency band has attracted increasing attention. A variety types of nonlinear oscillators including hardening, softening, and bistable oscillators are reported in literature.

One of the most important requirements in the practical design of nonlinear wide-band vibration energy harvester is to widen the resonance band while remaining the peak performance at the specific target frequency. Theoretical analysis using Krylov-Bogoliubov averaging method has indicated that it is required to make the backbone curve, i.e., the equivalent natural frequency of the nonlinear oscillator, to reach the maximum possible stroke at the resonance frequency of the reference linear oscillator [3]. In this study, a pair of mechanical stoppers and a pair of repulsive magnets are introduced to try to achieve this requirement. Numerical studies and experiments using a cantilever oscillator with a thin



piezoelectric bimorph are carried out to clarify the feasibility of the idea of tailoring the restoring force of the oscillator by introducing these mechanical elements.

## 2. Band-widening by introducing nonlinear oscillator

Let us consider a cantilever oscillator consisting of a piezoelectric bimorph and a tip mass. Suppose that there are a pair of stoppers which limits the motion of the tip mass in a certain stroke. Also, suppose a pair of repulsive magnets, one of which is fixed at the end of the tip mass, while the other is fixed on the base to induce axial compressive force in the cantilever at the neutral position. Assuming that the cantilever always deforms in a single mode shape (in the lowest vibration mode), the governing equations of the cantilever is derived as

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + F_{stopper}(x) + F_{magnet}(x) + \theta v_p(t) = f(t) \quad (1)$$

and

$$C_p v_p(t) = \theta x(t) - q(t) \quad (2)$$

where  $x(t)$ ,  $v_p(t)$ ,  $f(t)$ , and  $q(t)$  are the relative displacement of the tip mass, the voltage across the piezoelectric layers, the external force given by a base excitation, and the electrical charge, respectively. The constants  $m$ ,  $c$ ,  $k$ ,  $\theta$ , and  $C_p$  are the equivalent mass of the cantilever including the tip mass, the mechanical damping coefficient, the equivalent stiffness of the cantilever in the short circuit configuration the electromechanical coupling factor, and the clamped capacitance of the piezoelectric layer, respectively.

The force function  $F_{stopper}$  is an additional restoring force induced by the collision of the tip mass with the stopper, while the function  $F_{magnet}$  is a negative restoring force induced by the repulsive force between the magnets. Considering that these forces are well localized at the position of the stoppers and around the neutral position, respectively, they give a hardening effect to the oscillator by harden the oscillator at the stroke limits by  $F_{stopper}$ , and by soften the oscillator around the neutral position by  $F_{magnet}$ . The basic concept of this study is to tailor the oscillator's restoring force as desired by adjusting the parameters of these mechanical components, i.e., the stopper position, stopper material, specification of the magnets, and the gap between the magnets.

The piezoelectric layer in this study is assumed to be connected directly to a pure resistance  $R$  without any interface circuits, thus, the boundary condition for the piezoelectric voltage and charge is given by

$$v_p(t) = R\dot{q}(t) \quad (3)$$

The instantaneous harvested power, which is evaluated as the power consumption at the load resistance  $R$ , is given by

$$P(t) = R\dot{q}(t)^2 \quad (4)$$

Let us suppose the sinusoidal base excitation with a frequency of  $\omega$ . Assuming sinusoidal variation of all the state variables, the equation of motion of the cantilever is expressed as follows by utilizing equivalent linearization method (or equivalently, via Krylov-Bogoliubov averaging method):

$$m\ddot{x}(t) + c\dot{x}(t) + K_e(x_0)x(t) + \theta v_p(t) = f_0 \cos \omega t \quad (5)$$

where  $K_e$  is the equivalent stiffness in function of the displacement amplitude  $x_0$ .

The design objective of the nonlinear restoring force in this study is to achieve widening of the resonance band of the harvester while remaining the peak performance compared with a reference linear harvester. This requires

$$K_e(x_{\max}) = k_r \quad (6)$$

and

$$K_e(0) = k_0 < k_r, \quad \frac{dK_e}{dx_0} \geq 0 \quad (7)$$

where  $k_r$ ,  $x_{\max}$ , and  $k_0$  are the stiffness of the reference linear oscillator, its maximum displacement at the resonance peak for a specific level of excitation, and the stiffness of the nonlinear oscillator linearized at the neutral position, respectively. Equation (6) is the requirement to maintain the peak performance, while equation (7) is for the band widening by hardening.

### 3. Numerical and experimental studies

#### 3.1. Piezoelectric bimorph cantilever

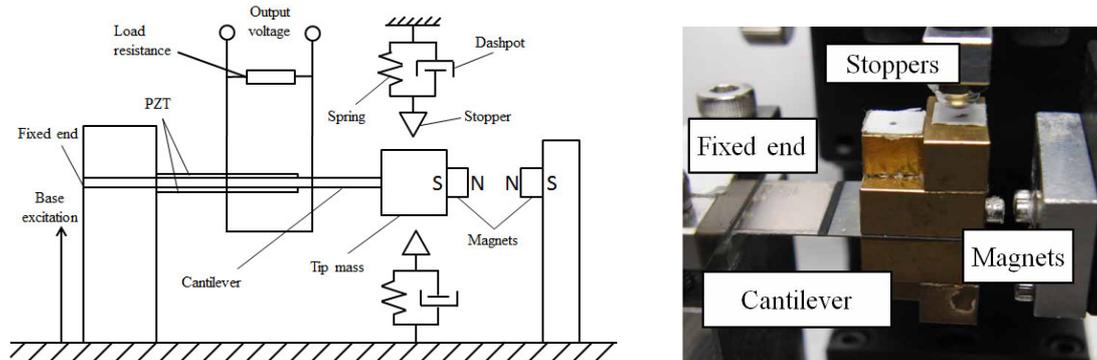
The piezoelectric bimorph cantilever used in this study is made with screen-printing technique to form a thick film of piezoelectric ceramics (PZT) directly on a stainless steel substrate [4]. The thickness of the PZT layer is  $30\ \mu\text{m}$  and the thickness of the based stainless layer is 0.2 mm. Different from a conventional piezoelectric element which is easily damaged when undergoes large tensile deformation, this element withstand such deformation because of its large residual compression stress (approx. 500 MPa). Also, it shows remarkable long life under repetitive plucking test over 10 million times. Other specifications of the bimorph are listed in table 1.

#### 3.2. Experimental Setup

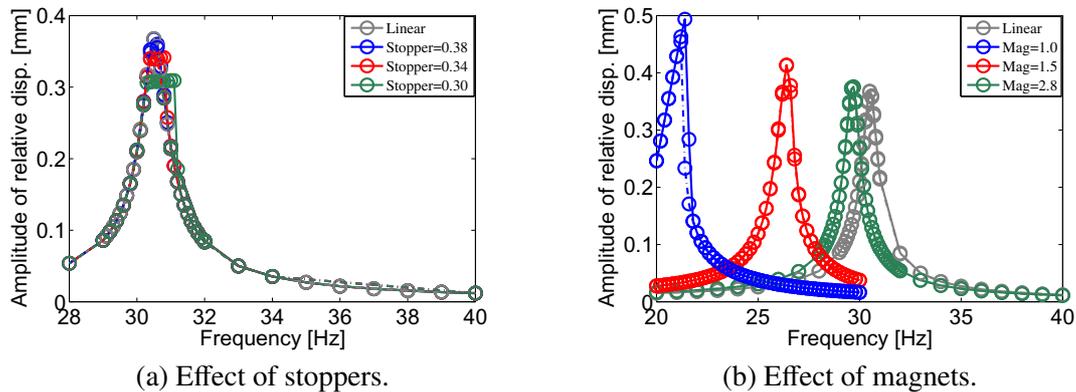
Figure 1 shows a schematic illustration of an experimental setup. A tip mass of 28.6 g made of copper was attached at the end of the bimorph cantilever. The resulting short circuit natural frequency was 30.3 Hz. At the end of the stroke of the tip mass, a pair of hemisphere stoppers were mounted on fine stages which enable to adjust the stoppers' vertical position. A pair of neodymium permanent magnets with a diameter of 3 mm and a height of 2 mm were adhered on the tip mass and on the base with the same polarity facing each other. The whole setup was mounted on a vertical shaker and the displacement of the tip mass and the base were measured by laser displacement sensors. A sinusoidal excitation was applied to see the steady-state response of the harvester. The input acceleration was controlled as  $0.5\ \text{m/s}^2$  during the numerical simulations and the experiments.

**Table 1.** Specifications of piezoelectric bimorph.

Effective length of substrate	25	[mm]
Width of substrate	14	[mm]
Thickness of substrate	0.2	[mm]
Density of substrate	7900	[kg/m <sup>3</sup> ]
Young's modulus of substrate	200	[GPa]
Capacitance of piezoelectric layer	22.5	[nF]



**Figure 1.** Experimental setup.



**Figure 2.** Numerical results showing the effect of independent use of stoppers and magnets.

First, the stoppers and magnets were removed, and the load resistance was determined so that the output power of the linear harvester is maximized. The resultant values of the resistance were  $100 \text{ k}\Omega$  for the numerical simulations and  $90 \text{ k}\Omega$  for the experiments.

In the following numerical calculations, the stopper force was modeled as an extra set of spring and dashpot as presented in the figure. The spring constant was set to 100 times of that of the linear oscillator, and the damping coefficient was determined so that the damping ratio  $\zeta_2$  during the collision of the tip mass with the stopper was to be 0.35 which was identified from the experiment. The magnetic force was modeled by following the formulation presented by Stanton et al. [5].

## 4. Results and discussions

### 4.1. Effect of independent use of stoppers and magnets

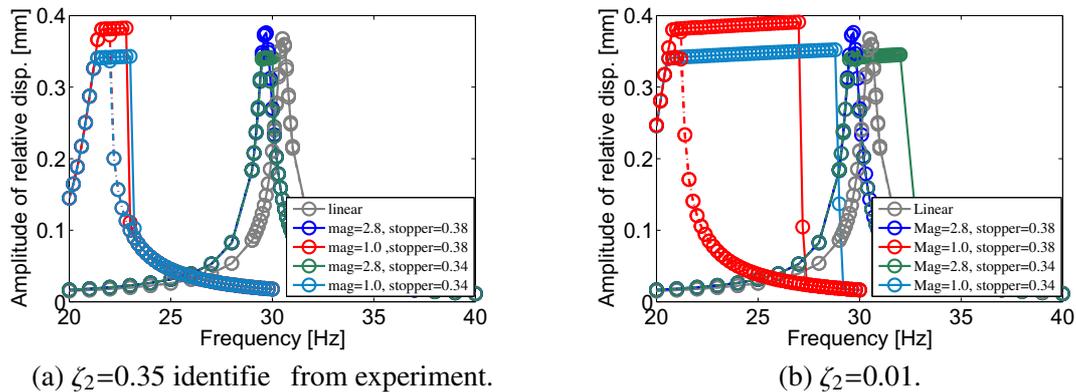
Figure 2 (a) shows the numerically calculated steady-state displacement amplitude for the case with stoppers at 0.38 mm, 0.34 mm, and 0.30 mm, compared with the case without stoppers and magnets (linear). From these results, one can figure out that the introduction of the stoppers folds the backbone curve, i.e., the center line of the resonance peak, toward the higher frequency, and the band widening effect becomes stronger when the stopper location becomes closer.

### 4.2. Effect of magnets

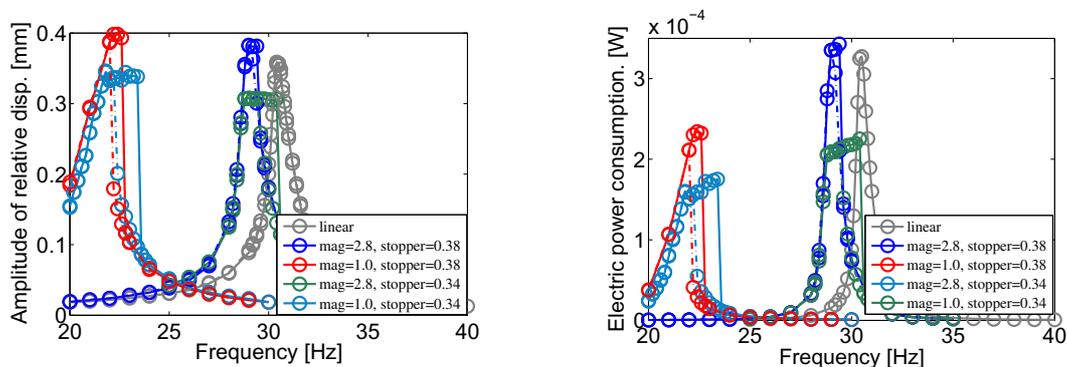
Figure 2 (b) shows the numerically calculated steady-state displacement amplitude for the case with magnets with the gap of 1.0 mm, 1.5 mm, and 2.8 mm, compared with the case without stoppers and magnets (linear). These plots indicate that the effect of introducing the magnets is to shift the backbone curve toward the lower frequency and add a slight effect of hardening. The frequency shift effect becomes stronger when the gap between the magnets becomes closer.

### 4.3. Effect of simultaneous use of stoppers and magnets

Figures 3 and 4 show the combination effect of the stoppers and magnets. Figure 3 shows the numerically calculated steady-state displacement amplitudes while figure 4 shows the experimental results of the steady-state displacement amplitudes and the averaged harvested power. In figure 3 (a),  $\zeta_2$ , the damping ratio while the tip mass contacting with the stopper, is set to 0.35 which is identified from the experimental results shown in figure 4. These results suggest that using the magnets to shift the resonance peak toward the lower frequency and using stoppers to expanding the resonance band toward the higher frequency can broaden the resonance band effectively maintaining the peak response. Figure 3 (b), in which  $\zeta_2$  is decreased to 0.01, implies that, to enhance the band widening, it is effective to reduce the damping due to the contact of the stopper. This may be realized by appropriately designing the shapes and materials of the contacting part of the stopper and the tip mass as well.



**Figure 3.** Numerical results showing the effect of simultaneous use of stoppers and magnets.



**Figure 4.** Experimental results showing the effect of simultaneous use of stoppers and magnets.

## 5. Conclusion

The feasibility of the band widening of vibration energy harvester by introducing stoppers and magnets have been verified in this paper. The numerical and experimental studies also suggested the importance of suppressing the energy loss during the contact of the tip mass with the stopper to enhance the band widening effect.

## 6. Reference

- [1] Tang L, Yang Y and Soh C K 2010 Toward broadband vibration-based energy harvesting, *Journal of Intelligent Material Systems and Structures* 21(18), pp 1867–1897.
- [2] Zhu D, Tudor M J and Beeby S P 2010 Strategies for increasing the operating frequency range of vibration energy harvesters: a review, *Measurement Science and Technology*, Vol. 21, 022001, pp 1-29.
- [3] Masuda A, Senda A, Sanada T, and Sone 2013 A Global stabilization of high-energy response for a Duffing-type wideband nonlinear energy harvester via self-excitation and entrainment, *Journal of Intelligent Material Systems and Structures*, Vol.24, No.13, 1589–1612.
- [4] Oishi A, Okumura H, Katsumura H and Kagata H 2014 The power generation characteristics and durability of a vibration power generation device using piezoelectric thick film formed directly by screen printing on a stainless steel substrate, *Journal of Physics: Conference Series, PowerMEMS 2014*, 557-1.
- [5] Stanton S C, McGehee C C and Mann B P 2010 Nonlinear dynamics for broadband energy harvesting: Investigation of a bistable piezoelectric inertial generator, *Physica D*, Vol. 239, No. 10, pp 640-653.