

# Autoparametric Resonance Systems for Vibration-Based Energy Harvesters

L. Kurmann<sup>1</sup>, D. Hoffmann<sup>2</sup>, B. Folkmer<sup>2</sup>, Y. Manoli<sup>2,3</sup>, P. Woias<sup>4</sup>, R. Andereggi<sup>1</sup>

<sup>1</sup>University of Applied Sciences and Arts Northwestern Switzerland, School of Engineering, Institute of Automation, Klosterzelgstrasse 2, 5210 Windisch, Switzerland

<sup>2</sup>Hahn-Schickard, Wilhelm-Schickard-Str. 10, 78052 Villingen-Schwenningen, Germany

<sup>3</sup>Fritz Huettinger Chair of Microelectronics, Department of Microsystems Engineering – IMTEK, University of Freiburg, Georges-Köhler-Allee 102, 79110 Freiburg, Germany

<sup>4</sup>Laboratory for the Design of Microsystems, Department of Microsystems Engineering – IMTEK, University of Freiburg, Georges-Köhler-Allee 102, 79110 Freiburg, Germany

E-mail: lukas.kurmann@fhnw.ch

**Abstract.** Motivation for this paper is the creation of a new kind of (vibration) kinetic energy harvester systems that can effectively transfer environmental mechanical vibrations into electrical energy over a wider frequency bandwidth than conventional devices. This paper presents a potential improvement in the 1DoF vibration transducer class and examining therefore analytically the behavior of such systems using strong nonlinear springs. Then a new 2DoF class of vibration transducer is presented having a strong nonlinear characteristic which is well suited for autoparametric resonance vibrations.

## 1. Introduction

Energy Harvesting is a technology for capturing non-electrical energy from ambient energy sources, converting it into electrical energy and storing it to power wireless electronic devices. The process of capturing mechanical energy such as shocks and vibrations is a particular field of energy harvesting requiring specific types of energy harvesting devices, so called kinetic energy harvesters. Conventional, first generation types of such transducers can harvest mechanical vibration energy effectively only in a narrow frequency window. Over time many different types of systems have been analytically characterized, designed and tested. Most of these systems show only small improvements with respect to their bandwidth. None of those systems can transfer mechanical vibration power into electrical energy over a wide frequency band. The main requirements of such a kinetic harvester system shall be a simple mechanical structure as well as a wide vibration frequency range for which the system can transfer very effectively environmental mechanical vibrations into electrical energy.

Chapters 2. and 3. deal with 1DoF vibration harvester systems using nonlinear springs. This idea has been proposed recently, by using springs with nonlinear hardening or softening characteristics; some propositions made in [1] and [2] lead to the duffing equation (cubic displacement of path proportional to the restoring force). Based on the duffing oscillator we create in these two chapters, an

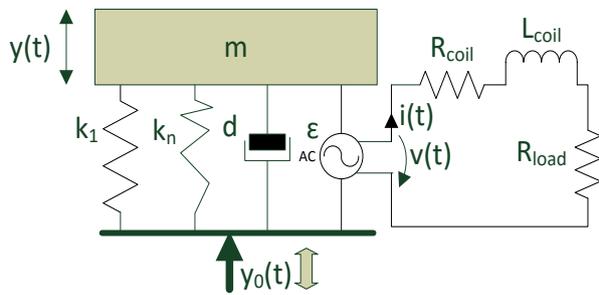


analytical model and measure its accuracy using numerical solutions. Then we explore the behavior of such 1DoF energy harvester systems by experimenting with larger nonlinear of springs.

In chapters 4. and 5. we deal with 2DoF vibration harvester systems and propose a new kind of vibration energy harvester, capable of autoparametric resonance. A dimensionless system is modeled and numerical solutions show the stability and energy transfer of such a transducer system.

## 2. Investigation in 1DoF Systems

In this chapter we deal with analytical and numerical simulations of vibration energy harvesters with one single degree of freedom (1DoF). The inertia of the oscillating proof mass results in a relative displacement when the frame experiences acceleration. Once the proof mass is displaced from its equilibrium position, the restoring force of the suspension starts to act on the proof mass and oscillations occur. The oscillations of the proof mass can be damped by a suitable transducer mechanism and thus kinetic energy is converted into electrical energy [1].



**Figure 1.** System model elements: a nonlinear oscillator with mass  $m_1$ , a damping factor  $d_1$ , a linear  $k_1$  and a nonlinear spring  $k_n$ . An external basepoint excited harmonic force with amplitude  $y_0(t)$ ; electromagnetic coupling elements: inductance  $L$ , resistive load  $R$ . System coordinates: mechanical displacement  $y(t)$  of mass  $m$ , current  $i(t)$  through the inductor, and the voltage  $v(t)$  generated with the coupling factor  $\varepsilon$ .

### 2.1. Lumped parameter model of the electromagnetic 1DoF system

The model comprises a mechanical and an electrical domain which interacts with each other through a coupling element  $\varepsilon$ , see also system elements in Figure 1. System parameters are the oscillating mass  $m$ , the springs  $k_1$  and  $k_n$  with the degree of nonlinearity  $n$  and the damping  $d$ . The amplitude of the ambient vibration is  $A$  and its frequency  $\omega$ ; assuming the system is excited by a harmonic function:

$$\ddot{y}_0 = \frac{d^2}{dt^2}(A \cos \omega t) = -A\omega^2 \cos(\omega t) \quad (1)$$

The equation of motion for such a system is given by

$$-m g - k_1(y_1 - y_0) - k_n(y_1 - y_0)^n - d(\dot{y}_1 - \dot{y}_0) + \varepsilon i = m \ddot{y}_1 \quad (2)$$

Changing equation by adding  $-m \ddot{y}_0$  and substituting resulting differences with  $y$ ,  $\dot{y}$  and  $\ddot{y}$ , e.g.  $y = y_1 - y_0 + c_0$ ,  $\dot{y} = \dot{y}_1 - \dot{y}_0$ ,  $\ddot{y} = \ddot{y}_1 - \ddot{y}_0$  we obtain a DE (of accelerations):

$$\ddot{y} + \frac{d}{m}\dot{y} + \frac{k_1}{m}y + \frac{k_n}{m}y^n + \frac{\varepsilon}{m}i = \ddot{y}_0 = -A\omega^2 \cos(\omega t) \quad (3)$$

Introducing the coil to obtain a voltage for an electrical resistive load, Faraday's law of induction needs to be applied (see also [1] for a more detailed treatment); equation for the generated voltage ( $R = R_{coil} + R_{load}$ ):

$$v(t) = \varepsilon \dot{y} = L \frac{di}{dt} + R i \quad (4)$$

The set of equation (3) and (4) models the system of motion. The parameter  $\varepsilon$  is the electromagnetic coupling constant (transduction factor) [ $Vs/m$ ], and is measured for the electromagnetic current  $i$  [ $A$ ] that is generated upon velocity of deflection  $y$ , assuming having resistive load  $R$  in the electrical system domain. To obtain a more general solution, the equations of motion are written in the dimensionless form:

$$u'' + 2\delta u' + u + \beta u^n + \kappa \lambda \zeta = -\Omega^2 \cos(\Omega \tau) \quad (5)$$

$$\kappa u' = \zeta' + \lambda \zeta \quad (6)$$

Using following parameters for nondimensionalization:

$$y_0 = A; i_0 = \frac{\varepsilon y_0}{R}; \omega_0^2 = \frac{k_1}{m}; \beta = \frac{k_n}{k_1} y_0^2; \tau = t \omega_0; \delta = \frac{d}{2m\omega_0}; \Omega = \frac{\omega}{\omega_0}; \kappa = \frac{\varepsilon^2}{L d \omega_0}; \lambda = \frac{R}{L \omega_0}; \quad (7a)$$

$$\text{path } u(\tau) = \frac{y(t)}{y_0} \text{ and current } \zeta(\tau) = \frac{i(t)}{i_0} \quad (7b)$$

In the equation system of (5) and (6),  $\delta$  is the damping ratio and  $\beta$  the nonlinearity constant together with the exponent  $n$  in  $u^n$  indicating the system's degree of nonlinearity. A well-known case with  $n=3$  describes the duffing oscillator. The parameter  $\kappa$  is called transduction factor (and assumed to be constant in this analytical treatment) and the load matching constant  $\lambda$ . In case of resistive load matching, the resistance is equal to the apparent impedance of the inductance:

$$\lambda = \frac{|Z(L)|}{L \omega_0} = \frac{\omega L}{L \omega_0} = \Omega \quad (8)$$

For a nonlinear system it is often sufficient to use a limited set of harmonics to obtain a sufficient accurate solution for the system, see also [5]; here only the fundamental frequency is considered:

$$u(\tau) = u_c \cos(\Omega\tau) + u_s \sin(\Omega\tau) \quad (9a)$$

$$\zeta(\tau) = \zeta_c \cos(\Omega\tau) + \zeta_s \sin(\Omega\tau) \quad (9b)$$

The ansatzfunction (9a) is inserted in equation (6) and its analytical solution is obtained:

$$\zeta(\tau) = e^{-\lambda\tau} c_1 - \frac{\Omega}{\lambda^2 + \Omega^2} ((u_s \lambda + u_c \Omega) \cos(\tau\Omega) + (-u_c \lambda + u_s \Omega) \sin(\tau\Omega)) \quad (10)$$

Here only the nonhomogeneous solution is of interest, as for large  $\tau$  only this solution will remain. Inserting again (9a) and its derivative in (10) yield the relation between displacement and voltage:

$$\zeta(\tau) = \frac{\lambda}{\lambda^2 + \Omega^2} u' - \frac{\Omega^2}{\lambda^2 + \Omega^2} u \quad (11)$$

With this relation, equation (5) can be rewritten:

$$u'' + 2\delta u' + u + \beta u^n + \frac{\kappa \lambda^2}{\lambda^2 + \Omega^2} u' - \frac{\kappa \lambda \Omega^2}{\lambda^2 + \Omega^2} u = -\Omega^2 \cos(\Omega\tau) \quad (12)$$

Energy harvesters are typically characterized by the three different parameters, the open-circuit voltage  $\lambda \rightarrow 0$  ( $d_{EM} = 0, k_{EM} = 0$ ) the short circuit current  $\lambda \rightarrow \infty$  ( $d_{EM} = \kappa, k_{EM} = 0$ ) and the maximum power output (load matching)  $\lambda \rightarrow \Omega$  ( $d_{EM} = \kappa/2, k_{EM} = -\kappa\Omega/2$ ). These three load cases correspond to the terms of  $d_{EM}, k_{EM}$  in equation (12). In theory such considerations are reasonable, as for many electrical circuits load matching is used. For electromagnetic harvesters, the influence of the reactance is below 10% of the resistance and optimal load is obtained of adding coil resistance plus the quadratic transduction factor divided by the parasitic mechanical damping  $d$ , see also [1].

### 3. Comparison of frequency response for electromagnetic 1DoF vibration harvesters with different degrees of nonlinearity

Equation (12) gives a general relation for basepoint excited (nonlinear) electromagnetic harvester systems; if  $\beta$  is zero, no nonlinear spring is present and we deal with a linear transducer system. If  $\beta$  is nonzero, we examine nonlinear springs with nonlinearity exponent  $n = 3$  (duffing case) and  $n = 5$ .

#### 3.1. Frequency response with nonlinear spring $n=3$ (duffing case)

A sufficient accurate approximation for equation (12) can be obtained using the harmonic balance (HB) method. As already stated, we neglect the influence of higher harmonics and assume the ansatzfunction (9a); the cubic term  $u^3$  is approximated by:

$$u^3 = \frac{3}{4} \approx \frac{3}{4}(u_c^2 + u_s^2) u = \frac{3}{4} \hat{u}^2 u \quad (13)$$

Two linear equations for sine and cosine balance term are formed following the HB procedure. An implicit relation between displacement amplitude and excitation frequency can here be obtained:

$$\hat{u}^2 \left( \left( -\Omega^2 + 1 + \frac{3}{4}\beta \hat{u}^2 - \frac{\kappa\lambda\Omega^2}{\lambda^2 + \Omega^2} \right)^2 + \left( 2\delta\Omega + \frac{\kappa\lambda^2}{\lambda^2 + \Omega^2} \right)^2 \right) = 1 \quad (14)$$

As proposed by Neiss et al in [4], we also limit our investigation of the maximum power output to weakly coupled systems, i.e.  $\kappa \leq 4\delta\Omega$  and follow also by using a quadratic approximation of the frequency around the linear resonance frequency by applying:

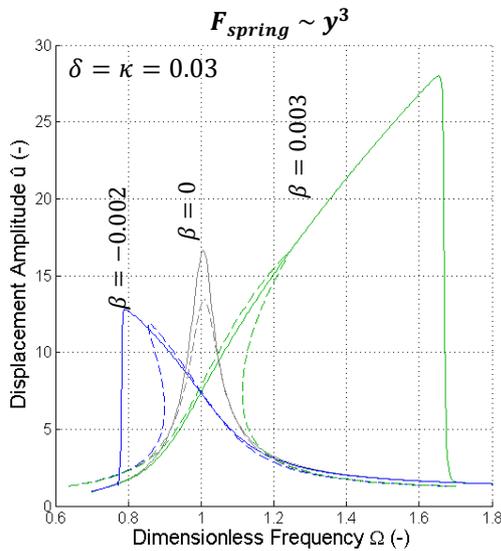
$$\left( 2\delta\Omega + \frac{\kappa}{2} \right)^2 \approx \left( \delta + \frac{\kappa}{4} \right) (\kappa + 4\delta\Omega^2) \quad (15)$$

By therefore replacing the damping and stiffness and inserting (15) into (14) we obtain:

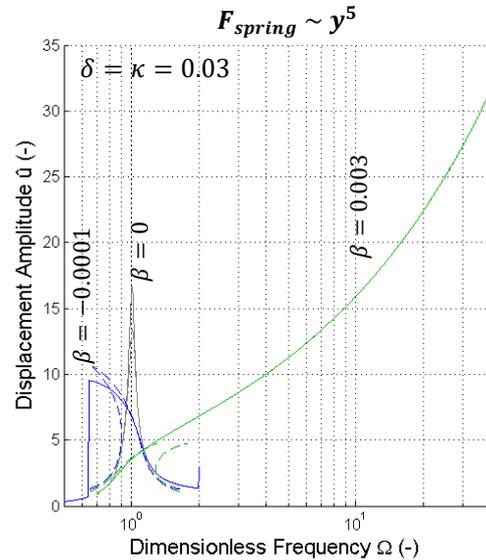
$$\hat{u}^2 \left( \left( -\Omega^2 + 1 + \frac{3}{4}\beta \hat{u}^2 - \frac{\kappa}{2} \right)^2 + \left( \delta + \frac{\kappa}{4} \right) (\kappa + 4\delta\Omega^2) \right) = 1 \quad (16)$$

This simplified implicit frequency response is now a quadratic polynomial which can be elegantly solved for  $\Omega^2$ :

$$\Omega_{1,2} = \frac{1}{2} \left( 3\hat{u}^2\beta + 4 - 8\delta^2 - 2\kappa - \delta\kappa \pm \frac{2}{\hat{u}^2} \left( 4\hat{u}^2 - 3\hat{u}^6\beta\delta(4\delta + \kappa) + \hat{u}^4(4\delta + \kappa)(4\delta(-1 + \delta^2) + (-1 + \delta(2 + \delta))\kappa) \right)^{1/2} \right)^{1/2} \quad (17)$$



**Figure 2.** Exemplary frequency responses linear (grey), softening (blue) and hardening (green) using numerical simulation (solid lines) and analytical solution (dashed lines).



**Figure 3.** Exemplary frequency responses linear (grey), softening (blue) and hardening (green) using numerical simulation (solid lines) and analytical solution (dashed lines).

A selection for the analytic solution from equation (17) along with the numerical solutions obtained by solving the coupled DE system in (5) and (6) is shown in Figure 2. The solution errors between analytical and numerical analysis show that the backbone curve deviation is small (in the range of ca. <1% over a large parameter range). The amplification errors for the analytical solutions are larger, best matches at ca. 3%. Jump-up and jump-down point identification, maximum power output and bandwidth calculations are not presented, as the scope of the paper is not limited to this special nonlinear case.

### 3.2. Frequency response with nonlinear spring $n=5$

As stated for the duffing case ( $n=3$ ) we approximate equation (12) again by using the harmonic balance (HB) method and neglecting again the influence of higher harmonics assuming the ansatzfunction (9a); the stiffness term with  $u^5$  is approximated by:

$$u^5 \approx \frac{5}{8} \hat{u}^4 u \tag{18}$$

Using again the same approach as presented in chapter 3.1. leads to the analytical simplified implicit frequency response; its quadratic solution for  $\Omega$  yields:

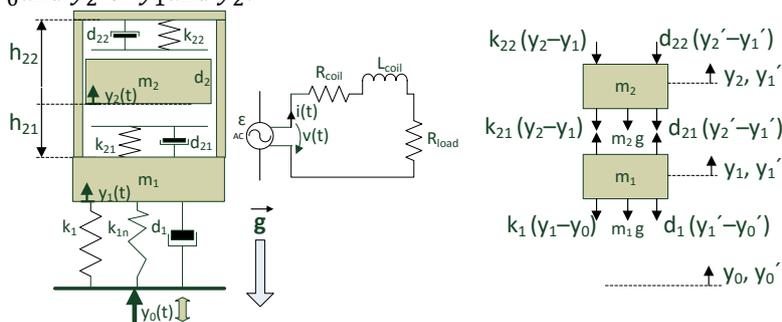
$$\Omega_{1,2} = \frac{1}{2} \left( \frac{5u_0^4 \beta}{2} - 2(-2 + \kappa + \delta(4\delta + \kappa)) \pm \frac{\sqrt{2}}{u_0^2} \left( u_0^2 (8 + u_0^2(4\delta + \kappa)(-2\kappa + \delta(-8 - 5u_0^4 \beta + 8\delta^2 + 2(2 + \delta)\kappa))) \right)^{1/2} \right)^{1/2} \tag{19}$$

Figure 3 shows exemplary frequency responses for the analytic solution from equation (19) together with the numerical solution of the coupled DE system. For the hardening case, numerical simulation show that the displacement amplitude is growing indefinitely, until it reaches so large amplitudes that the system becomes unstable ( $\Omega > 148 @ \hat{u} \approx 150$ ) and the jump-down phenomenon occurs therefore very late; the analytical solution shows no jump-down phenomenon, as the amplitude reaches a complex solution for frequencies  $\Omega > 1$ . Furthermore in the analytical solution no such amplitude amplification can be observed. It is important to note for all numerical simulations that the generated basepoint excitation is a chirp signal that is slowly ramped up for hardening systems, e.g.  $\beta > 0$  and slowly ramped down for softening systems ( $\beta < 0$ ); if a quasi-static excitation is used for each frequency (always with initial conditions set to zero), no large amplitude amplifications can be observed.

The analytical frequency response grows with a higher power. For the backbone deviation in the  $u^5$  solution is ca. 3%; the amplification deviation is larger, best matches at ca. 5%. These mismatches can be attributed to the fact of having only the fundamental harmonics introduced for an acceptable manageable analytical expression. Making a spring with  $u^3$  is feasible, see also [1]; it needs further investigation whether a spring with  $u^5$  can be realized without introducing too much of damping!

## 4. Investigation in 2DoF Systems

In this chapter we deal with models and numerical simulations of vibration energy harvesters with two degrees of freedom (2DoF). Such systems are capable of autoparametric resonances, e.g. they stabilize their frequency response in a limited window of excitation, see also [6] and [7]. Figure 4 shows one configuration of a new basepoint excited vibration harvester system. The corresponding electrical network is deliberately drawn disconnected, as the transducer can be placed between  $y_0$  and  $y_1$  or  $y_0$  and  $y_2$  or  $y_1$  and  $y_2$ .



**Figure 4.** Lumped parameter model of the electromagnetic 2DoF vibration energy harvester; the electrical network showing the transducer unit can be placed (in theory) at three locations: between  $y_0$  and  $y_1$  or  $y_0$  and  $y_2$  or  $y_1$  and  $y_2$ . On the right hand side the cut free system.

Mass  $m_2$  is free flying, but bounded within the region of  $h_{21} + h_{22}$ . If the drawn spring  $k_{21,22}$  elements are present, mass  $m_2$  has soft impact, otherwise hard impact behavior. The mechanical damping elements  $d_{21,22}$  are always present. The input vibration is  $y_0$  and like in the 1DoF system, a

linear and also a nonlinear spring might be present, plus the unwanted mechanical dampings  $d_1$  plus the sketchily drawn system wall damping  $d_2$ .

#### 4.1. Lumped parameter model of the electromagnetic 2DoF system

The lumped parameter model is shown in Figure 4. The proposed harvester can be thought as 2DoF upgrade of the 1DoF model shown in Figure 1 using the same nomenclature as already introduced in chapter 2. (with additionally on top of mass  $m_1$  a second bounded in  $h_{21} + h_{22}$  free flying mass  $m_2$ ). Following the same nondimensionalization procedure as shown in chapter 2.1. will lead to the three dimensionless coupled differential equations for the system dynamic:

$$u_1'' + \frac{g}{y_0 \omega_0^2} - s_a(\lambda_{K1} \Delta u + 2\delta \lambda_{D1} \Delta u') - s_b(\lambda_{K2} \Delta u + 2\delta \lambda_{D2} \Delta u') - s_c(\lambda_{K1}(\Delta u - \rho) + 2\delta \lambda_{D1} \Delta u') = -\Lambda(\tau) \quad (20)$$

$$u_2'' + \frac{\lambda_M g}{y_0 \omega_0^2} + s_a(\lambda_{K1} \Delta u + 2\delta \lambda_{D1} \Delta u') + s_b(\lambda_{K2} \Delta u + 2\delta \lambda_{D2} \Delta u') + s_c(\lambda_{K1}(\Delta u - \rho) + 2\delta \lambda_{D1} \Delta u') + \kappa \lambda_E \zeta = 0 \quad (21)$$

$$\kappa u' = \zeta' + \lambda_E \zeta \quad (22)$$

with following supplementary equations for (21) and (20):

$$\Delta u = u_2 - u_1; \Delta u' = u_2' - u_1'; s_a = \sigma(-\Delta u); s_c = \sigma(\Delta u - \rho); s_b = 1 - s_a - s_c \quad (23)$$

and with the inserted basepoint excitation  $A \cos(\Omega \tau)$  the stimulation  $\Lambda$  can be written as:

$$\Lambda(\tau) = u_1 + \beta u_1^n - A \cos(\Omega \tau) + 2\delta(u_1' + A \Omega \sin(\Omega \tau)) \quad (24)$$

The activation of according stiffness and damping depending on freefall ( $s_b$ ), lower impact ( $s_a$ ) and upper impact ( $s_c$ ) are switched on and off via only path depending Heaviside functions.

### 5. Comparison of frequency response for electromagnetic 2DoF vibration harvesters

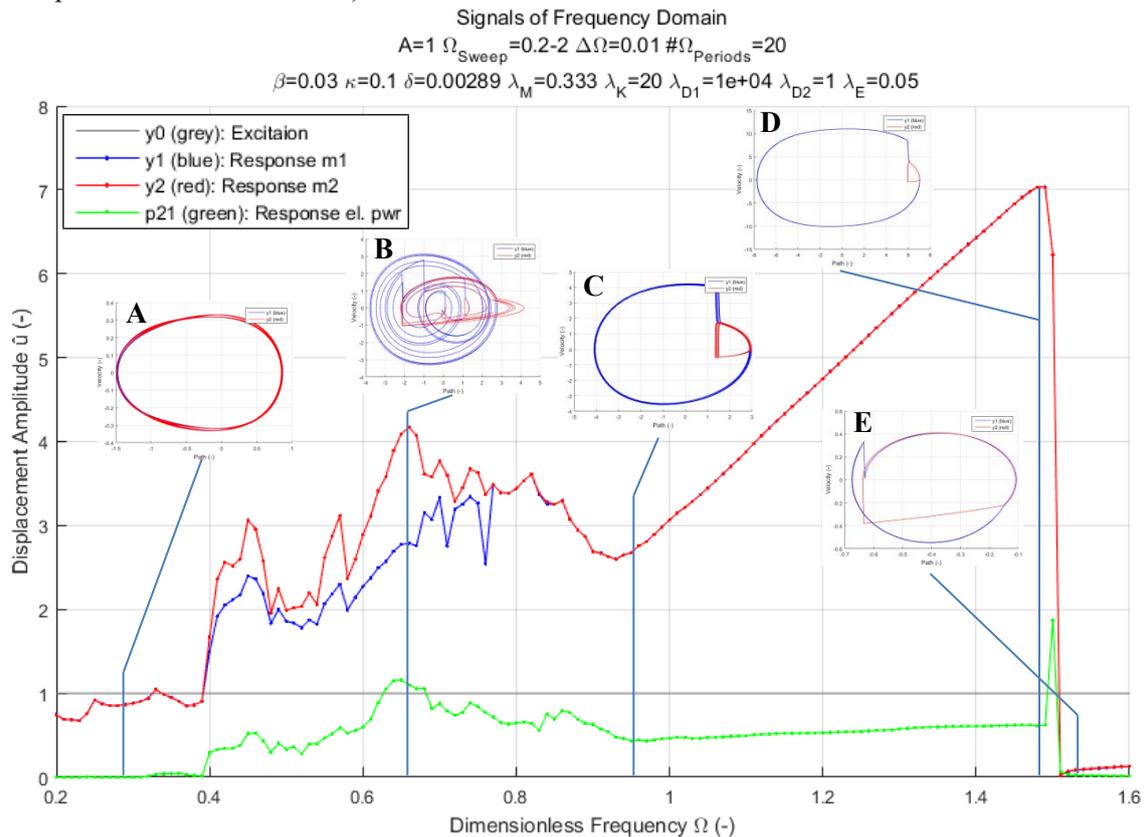
Frequency response of the three dimensionless coupled differential equations (20), (21) and (22) are depicted in Figure 5, using numerical simulation tools. The nonlinearity of the spring  $k_{1n}$  is cubic (using a weak nonlinearity with  $\beta = 0.03$ ), e.g. exponent  $n = 3$  in the stimulation function  $\Lambda$ . The springs  $k_{21} = k_{22}$  are linear, the damping for impact  $d_{21} = d_{22}$  kept also equal (see also Figure 4), resulting in dimensionless constants  $\lambda_{K1} = \lambda_K = 20$  and  $\lambda_{D1} = 10^4$ . The free fall damping  $\lambda_{D2} = 1$  is weak (free fall stiffness  $\lambda_{K2} = 0$ ) and the proportional factor for masses  $m_1$  and  $m_2$  is set to  $\lambda_M = 1/3$ . In the electrical domain the dimensionless constants are the transduction factor  $\kappa$  (also assumed to be constant) and the resistive load is assumed to be weak  $\lambda_E = 0.05$ . For the upper mass  $m_2$  bounded path limits are set to  $h_{21}=0$  and  $h_{22}$  dimensionless transformed constant set large to  $\rho = 50$  (resulting in no upper impact).

Exciting the system with a frequency  $\Omega < 4$  results in a linear frequency response (phase-plot in detail A), as for such frequencies the resulting acceleration for  $m_2$  is too weak (in comparison to the gravitational acceleration) and therefore it will not lift off. Increasing the frequency further,  $m_2$  starts intermittently to lift off, this is shown in phase-plots details B-E. Especially interesting is the behavior of mass  $m_2$  in interval  $0.8 < \Omega < 1.5$  (phase-plot details C and D are similar) – over this large interval, we have an auto-stabilization frequency effect of mass  $m_2$  (autoparametric resonance).

### 6. Conclusions

New analytical models for the set of DE given in (5) and (6) have been created using a well-studied nonlinear exponent ( $n = 3$ ) and a new ( $n = 5$ ) for 1DoF vibration energy harvesters leading to solutions presented in (17) and (19); these analytical models provide a qualitative estimation of the nonlinear frequency response (compared to the numerical solutions). The larger the exponent grows, the larger the estimated analytical error becomes. Furthermore we showed, that introducing a very strong nonlinear spring with exponent  $n = 5$ , we stay almost indefinitely in resonance for hardening systems (Figure 3).

We then proposed and modeled a new class of 2DoF vibration harvester systems and showed a first none exhaustive analysis of the energy transfer using a dimensionless model, showing autoparametric resonance for both masses  $m_1$  and  $m_2$  over a large frequency range (see also Detail C and D, Figure 5 and chapter 5. for further details).



**Figure 5.** Dimensionless frequency responses  $y_1$  (blue) and  $y_2$  (red) and electrical power (green) having weak electrical load (see also chapter 4. ) using an excitation chirp signal  $y_0$  (grey).

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