# Multi-physics modelling approach for oscillatory microengines: application for a microStirling generator design

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**Abstract**. An electrical circuit equivalent (ECE) approach has been set up allowing elementary oscillatory microengine components to be modelled. They cover gas channel/chamber thermodynamics, viscosity and thermal effects, mechanical structure and electromechanical transducers. The proposed tool has been validated on a centimeter scale Free Piston membrane Stirling engine [1]. We propose here new developments taking into account scaling effects to establish models suitable for any microengines. They are based on simplifications derived from the comparison of the hydraulic radius with respect to the viscous and thermal penetration depths ($\delta_\nu = \sqrt{2\mu/(\bar{\rho}\omega)}$ and $\delta_k = \sqrt{2k/(\bar{\rho}C_p\omega)}$ respectively).

## 1. Introduction

Multi-physics coupling is a critical issue for microengines design. On the one hand, the electrical analogy is a useful strategy as it allows multiple physics to be described in a common language. On the other hand, micromachines can be usually divided into elementary components/functions (transducers, membrane/piston, thermal insulation…) and geometries (heat exchangers ducts, chambers…). Apart from the microturbines, they present common features: oscillatory operation, membranes, piezoelectric transducers [2]. Considering the gas flow, linear thermoacoustics theory [3] offers a framework for small harmonic variations that defines equivalent electrical models in which the pressure is the potential and the fluid flow is the current. Thus, we can develop the model of each component of an engine from a simplified 1D scheme of the fluid flow along the fluid flow axis ($\vec{x}$). Moreover, the assumed simplified geometry consists in two parallel plates for which in plane dimension along one direction is much greater than the transverse dimension ($\vec{y}$). For preliminary design, the presented approach is deemed accurate enough as long as it has been successfully run for a centimeter scale Free Piston membrane Stirling engine [1].

The variations of the pressure *p(x,t)*, the volume *V(t)*, the temperature *T(x,y,t)* and the velocity *u*(*x*, *y*, *t*) are assumed to be small with respect to their mean value over a cycle which are noted $\bar{p}$, $\bar{V}$, $\bar{T}$ respectively (the mean velocity is zero).

The equilibrium equations (1-3) for the mass, the momentum and the energy conservations respectively are the starting point of the ECE developments:

$$\frac{1}{\gamma r\bar{T}}\left(\frac{\partial p}{\partial t} - (\gamma-1)\Phi\right) + \bar{\rho}\frac{\partial u}{\partial x} = 0 \quad (1)$$

$$\bar{\rho}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{2}\mu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\bar{\rho}C_p\frac{\partial T}{\partial t} = \frac{\partial p}{\partial t} + k\frac{\partial^2 T}{\partial y^2} \tag{3}$$

In which $\Phi$ is the thermal heat flux per unit volume, $k$ the thermal conductivity, $\gamma$ is the ratio of specific heats and $C_p$ the thermal capacity.

## 2. Developments of electrical equivalent models for basic components

### *2.1 Chambers*

Chambers are defined as 0D volumes such as unique pressure, temperature and density stand for the state of the gas within the chamber. Figure 1a presents the generic schematic of the studied geometry. The adiabatic assumption is usually retained when the reaction time of the heating process is much longer than the period of oscillation. Besides, technological constraints may lead to thick walls to endure the large pressure solicitation. Considering microengines, the downsizing induces large heat transfer so the isothermal assumption is also of relevance and considered here.

From eq. (1), the adiabatic and isothermal assumptions lead to the ECE model of figures 1b and 1c respectively. The expression of the capacitors are given in table 1.

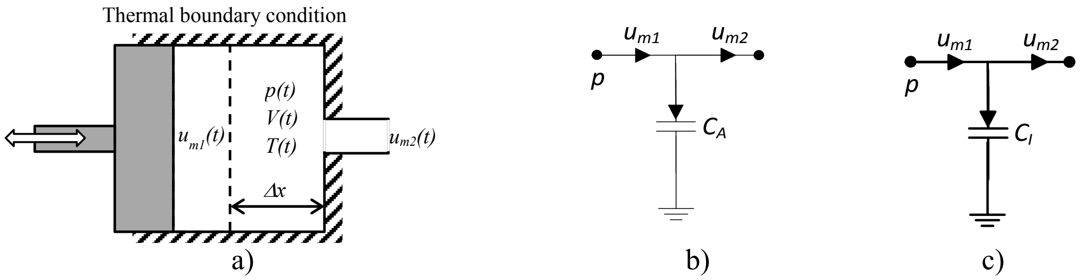



**Figure 1.** a) basic scheme for the study of compression/expansion chamber, b) ECE model for adiabatic chamber c) ECE model for isothermal chamber

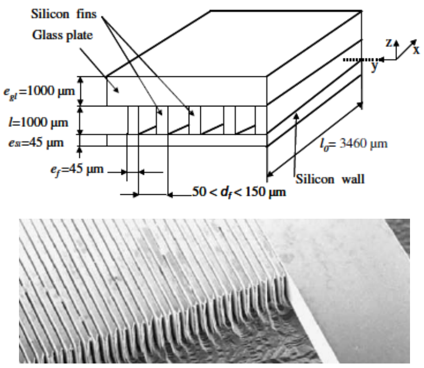
**Table 1.** Expressions for the electrical components for chambers and heat exchangers

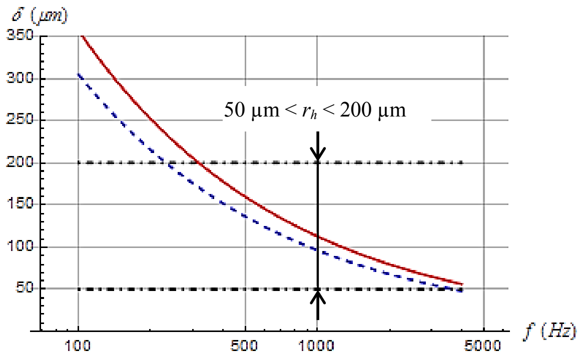
| | Boundary | Electrical circuit equivalent components | |
|---|---|---|---|
| | | $r_h < \delta$ | $r_h > \delta$ |
| Chamber | Isothermal | $C_I = \Delta x/\bar{p}$ | |
| | Adiabatic | $C_A = \Delta x/\gamma\bar{p}$ | |
| Heat exchanger | Isothermal | $R_{vS} = 3\mu\,\bar{\rho}\frac{\Delta x}{{r_h}^2}$<br>$C_{IS} = \Delta x\,(2-\gamma)/(\gamma\bar{p})$ | $R_{vL} = \frac{1}{2}\,\bar{\rho}\,\omega\frac{\delta_v}{r_h}\Delta x$ , $L_{vL} = \bar{\rho}\left(1+\frac{1}{2}\frac{\delta_v}{r_h}\right)\Delta x$<br>$C_{IL} = \Delta x/(\gamma\bar{p})$ , $R_{ILk} = 2\bar{p}r_h\gamma/\left(\omega\delta_k\Delta x(\gamma-1)\right)$ |
| | Generic heat exchange | $R_{vS} = 3\mu\,\bar{\rho}\frac{\Delta x}{{r_h}^2}$<br>$C_{HS} = \Delta x\left(1+2(\gamma-1)\frac{1}{\omega}\frac{h}{Nu}\frac{\bar{T}}{\bar{\rho}C_p}\frac{r_h}{\delta_k}\right)/(\gamma\bar{p})$ | $R_{vL} = \frac{1}{2}\,\bar{\rho}\,\omega\frac{\delta_v}{r_h}\Delta x$ , $L_{vL} = \bar{\rho}\left(1+\frac{1}{2}\frac{\delta_v}{r_h}\right)\Delta x$<br>$C_{HL} = \Delta x/(\gamma p_m)$ , $R_{HLk} = C_p\bar{T}\bar{p}\bar{\rho}\gamma/\left(h\Delta x(1-\gamma)\right)^*$<br>* high heat transfer coefficient |

### *2.2 Heat exchangers*

The heat exchangers model has to take into account viscous dissipation along the $\vec{x}$ axis as well as various standard thermal boundary conditions: isothermal walls ($T_w$ = cste) and generic heat transfer for which the mean thermal heat flux is $\bar{\Phi} = h/r_h(\bar{T} - T_{ext})$ and the instantaneous heat flux is $\Phi = h/r_h\,T$.

Figure 2a presents an architecture of a micro-heat exchanger [4]. From its dimensions, assuming air as the working fluid, the thermal and viscous penetration depths are plotted as a function of the operating frequency in the range [100 Hz, 4000 Hz] (figure 2b). Dashed dotted horizontal lines are the upper and lower bounds of the considered width of the channel from [4]. It can be inferred from figure 2b that both small and large hydraulic radius to penetration depths ratio cases have to be considered.

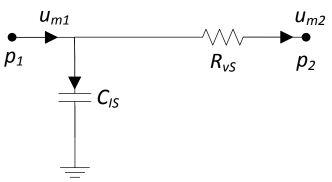


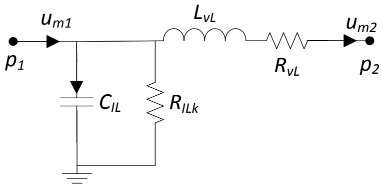




a) b)

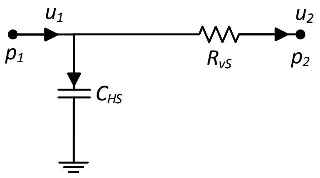
**Figure 2.** a) LIGA technology micro-heat exchanger, b) evaluation of $\delta_v$ and $\delta_k$

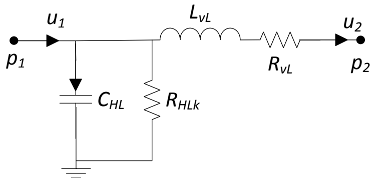
For isothermal wall conditions, we obtain the ECE models shown in figure 3a and 3b for small and large penetration depth to hydraulic radius ratio respectively. For generic heat transfer, we obtain the scheme of figure 3c-3d. Table 1 gives the values for the electrical components.




a) b)



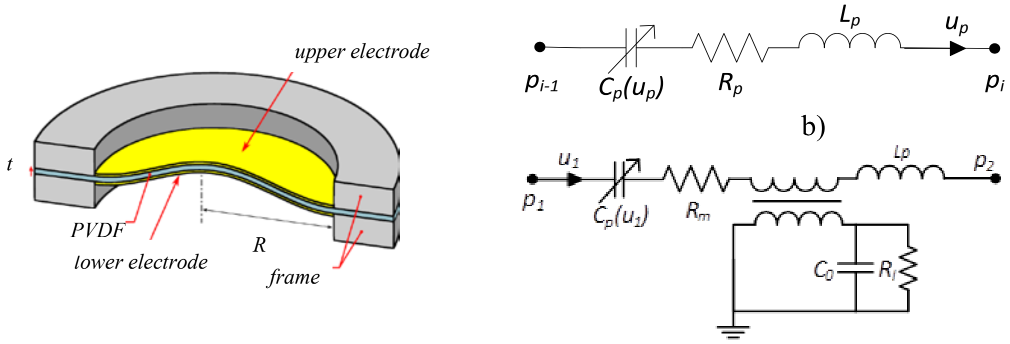

c) d)

**Figure 3.**ECE for heat exchangers considering isothermal walls: a) $r_h < \delta$ and b) $r_h > \delta$; generic heat transfer: c) $r_h < \delta$ and d) $r_h > \delta$

*2.3- Membrane and electromechanical convertor*

Oscillating membranes are usually preferred compared to sliding pistons as they are compatible with planar process and provide perfect sealing with controlled dissipation. Yet, their dynamic behavior has to be accurately modelled including potential large deformation effects. Indeed, micro-engines are free pistons mechanical arrangement so the steady state operation stroke arises from nonlinear effects. Large deformation analytical membranes models are developed in the literature and their integration in the electrical model framework has been demonstrated in [1]. The associated ECE model is presented in figure 4b.

As an example of an integrated piezoelectric the single layered PVDF membrane such as shown in figure 4a can be used for electromechanical conversion. Large deformation effects induces tension stress which produces electrical charge on the electrodes [5]. The ECE model of the piezoelectric membrane including large deformation effect is given in figure 4c in which $C_0$ is the actual electrical capacitance of the PVDF membrane and $R_l$ the actual electrical resistance load.




a) c)

**Figure 4.** a) PVDF membrane [5], b) ECE mechanical model, c) ECE electromechanical model

## 3. Design test cases: Multiphase micro-Stirling engine

This modeling approach has been used to design a miniature planar free-piston Stirling generator for low temperature waste heat recovery (figure 5). It consists of three coupled elementary phases, each composed of: *i)* a membrane (∅ 5 mm, thickness $h$ = 40 µm) which acts as a piston and transducer integrating a piezoelectric planar spiral, *ii)* an expansion chamber ($T_e \approx 120°C$), *iii)* a compression chamber ($T_c \approx 30°C$) linked by *iv)* a regenerator.

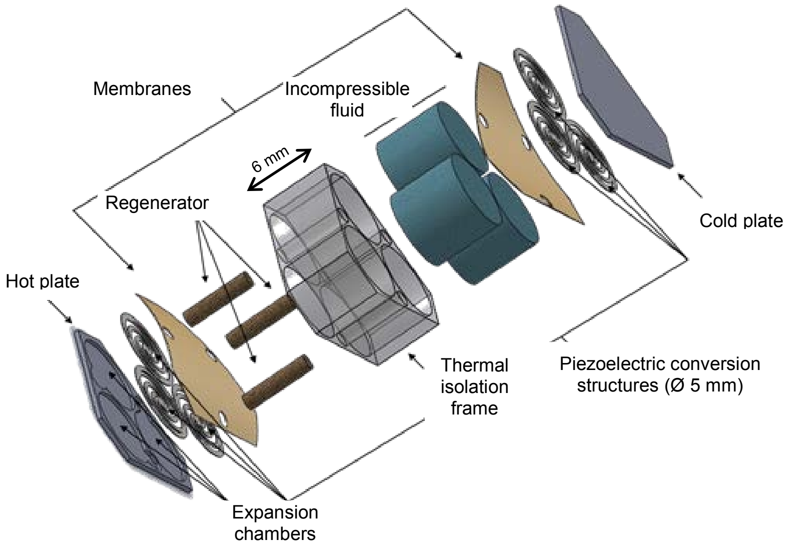
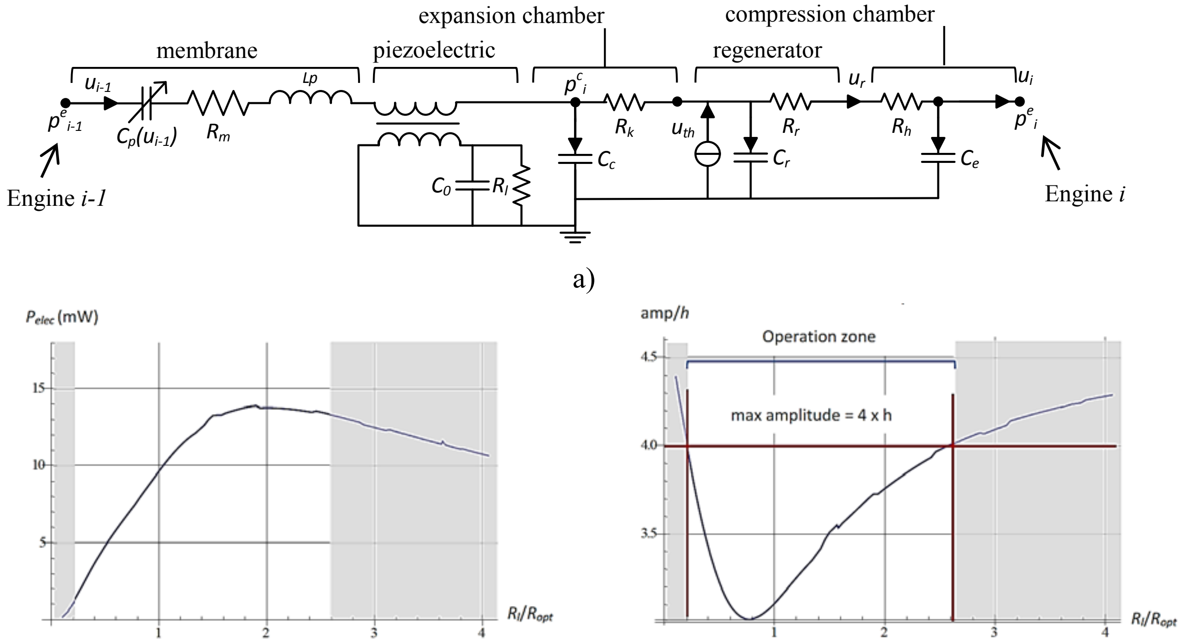



**Figure 5.** Exploded CAD view of the studied microStirling generator

The associated ECE model for a single phase is shown in figure 6a. The relationship between the pressures from one engine to the other ($p^e_{i-1}$, $p^e_i$) and fluid flow rate ($u_{i-1}$, $u_i$) can then be easily established [1]. The condition of operation are a 120° phase angle between the variables and a magnitude equals to unity. Therefore, its operation is simulated and the output electrical power derived. Figure 6b and 6c present the evolution of the electrical power and the membrane flexion amplitude as a function of the electrical load, $R_l$. As the load increases, the power rises (figure 6b) but the harvested energy induces a higher damping, hence reducing the amplitude (figure 6c) and the operating frequency. Design of a microStirling was also proposed with power generation in the range 60-80 mW/cm$^2$.

These results underline the ability of the proposed ECE modeling approach for the design and optimization of oscillating microengines, accounting for coupled multi-physics and dynamic behavior.




b) c)

**Figure 6.** a) Equivalent model for one phase, b) Evolution of the output power with respect to the load resistance, evolution of the stroke with respect to the electric charge

## 4. Conclusion and perspectives

A novel modelling approach dedicated to oscillating microengines has been proposed. The availability of a model for the main microengine's components allows new concept to be theoretically evaluated and designed. As the phase change thermodynamic cycles prove to be especially relevant for power density, future works will aim at the development and validation of micro-evaporator and micro-condenser modelling in the spirit of the electrical circuit equivalent (ECE) approach.

## Acknowledgment

This work has been funded by the French National Research Agency (ANR) under grant no. ANR-12-SEED-0005-01 (MISTIC Project).

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