

Active fault detection: A comparison of probabilistic methods

Jan Škach, Ivo Punčochář

Department of Cybernetics and NTIS - New Technologies for the Information Society, Faculty of Applied Sciences, University of West Bohemia, Technická 8, 30614 Plzeň, Czech Republic

E-mail: janskach@kky.zcu.cz, ivop@kky.zcu.cz

Abstract. The paper deals with probabilistic methods for designing the active fault detectors that improve the quality of detection using an auxiliary input signal. Two probabilistic methods that assume a similar stochastic model of a monitored system are considered and compared with a special attention to various difficulties associated with active fault detector designs. The active fault detector design based on a general detection cost function is compared with the model sequence selection error minimization design in terms of assumptions and theoretical properties. Practical aspects of both methods are also considered and demonstrated through a numerical example.

1. Introduction

Over the last decades fault diagnosis has received an increased attention [1]. With growing demands on the quantity and quality of production, automated systems have become more complex and less predictable and protected at the same time. Fault diagnosis allows a monitored system to be supervised in order to achieve a required safety integrity level.

There are two approaches to fault detector design: passive approach and active approach. The difference between these two approaches is illustrated in the block diagrams in Figure 1. In the case of the passive approach [2, 3] the input-output data of the monitored system are measured, analyzed for faulty behavior, and consequently a decision about a fault is made based on this analysis. Passive detectors are designed using various methods ranging from signal based methods to model based methods or knowledge based methods [4, 5, 6]. Contrary to the passive approach that is mature and commonly used in many today's applications, the active approach [7, 8, 9, 10, 11] is historically younger, generally more complex, and still under development. In addition to generating decisions, an active fault detector interacts with the monitored system by means of a suitably designed auxiliary input signal that is injected into the system to probe it for faults. This auxiliary input signal can then increase the quality of fault diagnosis.

Several methods with different levels of generality have been developed in the area of active fault detection (AFD). They can be classified into two groups based on the description of uncertainties in the monitored system. Methods in the first group can be called probabilistic as the uncertainties are described by stochastic processes with known properties. A common practice is to assume that the initial state, state noise, and measurement noise are white random processes with known probability density functions (pdfs). More information on the probabilistic



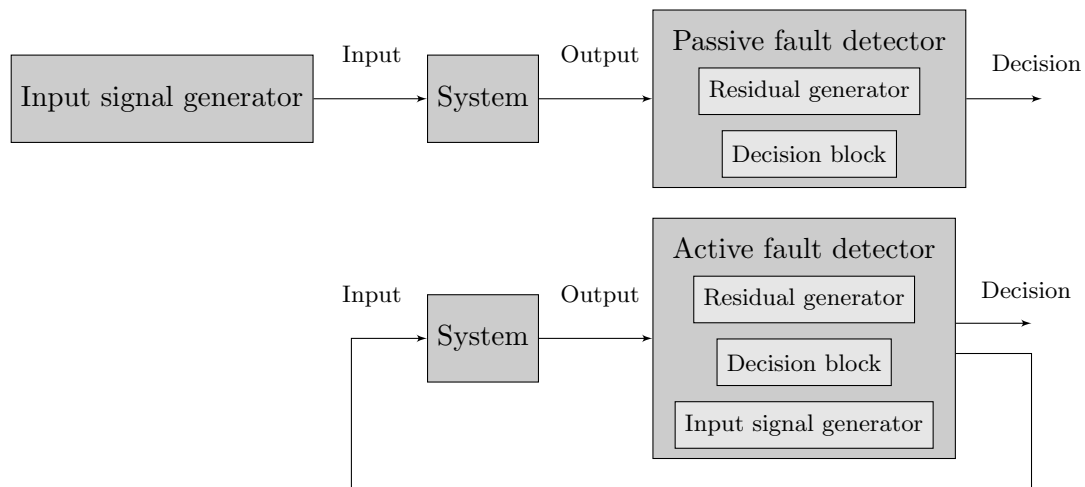


Figure 1. General schemes for passive and active fault detection.

methods can be found in [8, 12, 13, 11, 14]. The second group consists of deterministic methods which assume that the uncertainties are described as bounded signals. More specifically, the initial state, state noise, and measurement noise belong to particular bounded sets. Contrary to the probabilistic methods a perfect diagnosis can be achieved on a finite time horizon if input-output trajectories can be separated by a suitably designed auxiliary input signal. Deterministic methods based on an integrated design of a controller with four degrees of freedom using the \mathcal{H}_2 and \mathcal{H}_∞ theory can be found in [15, 10, 16]. The other group of deterministic methods based on an idea of bounding system uncertainties is investigated, e.g., in [9, 17, 18, 19].

The AFD methods are generally more complex than the passive methods and finding a suitable auxiliary input signal can be a very challenging task. The methods usually struggle with similar design problems and throughout the solution may employ various practices from different technical fields such as filtering, estimation, optimization, artificial intelligence, etc. In this paper, two probabilistic methods will be discussed in order to show common features as well as different characteristics and difficulties of the probabilistic methods. This can be useful when closely studying the AFD. Moreover, a numerical example will be given to demonstrate the performance and to provide a better insight into the theoretical background of finding the auxiliary input signal. To the best of authors' knowledge, no detailed comparison of probabilistic AFD methods accompanied by a numerical example has been done.

In the first method, the design of an active fault detector is formulated as an optimization problem where a criterion penalizing wrong decisions is minimized. The method was presented both for a finite time horizon [11, 20] and an infinite time horizon [21, 22]. Note that the formulation contains three special cases based on the form of the criterion, i.e. AFD, active fault detection and control (AFDC), and dual control (DC). In this paper, only the AFD case is considered. A solution can be found by approximate dynamic programming (ADP) techniques [23]. The second method considered in this paper is based on finding an auxiliary input signal that minimizes the probability of the model selection error [13]. A solution to the finite horizon optimization problem can be found using sequential quadratic programming (SQP) techniques [24]. Moreover, a given control task can be achieved when considering constraints on admissible system state behavior.

The paper is organized as follows. In Section 2, a general formulation of the AFD problem for linear discrete-time stochastic systems and a state estimation problem is presented. Active fault detector design formulations and corresponding problem solutions of the two probabilistic

methods are presented and comprehensively compared in Section 3. A numerical example in Section 4 demonstrates performance of designed active fault detectors.

2. Problem formulation

Since two considered probabilistic methods make a slightly different assumptions about the system and the aim of fault detection, this section presents only the part of the problem formulation that is common to both methods. At first, the multiple-model description of the system is presented. Then an approximate solution to the state estimation problem for such a model is shown. The other assumptions are discussed in Section 3.

2.1. System description

The AFD methods usually employ the multiple-model approach to represent the behavior of the monitored system under fault-free and faulty conditions. Therefore, it is considered that a monitored system can be described at each time step k by one of the following linear discrete-time stochastic dynamic models

$$\mathbf{x}_{k+1} = \mathbf{A}_{\mu_k} \mathbf{x}_k + \mathbf{B}_{\mu_k} \mathbf{u}_k + \mathbf{w}_k, \quad (1a)$$

$$\mathbf{y}_k = \mathbf{C}_{\mu_k} \mathbf{x}_k + \mathbf{v}_k, \quad (1b)$$

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is an unknown continuous common part of the hybrid state $\mathbf{x}_k^a = [\mathbf{x}_k^T, \mu_k]^T \in \mathbb{R}^{n_x} \times \mathcal{M}$, $\mu_k \in \mathcal{M} = \{1, 2, \dots, N_\mu\}$ is an unknown index of the model, N_μ is the number of models with one fault-free model and $N_\mu - 1$ faulty models, $\mathbf{u}_k \in \mathcal{U} \subset \mathbb{R}^{n_u}$ is the input, $\mathbf{y}_k \in \mathbb{R}^{n_y}$ is the output measurement, $\mathbf{w}_k \in \mathbb{R}^{n_x}$ and $\mathbf{v}_k \in \mathbb{R}^{n_y}$ are corresponding mutually independent zero-mean Gaussian white noises with known covariances \mathbf{Q}_{μ_k} and \mathbf{R}_{μ_k} , respectively. The matrices \mathbf{A}_{μ_k} , \mathbf{B}_{μ_k} , \mathbf{C}_{μ_k} are known and have matching dimensions. The model index μ_k is described by a finite-state Markov chain with a known transition probability matrix $\mathbf{P}_{j,i} = P(\mu_{k+1} = j | \mu_k = i)$. The initial hybrid-state distribution is described by probability $P(\mu_0)$ of the initial model index μ_k and initial condition on continuous part \mathbf{x}_0 such that it corresponds to the Gaussian distribution with a given mean value $\hat{\mathbf{x}}_{0|-1}$ and a symmetric positive definite covariance matrix $\Sigma_{0|-1}^x$.

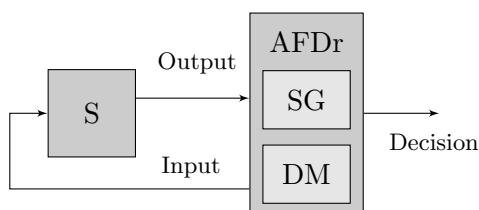


Figure 2. A block diagram of active fault detection.

Active fault detector (AFDr) consists of a decision making block (DM) that generates a decision about faults in the system (S) and an auxiliary input signal generator (SG) that probes the system, as shown in Figure 2. A goal of the AFD problem is to design AFDr such that a certain criterion will be optimized. In Section 3, an overview of two different design methods of the probabilistic approach to AFD will be covered.

2.2. State estimation

The multiple model approach brings a problem of state estimation which is described by a stochastic linear Markovian switching system. The question can be either to determine a conditional probability of model $P(\mu_k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ or a conditional probability of hybrid-state $P(\mathbf{x}_k^a | \mathbf{y}_0^k, \mathbf{u}_0^{k-1})$. Both probabilities can be expressed using a sum over all of $(N_\mu)^k$ possible

model sequences μ_0^{k-1} , i.e.

$$P(\mu_k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}) = \sum_{\mu_0^{k-1}} P(\mu_0^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}), \quad (2a)$$

$$P(\mathbf{x}_k^a | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}) = \sum_{\mu_0^{k-1}} P(\mu_0^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}) P(\mathbf{x}_k | \mu_0^k, \mathbf{y}_0^k, \mathbf{u}_0^{k-1}). \quad (2b)$$

Note that the conditional probability of the model sequence μ_0^k can be determined by

$$P(\mu_0^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}) = \frac{p(\mathbf{y}_k | \mu_0^k, \mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-1}) P(\mu_0^k | \mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-2})}{\sum_{\mu_0^k} p(\mathbf{y}_k | \mu_0^k, \mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-1}) P(\mu_0^k | \mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-2})}, \quad (3)$$

where the predictive probability of the model sequence is given by

$$P(\mu_0^k | \mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-2}) = \begin{cases} P(\mu_0) & \text{if } k = 0, \\ P(\mu_k | \mu_{k-1}) P(\mu_0^{k-1} | \mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-2}) & \text{if } k > 0. \end{cases} \quad (4)$$

For a given model sequence μ_0^k the optimal state estimate can be obtained by the Kalman filter. The conditional pdf $p(\mathbf{y}_k | \mu_0^k, \mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-1})$ has the Gaussian distribution with the mean value $\hat{\mathbf{y}}_{k|k-1}$ and the covariance matrix $\Sigma_{k|k-1}^{\mathbf{y}}$ that are given by

$$\hat{\mathbf{y}}_{k|k-1} = \mathbf{C}_{\mu_k} \hat{\mathbf{x}}_{k|k-1}(\mu_0^{k-1}), \quad (5a)$$

$$\Sigma_{k|k-1}^{\mathbf{y}} = \mathbf{C}_{\mu_k} \Sigma_{k|k-1}^{\mathbf{x}} \mathbf{C}_{\mu_k}^T + \mathbf{R}_{\mu_k}. \quad (5b)$$

Note that the initial values $\hat{\mathbf{x}}_{0|-1}$, $\Sigma_{0|-1}^{\mathbf{x}}$ are known. The filtering conditional pdf $p(\mathbf{x}_k | \mu_0^k, \mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ has the Gaussian distribution with the mean value $\hat{\mathbf{x}}_{k|k}(\mu_0^k)$ and the covariance $\Sigma_{k|k}^{\mathbf{x}}(\mu_0^k)$ given by

$$\hat{\mathbf{x}}_{k|k}(\mu_0^k) = \hat{\mathbf{x}}_{k|k-1}(\mu_0^{k-1}) + \mathbf{K}(\mu_0^k) [\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}(\mu_0^k)], \quad (6a)$$

$$\Sigma_{k|k}^{\mathbf{x}}(\mu_0^k) = \Sigma_{k|k-1}^{\mathbf{x}}(\mu_0^{k-1}) - \mathbf{K}(\mu_0^k) \mathbf{C}_{\mu_k} \Sigma_{k|k-1}^{\mathbf{x}}(\mu_0^{k-1}), \quad (6b)$$

where the Kalman gain $\mathbf{K}(\mu_0^k)$ can be found solving

$$\mathbf{K}(\mu_0^k) = \Sigma_{k|k-1}^{\mathbf{x}}(\mu_0^{k-1}) \mathbf{C}_{\mu_k}^T \Sigma_{k|k-1}^{\mathbf{y}}(\mu_0^k)^{-1}. \quad (7)$$

Finally, the predictive state mean value $\hat{\mathbf{x}}_{k+1|k}(\mu_0^k)$ and the predictive state covariance matrix $\Sigma_{k+1|k}^{\mathbf{x}}(\mu_0^k)$ are given by

$$\hat{\mathbf{x}}_{k+1|k}(\mu_0^k) = \mathbf{A}_{\mu_k} \hat{\mathbf{x}}_{k|k}(\mu_0^k) + \mathbf{B}_{\mu_k} \mathbf{u}_k, \quad (8a)$$

$$\Sigma_{k+1|k}^{\mathbf{x}}(\mu_0^k) = \mathbf{A}_{\mu_k} \Sigma_{k|k}^{\mathbf{x}}(\mu_0^k) \mathbf{A}_{\mu_k}^T + \mathbf{Q}_{\mu_k}. \quad (8b)$$

It can be seen that the conditional probabilities (2) can be computed using the equations above. However, with the higher k , the number of model sequences grows exponentially and it would be computationally intractable to compute the statistics for all the sequences. One of the common suboptimal solutions that prevents the problem of increasing number of all possible model sequences with time consists of tracking only a h -step history of the model sequences.

It means that when $k \geq h$, the conditional probability of the model sequence μ_{k-h+1}^k can be computed as

$$P(\mu_{k-h+1}^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}) = \sum_{\mu_{k-h}} P(\mu_{k-h}^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}) \quad (9)$$

and the conditional pdf $p(\mathbf{x}_k | \mu_{k-h+1}^k, \mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ has a form of Gaussian sum replaced by a single Gaussian pdf with the first and second moments computed by a moment matching technique. This h -step history tracking corresponds to the Generalized Pseudo Bayes algorithm [25].

3. Active fault detector design

Two methods for designing an active fault detector will be briefly summarized and then compared in this section. Both methods assume the same model of the system described in Section 2.1. Also the approximate solution to the state estimation problem presented in Section 2.2 is employed by these methods. Besides some slight differences between methods, the key difference is the way the available information is utilized in those two methods. The first method was proposed in [22] and it uses so called closed loop information processing strategy. The second method was presented in [13] and it basically applies the open loop feedback information processing strategy. Without use of any approximations the closed loop information processing strategy will always result in a lower or the same value of the criterion compared to the open loop information processing strategy. Unfortunately, when approximations are used through the solution, it is almost impossible to tell in advance whether the use of the closed loop information processing strategy will be beneficial and to what extent. In such a case, a numerical comparison for a particular example can provide some insight into benefits of the closed loop information processing strategy.

3.1. Optimal active fault detector design

The design of an optimal active fault detector that minimizes a discounted sum of decision costs over an infinite time horizon assuming that the common continuous part of the hybrid state is known was considered in [22]. However, this part of the hybrid state is rather observed through noisy measurements in most of the real-world systems. A method for designing an active fault detector for such systems was proposed in [26]. Although the infinite time horizon formulation is theoretically more challenging, it has two advantages when compared to the finite time horizon formulation. First, the optimal active fault detector is represented by a time-invariant function that maps the pdf of the hybrid state into decisions and auxiliary inputs. Second, the actual time horizon need not be known in advance and the obtained active fault detector can be used even for a finite time horizon as an approximation of the optimal solution. The h -step history tracking is employed to limit the increasing number of sequences with time. In the following part a problem formulation of finding the active fault detector will be given.

The first step of the design procedure is the problem reformulation using a hyper-state that includes the results produced by the approximate state estimation procedure. The hyper-state allows the original model to be rewritten into a new model that basically includes the original model together with the approximate state estimator. Thus the monitored system coupled with the approximate state estimator can be described at each time step k by the following model

$$\boldsymbol{\xi}_{k+1} = \phi(\boldsymbol{\xi}_k, \mathbf{u}_k, \mathbf{y}_{k+1}), \quad (10)$$

where $\boldsymbol{\xi}_k \in \mathcal{G} \subset \mathbb{R}^{n_\epsilon}$ is the hyper-state that consists of a finite number of sufficient statistics for the unknown hybrid state \mathbf{x}_k^a , $\phi : \mathcal{G} \times \mathcal{U} \times \mathbb{R}^{n_y} \mapsto \mathcal{G}$ is a time-invariant function that describes the monitored system coupled with the approximate state estimator, and \mathbf{y}_{k+1} is a random

variable with the known predictive conditional pdf $p(\mathbf{y}_{k+1}|\boldsymbol{\xi}_k, \mathbf{u}_k)$ that has the form of a Gaussian sum. The hyper-state $\boldsymbol{\xi}_k$ consists of the filtering estimates of the hybrid state, i.e., the mean values $\hat{\mathbf{x}}_{k|k}(\mu_{k-h+1}^k)$, covariance matrices $\boldsymbol{\Sigma}_{k|k}^{\mathbf{x}}(\mu_{k-h+1}^k)$, and probabilities $P(\mu_{k-h+1}^k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ for all possible model sequences μ_{k-h+1}^k . Note that the hyper-state dimension can be reduced simply by considering only the lower or upper triangular part of the covariance matrix $\boldsymbol{\Sigma}_{k|k}^{\mathbf{x}}(\mu_{k-h+1}^k)$ and only $(N_\mu)^h - 1$ conditional probabilities of model sequences since they sum up to one. Then the hyper-state dimension yields in $n_{\boldsymbol{\xi}} = (N_\mu)^h(n_{\mathbf{x}}^2 + 3n_{\mathbf{x}} + 2)/2 - 1$.

This reformulation of the original problem enables the active fault detector that depends only on the hyper-state $\boldsymbol{\xi}_k$ to be designed. The active fault detector is assumed to be a time-invariant mapping of the hyper-state into a decision and auxiliary input

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \boldsymbol{\rho}(\boldsymbol{\xi}_k) = \begin{bmatrix} \sigma(\boldsymbol{\xi}_k) \\ \gamma(\boldsymbol{\xi}_k) \end{bmatrix}, \quad (11)$$

where $\boldsymbol{\rho} : \mathcal{G} \mapsto \mathcal{M} \times \mathcal{U}$ is an unknown function that represents the active fault detector with components $\sigma : \mathcal{G} \mapsto \mathcal{M}$ generating the decisions $d_k \in \mathcal{M}$ and $\gamma : \mathcal{G} \mapsto \mathcal{U}$ generating the auxiliary input signal \mathbf{u}_k . The active fault detector is designed such that the design criterion

$$J^{\text{OAFD}}(\boldsymbol{\rho}) = \lim_{F \rightarrow \infty} \mathbb{E} \left\{ \sum_{k=0}^F \lambda^k L^{\text{d}}(\boldsymbol{\xi}_k, d_k) \right\} \quad (12)$$

is minimized. Note that $L^{\text{d}} : \mathcal{G} \times \mathcal{M} \mapsto \mathbb{R}^+$ refers to a general detection cost function that penalizes wrong decisions about faults in the monitored system and $\lambda \in (0, 1)$ is a discount factor that influences the future detection costs. Since the detection function L^{d} is chosen in advance the designer must understand the system behavior. In case of the AFDC problem the criterion (12) must include a component that penalizes system state behavior.

The introduced problem reformulation leads to a dynamic optimization problem with perfect state information that can be solved by dynamic programming (DP) [27]. It follows from DP that the design of an optimal active fault detector is reduced to solving the following discrete-time Bellman functional equation

$$V^*(\boldsymbol{\xi}_k) = \min_{d_k \in \mathcal{M}} \mathbb{E}\{L^{\text{d}}(\boldsymbol{\xi}_k, d_k)|\boldsymbol{\xi}_k, d_k\} + \min_{\mathbf{u}_k \in \mathcal{U}} \mathbb{E}\{\lambda V^*(\phi(\boldsymbol{\xi}_k, \mathbf{u}_k, \mathbf{y}_{k+1}))|\boldsymbol{\xi}_k, \mathbf{u}_k\}, \quad (13)$$

where $V^* : \mathcal{G} \mapsto \mathbb{R}$ is the unknown Bellman function that represents the expected detection costs over the infinite time horizon $[k, \infty)$ discounted by λ when starting at $\boldsymbol{\xi}_k$ and using the optimal policy $\boldsymbol{\rho}^*$. Note that the term "policy" is commonly used in DP and refers to a control law. Then the optimal detector and the optimal auxiliary input signal generator are given by

$$d_k^* = \sigma^*(\boldsymbol{\xi}_k) = \arg \min_{d_k \in \mathcal{M}} \mathbb{E}\{L^{\text{d}}(\boldsymbol{\xi}_k, d_k)|\boldsymbol{\xi}_k, d_k\} \quad (14a)$$

$$\mathbf{u}_k^* = \gamma^*(\boldsymbol{\xi}_k) = \arg \min_{\mathbf{u}_k \in \mathcal{U}} \mathbb{E}\{V^*(\phi(\boldsymbol{\xi}_k, \mathbf{u}_k, \mathbf{y}_{k+1}))|\boldsymbol{\xi}_k, \mathbf{u}_k\}. \quad (14b)$$

Although the formal solution described by (13) and (14) is obtained straightforwardly by DP, the design problem was only recast into solving the discrete-time Bellman functional equation. As an analytical form of the Bellman function is hard to find in general case, a suitable approximation of the Bellman function needs to be found using ADP [28, 23]. A common approximations to Bellman function include parametric approximations, nonparametric approximations, and other variants. Furthermore, the approximate Bellman function is found by an iterative algorithm such as the value iteration algorithm or the policy iteration algorithm. Finally, the conditional expected value in the discrete-time Bellman functional equation (13) must be approximated since the system function (10) is nonlinear. To approximate the expectation a sufficient number of Monte Carlo simulations or an unscented transformation can be used.

3.2. Model selection error minimization design

The design of an auxiliary input signal generator that minimizes the probability of the model sequence selection error at the end of a finite time horizon was presented in [13]. Although the time horizon is assumed to be finite, it can be very large. The exponentially growing number of model sequences must be then approximated using a pruning technique. Since the key idea of the auxiliary input signal design is to minimize the probability that the true model sequence will be pruned at the end of the finite horizon, a different approach is used to tackle this issue. The auxiliary input signal is designed in the open loop by considering only a reasonable number of the model sequences that are a priori the most probable. Thus, the model sequences with a low a priori probability are discarded and only a finite number of promising model sequences is tracked.

The auxiliary input signal is designed such that the following criterion

$$J^{\text{MSEM}}(\mathbf{u}_0^{F-1}) = P(\text{prune}|\mathbf{u}_0^{F-1}) = \sum_{\mu_0^{*F}} \int_{\mathbf{y}_0^F} P(\text{prune}|\mathbf{y}_0^F, \mathbf{u}_0^{F-1}) p(\mathbf{y}_0^F|\mu_0^{*F}) d\mathbf{y}_0^F P(\mu_0^{*F}) \quad (15)$$

is minimized. The criterion J^{MSEM} denotes the probability of pruning the true model sequence marginalized over the measurements and true model sequences. The event of pruning true model sequence can be expressed as

$$\exists i \geq K : P(\mu_0^{*F}|\mathbf{y}_0^F, \mathbf{u}_0^{F-1}) < P((\mu_0^F)_i|\mathbf{y}_0^F, \mathbf{u}_0^{F-1}), \quad (16)$$

where μ_0^{*F} is the true model sequence, K is a known number of tracked model sequences, the model sequences are sorted in descending order of probability, and F denotes the final time step of the finite time horizon.

However, the exact value of the probability of pruning the true model sequence cannot be determined analytically. Therefore, the method employs an approximation by a tractable upper bound of this probability. Then for the K tracked model sequences, the criterion (15) approximated by the upper bound has the following form

$$\bar{J}^{\text{MSEM}}(\mathbf{u}_0^{F-1}) = \sum_{i=1}^K \sum_{j>i}^K P((\mu_0^F)_i)^{\frac{1}{2}} P((\mu_0^F)_j)^{\frac{1}{2}} e^{-\kappa_{ij}}, \quad (17)$$

where κ_{ij} is a quadratic function of \mathbf{u}_0^{F-1} parametrized by predictive mean values and covariance matrices of the measurements conditioned by the corresponding tracked sequence $(\mu_0^F)_i$. The problem of finding K a priori most probable model sequences is not trivial, but it fits to a tree search problems known from computer science and artificial intelligence [29]. The tree search algorithms evaluate and expand tree nodes, i.e. the probabilities of model sequences. A^* search is a best-first search algorithm that features a memory and expands the tree based on a cost function of the known costs and a heuristic estimate. The tree search algorithm such as A^* should find the K most probable sequences efficiently.

The problem formulation leads to an optimization problem suitable to be solved using SQP [24]. When extending the AFD problem to AFDC, the demands on system state behavior can be expressed in form of linear constraints. Then the SQP methods find a solution to the equality-constrained problem

$$\mathbf{u}_0^{*F-1} = \arg \min_{\mathbf{u}_0^{F-1}} \bar{J}^{\text{MSEM}}(\mathbf{u}_0^{F-1}), \quad (18a)$$

$$\text{subject to } c_{\text{eq}}(\mathbf{u}_0^{F-1}) = 0, \quad c_{\text{iq}}(\mathbf{u}_0^{F-1}) \leq 0, \quad (18b)$$

where c_{eq} and c_{iq} are functions of equality and inequality constraints, respectively, e.g. constraints on the control inputs $\mathbf{u}_{\min} \leq \mathbf{u}_i \leq \mathbf{u}_{\max}$ for all $i = 0, 1, \dots, F-1$ or expected system state $\mathbf{x}_{\min} \leq \mathbb{E}[\mathbf{x}_i | \mathbf{y}_0^i, \mathbf{u}_0^{i-1}] \leq \mathbf{x}_{\max}$ in case of the AFDC problem. Note that \mathbf{u}_{\min} , \mathbf{u}_{\max} , \mathbf{x}_{\min} , and \mathbf{x}_{\max} are corresponding extreme values of the input and system state, respectively.

3.3. Comparison

A theoretical comparison of the two probabilistic methods is covered in Table 1. Both methods consider analogous system description and state estimation techniques that encouraged their joint comparison. However, one of the substantial difference lies in a type of the information processing strategy used. The closed loop information processing strategy should be advantageous because all available and the future information is considered in the design. Another significant difference can be seen in the criteria defining the auxiliary input signals. Therefore, this should be considered when evaluating a quality of the detectors.

Since the analytical solution to the formulated active fault detector designs is impossible to find, various approximations are used. A reformulation as a perfect state information problem is employed in case of OAFD. An analytical solution of the reformulated problem is found and approximated afterwards. In [22], the continuous state space is discretized using a uniform grid. The Bellman function is evaluated in the grid points. Then, the auxiliary input signal is generated for an arbitrary continuous state using an aggregation function that projects the state to the grid. The state-space quantization is used in [30] to transform the original problem into a Markov decision problem that can be solved using DP. A different approach is considered in [26] where a nonparametric local Bellman function is first trained using sampled data along the system trajectory and then employed to approximate the active fault detector. On the other hand, a problem reformulation in the MSEM design is used to propose an approximate solution. Such a reformulated problem is then solved by SQP [31, 13]. These approximations can largely influence the quality of suboptimal solutions as well as computational demands. As a consequence, different structures of auxiliary input signal generators can largely influence selection of the method for online detection.

4. Numerical example

Both probabilistic AFD methods follow a similar framework for the auxiliary input signal design as shown in Section 3. In this section, particular features of the methods are highlighted by means of a numerical example. Basically, three information processing strategies are used in the numerical example: the MSEM designed using open loop and open loop feedback information processing strategy and OAFD designed using the closed loop information processing strategy.

To simplify the design of the active fault detectors, it is assumed that the monitored system can be described by one of three first-order models. The matrices of these models are the following

$$\begin{aligned} \mathbf{A}_1 &= 0.8187, \mathbf{A}_2 = 0.8106, \mathbf{A}_3 = 0.8959, \mathbf{B}_1 = 0.1813, \mathbf{B}_2 = 0.1894, \mathbf{B}_3 = 0.1042, \\ \mathbf{C}_1 &= \mathbf{C}_2 = \mathbf{C}_3 = 1, \mathbf{Q}_1 = \mathbf{Q}_2 = \mathbf{Q}_3 = 1 \cdot 10^{-6}, \mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}_3 = 1 \cdot 10^{-4}. \end{aligned} \quad (19)$$

Two different lengths of the finite time horizon $F = 3$ and $F = 20$ are considered. The switching between models over time is given by the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 0.98 & 0.2 & 0.1 \\ 0.01 & 0.6 & 0.3 \\ 0.01 & 0.2 & 0.6 \end{bmatrix}. \quad (20)$$

The probability of the initial model is $P(\mu_0) = [0.1, 0.5, 0.4]^T$. The initial state mean value is $\hat{\mathbf{x}}_{0|-1} = 0$ and the covariance matrix is $\Sigma_{0|-1}^{\mathbf{x}} = 2 \cdot 10^{-4}$. The set of admissible inputs is

	OAFD	MSEM
System	Linear discrete-time stochastic systems described by state-space models are considered. System uncertainties are described by known pdfs.	Linear discrete-time stochastic systems described by state-space models are considered. System uncertainties are described by known pdfs.
Statistics	The Kalman filter and Bayesian theory are used for hyper-state estimation.	The Kalman filter and Bayesian theory are used for hyper-state estimation.
Information processing strategy	The method uses the closed loop information processing strategy.	The method can either employ the open loop or open loop feedback information processing strategy.
Model sequences tracking	A finite history of model sequences are tracked.	The aim is to track a finite number of model sequences that contain the true model sequence.
Time horizon	The infinite time horizon formulation is presented due to its universal use. The finite time horizon case is presented in [11].	The auxiliary input signal is designed for a finite time horizon. A rolling horizon technique can be used for the infinite time horizon.
Fault detection	A general detection aim can be expressed using the detection cost function.	Penalization of true and false decisions is done only by zero and one, respectively.
System control	A control aim can be included into the criterion.	A control aim can be expressed using state constraints.
Criterion	The auxiliary input signal minimizes the given general design criterion.	The auxiliary input signal minimizes the upper bound of losing track of the true model sequence.
Approximations in formulation	The original problem is reformulated using hyper-state.	Upper bound on the probability of model selection error is considered.
Solution	The optimization problem is solved by ADP. The Bellman function is replaced by a local non-parametric function approximation.	The optimization problem is solved by SQP.
Auxiliary signal	The auxiliary input signal generator is a time invariant function of the hyper-state.	The auxiliary input signal is a given sequence independent of system state and measurements.

Table 1. Comparison of the optimal active fault detector design (OAFD) method and the model selection error minimization design (MSEM) method in terms of their features.

$\mathcal{U} = \{-1, 0, 1\}$. The GPB algorithm is used with parameter $h = 1$ to compute the approximate state estimate that consists of three mean values $\hat{\mathbf{x}}_{k|k}(\mu_k)$, three covariances $\Sigma_{k|k}^{\mathbf{x}}(\mu_k)$ and two probabilities $P(\mu_k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1})$. As a result, the hyper-state ξ_k is an eight dimensional vector

$$\xi_k = [\hat{\mathbf{x}}_{k|k}(1), \hat{\mathbf{x}}_{k|k}(2), \hat{\mathbf{x}}_{k|k}(3), \Sigma_{k|k}^{\mathbf{x}}(1), \Sigma_{k|k}^{\mathbf{x}}(2), \Sigma_{k|k}^{\mathbf{x}}(3), P(\mu_k = 1 | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}), P(\mu_k = 2 | \mathbf{y}_0^k, \mathbf{u}_0^{k-1})].$$

Note that $P(\mu_k = 3 | \mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ can be simply computed from the hyper-state ξ_k .

In order to make the detection aim as similar as possible in both methods, the detection cost function for OAFD method is chosen to be

$$L^d(\xi_k, d_k) = E\{L(\mu_k, d_k) | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}\} = \begin{cases} 1 - \xi_{k,7} & \text{if } d_k = 1, \\ 1 - \xi_{k,8} & \text{if } d_k = 2, \\ \xi_{k,7} + \xi_{k,8} & \text{if } d_k = 3, \end{cases} \quad (21)$$

where $L : \mathcal{M} \times \mathcal{M} \mapsto \{0, 1\}$ is the zero-one cost function defined as

$$L(\mu_k, d_k) = \begin{cases} 0 & \text{if } \mu_k = d_k, \\ 1 & \text{otherwise.} \end{cases} \quad (22)$$

Since the OAFD, which assumes the infinite time horizon, is used as an approximate solution to a finite time horizon, the discount factor is computed as

$$\lambda = 10^{\frac{\log(z/100)}{F+1}}, \quad (23)$$

where $z = 1\%$ denotes the desired value of the tail costs with respect to the overall costs in percent. The Bellman function was approximated by a nonparametric approximation analogously to [26]. The samples in the hyper-state space that were used to solve the discrete-time Bellman functional equation approximately using the value iteration algorithm were obtained by simulating the monitored system with the following input signal generators:

- zero input signal generator,
- sine input signal generator, and
- MSEM OL input signal generator.

These simulations resulted into 378 sample points in the hyper-state space. The value iteration algorithm convergence error threshold was set to $1 \cdot 10^{-2}$ and the algorithm converged in 5 iterations for $F = 3$ and 16 iterations for $F = 20$.

The following setup was used for the MSEM. The number of the most probable sequences was chosen to be $K = 10$ and the optimization problem with a relaxed constraint $u_k \in [-1, 1]$ was solved using a trust region method. The starting auxiliary input sequence for the trust region method was chosen randomly such that it satisfies the relaxed constraint on the input. The simulation results show that the optimal solution is on the boundary of the constraint set and thus the applied inputs belong to the original constraint set \mathcal{U} .

The results of 1000 Monte Carlo (MC) simulations together with the time and memory requirements are presented in Table 2 and Table 3 for the length of the horizon $F = 3$ and $F = 20$, respectively. The first two columns show the estimate of the criterion and the variance of this estimate computed using a bootstrap technique. As the estimates of the criterion value for the considered methods are close to each other, the methods seem to have a comparable quality of detection in this particular example. The third column shows the off-line time required to design the active fault detector. Note that there is zero off-line time for the MSEM OLF because all computations are performed on-line. The increase in the off-line computation time for the OAFD and $F = 20$ is mainly caused by the change in the discount factor λ and thus the number of iterations of the value iteration algorithm. The fourth column presents the average on-line time requirements during simulations. In all cases it also includes time of simulating the system and performing the state estimation. It can be seen that for a short time horizon the OAFD is the most demanding. However, the MSEM OLF becomes the most time expensive when the time horizon gets longer because of an increased size of the optimization problems that are solved on-line. Moreover, contrary to the MSEM OL and OAFD that have steady on-line

Table 2. Results of MC simulations, computational and memory requirements of the methods for $F = 3$.

method	\hat{J}	var \hat{J}	off-line time [s]	on-line time [s]	memory [bytes]
MSEM OL	0.7116	$0.2730 \cdot 10^{-6}$	0.0334	0.0025	24
MSEM OLF	0.7144	$0.2730 \cdot 10^{-6}$	0	0.0513	≤ 7840
OAFD	0.7103	$0.2724 \cdot 10^{-6}$	91.0127	0.1900	27216

Table 3. Results of MC simulations, computational and memory requirements of the methods for $F=20$.

method	\hat{J}	var \hat{J}	off-line time [s]	on-line time [s]	memory [bytes]
MSEM OL	1.8865	$0.7285 \cdot 10^{-6}$	0.1295	0.0132	160
MSEM OLF	1.8496	$0.6970 \cdot 10^{-6}$	0	1.4449	≤ 255360
OAFD	1.8921	$0.7527 \cdot 10^{-6}$	291.2339	1.0298	27216

time requirements, the on-line time requirements of the MSEM OLF for a longer time horizon fluctuate significantly between 1 [s] and 5.5 [s] depending on the actual state estimate. The last column provides rough estimates of the memory requirements. The value for the MSEM OL corresponds to the memory requirements of storing the whole auxiliary input sequence. In case of the MSEM OLF, the value represents an estimate of the memory requirements when the SQP optimization problem is set up at the first time step. Finally, the memory requirements for the OAFD accounts for storing the sampled hyper-states and corresponding values of the Bellman function. The MSEM OL has the lowest memory requirements. The memory requirements of the MSEM OLF increases rapidly for a longer time horizon whereas it remains the same for the OAFD because the Bellman function approximation is independent of the time horizon length.

5. Conclusion

The paper deals with a comparison of two probabilistic methods for designing an active fault detector. The methods are intentionally chosen for a comparison because they share the same assumptions on the system description to a great extent. Nevertheless, the detection aims of the methods are different and the provided active fault detectors that solve the problem are designed only approximately. Therefore, the comparison of the detection quality is not definite and other factors need to be considered as well. The preliminary results of Monte Carlo simulations indicate that, on a finite time horizon, the OAFD can provide a similar detection quality when compared to the MSEM and may become advantageous for longer time horizons when the on-line time requirements need to be kept reasonably small.

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