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Rotor thermal stress monitoring in steam turbines

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Abstract. One of the issues of steam turbines diagnostics is monitoring of rotor thermal stress that arises from nonuniform temperature field. The effort of steam turbine operator is to operate steam turbine in such conditions, that rotor thermal stress doesn't exceed the specified limits. If rotor thermal stress limits are exceeded for a long time during machine operation, the rotor fatigue life is shortened and this may lead to unexpected machine failure. Thermal stress plays important role during turbine cold startup, when occur the most significant differences of temperatures through rotor cross section. The temperature field can't be measured directly in the entire rotor cross section and standardly the temperature is measured by thermocouple mounted in stator part. From this reason method for numerical solution of partial differential equation of heat propagation through rotor cross section must be combined with method for calculation of temperature on rotor surface. In the first part of this article, the application of finite volume method for calculation of rotor thermal stress is described. The second part of article deals with optimal trend generation of thermal flux, that could be used for optimal machine loading.

Notation

a	thermal diffusivity
c_r	specific heat capacity
n_j, n_i	normal vector
m	mass
$q(t), q^k$	heat flux
Q	heat
dQ	amount of obtained/lost heat
r	radius from rotor (cylinder) center
$R[i]$	radius from rotor center to node i
S	area
S_i, S_{i-1}	i-th control volume surfaces
t	time
Δt	sampling period
T	temperature
$dT, \Delta T$	temperature change
T_i^k	temperature in layer (node) i in time point k
T_n^k	measured temperature (in time point k)
$T\_filt_n^k$	filtered inlet temperature
TS^k	thermal stress of rotor
TS_{max}^k	upper limit of maximum of permissible TS^k
TMI^k	mean integral temperature
V	volume
$V[i]$	volume of element i
z	coordinate corresponding to rotor length
ρ	density of material
φ	rotation angle
τ	time constant of filter

1. Introduction

Currently, the power industry development is divided into two main areas. The first area is aimed to the problem of depletion of non-renewable resources associated with gradually replacement by renewable resources. The second area is aimed to achieving of higher efficiency, durability and reliability of machines producing the electricity power. Both areas are associated with requirement of monitoring and diagnostics of machines, because early detection of possible faults can prevent machine fault and minimize associated economic losses. Some serious problems of steam turbine operation, for example high vibrations due to bend rotor or rotor/stator rub are related to inappropriate heating of rotor during machine startup. The non-uniform thermal field around rotor surface is also associated with rotor thermal stress. Generally, the largest thermal stress occurs during the turbine startup from the cold state – i.e. turbine run-up on the operating speed and especially on the respective nominal power. In ideal situation the machine startup is archived as fast as possible, without threat of machine operation, but thermal stress limits must be taken account. The first aim of this article is development of method for evaluation of thermal stress of the rotor based on temperature measurement in stator part of machine, because it is not possible to measure rotor surface temperature directly. The assumption for evaluation of rotor surface temperature from measurement of temperature in stator part is the same thermal conductivity of rotor and stator part - effects of flowing steam are in this article neglected. Because the only one thermocouple was used for stator temperature measurement, therefore the analysis is made only for 2D case (i.e. rotor cross section). The second aim of this article is development of algorithm for calculation of optimal thermal flux which can be used for evaluation machine run-up RPM and power trend.

2. Calculation of thermal field and thermal stress of the rotor

2.1. Heat equation in cylindrical coordinates

Differential equation of the heat conduction in cylindrical coordinates (for rigid homogeneous substance without internal heat resources), which describes a temperature distribution in the space (i.e. in the cylinder – in the rotor) and across time is as follows [2], [4], [11]:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2}\right), \tag{1}$$

where T is a temperature, t is a time, a is a thermal diffusivity, r is a radius from the cylinder center (rotor), φ is a rotation angle and coordinate z corresponds to the rotor length. Because the subject of interest in our case is only temperature distribution in the cross section of the rotor, the term $\frac{\partial^2 T}{\partial z^2}$ in equation (1) is zero. For 3D case (also in the z-direction) it would be required to use an additional thermocouple. Assumption of rotor thermal symmetry leads to independency of temperature T on angle φ, and equation (1) can be rewritten in following form:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right). \tag{2}$$

Analytical solution of partial differential equations of heat conduction can be easily obtained only for simple cases and therefore we will use the numerical solution. By a spatial discretization – cross section of the rotor on each layer (i.e. annulus, see fig. 1, i-th layer), by a time discretization and by an application of finite volume method the system of algebraic equations for the calculation of temperatures in the individual layers in a given time can be obtained. For completeness the boundary conditions must be defined. The first boundary condition leads from rotor thermal field symmetry:

$$\frac{\partial T}{\partial r} = 0. \tag{3}$$

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The second boundary condition is given by information from thermocouple in stator part. For equal rotor and stator thermal conductivity one can expect the same temperature in appropriate rotor layer.

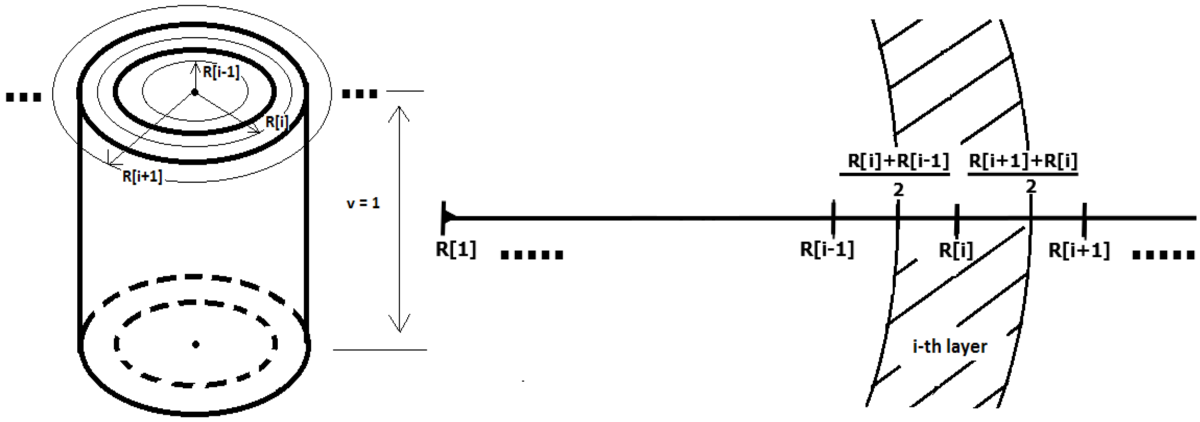

fig. 1: Spatial (rotor) discretization, *i*-th layer

2.2. Finite volume method

In this section will be derived equations for the temperature calculation in the *i*-th layer resp. in the *i*-th node (*i*=2,..,*(n-1)*) [5], [6], [9]. Relations for calculation of temperatures in nodes *1* and (*n+1)* will be derived later. Consider rotor like cylinder with unit height. Let divide this cylinder is into *(n+1)* elements – called control volumes (i.e. annulus with unit height, see fig. 1), which don't overlap each other. In the geometric center of each element (control volume) is located computing node (except for layer (*n+1)*). The temperature at any point of control volume is equal to the temperature in the computing node. Temperature calculation in the rotor cross section (at given time) runs across *(n+1)* nodes, where node *1* corresponds to the rotor geometric center, node *n* corresponds to the thermocouple location and node (*n+1)* corresponds to the rotor surface. Equation (2) can be written in the following form:

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right). \tag{4}$$

By integration over the volume V we get:

$$\int_V \frac{1}{a}\frac{\partial T}{\partial t}dV = \int_V \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)dV. \tag{5}$$

For further derivations the Gauss theorem will be used:

$$\int_V \frac{\partial \phi_j}{\partial x_j}dV = \int_S \phi_j n_j dS, \quad n_j = \left[n_x\, n_y\, n_z\right]^T, \tag{6}$$

where S is the area and n_j is the normal to the area. Term $\frac{\partial T}{\partial t}$ in equation (5) is approximated by the backward difference and on the right side the Gauss theorem is applied:

$$\frac{1}{a}\frac{T_i^k - T_i^{k-1}}{\Delta t}\int_V dV = \int_S \frac{1}{r}r\frac{\partial T}{\partial r}n_r\, dS, \quad n_r = \left[-n_{i-1}\, n_i\,\right]^T \tag{7}$$

Flow through the control volume is given by the sum over the surfaces (through one surface heat enters into the control volume, through another surface heat leaves the element). Previous equation can be rewriten into form:

$$\frac{1}{a}\frac{T_i^k-T_i^{k-1}}{\Delta t}V[i] = -\int_{S_{i-1}}\frac{\partial T}{\partial r}n_{i-1}\,dS + \int_{S_i}\frac{\partial T}{\partial r}n_i\,dS, \tag{8}$$

where Δt is the sampling period, $V[i]$ is the volume of element *i* (*i*=2,..,*(n-1)*) defined by:

$$V[i] = \pi\left(\left(\frac{R[i+1]+R[i]}{2}\right)^2 - \left(\frac{R[i]+R[i-1]}{2}\right)^2\right), \tag{9}$$

and S_i resp. S_{i-1} is the *i*-th control volume surface, through which heat into the element (for example) enters resp. leaves, i.e. it is a surface of the respective cylinder.

$$S_i = \underbrace{2\pi\frac{R[i+1]+R[i]}{2}}_{2\pi r}\underbrace{1}_{v},\ S_{i-1} = \underbrace{2\pi\frac{R[i]+R[i-1]}{2}}_{2\pi r}\underbrace{1}_{v}. \tag{10}$$

Surface integral of partial derivative of temperature in equation (8) can be approximated by a mean value of this function on a given surface:

$$\frac{1}{a}\frac{T_i^k-T_i^{k-1}}{\Delta t}V[i] = -\frac{\partial T_{S[i-1]}}{\partial r}S[i-1] + \frac{\partial T_{S[i]}}{\partial r}S[i] \tag{11}$$

By approximation of partial derivatives in (11) by backward differencies the final relation for calculation of temperature in layer *i* for time step *k* can be obtained:

$$T_i^k = T_i^{k-1} + \frac{a.\Delta t.\pi}{V[i]}\left[\frac{R[i+1]+R[i]}{R[i+1]-R[i]}\left(T_{i+1}^{k-1}-T_i^{k-1}\right) - \frac{R[i]+R[i-1]}{R[i]-R[i-1]}\left(T_i^{k-1}-T_{i-1}^{k-1}\right)\right] \tag{12}$$

2.2.1. *Temperature calculation in the rotor geometrical center*

In following part of the article, the relation for evaluation of temperature in rotor geometric center will be derived. From the assumption rotor thermal symmetry follows that for any time step the temperature over all layers has local or global extrema in the rotor center. The mechanism for derivation of temperature in the rotor center is the same as for inner rotor layers, but now we use simplified partial differential equation:

$$\frac{\partial T}{\partial \tau} = a\frac{\partial^2 T}{\partial r^2}. \tag{13}$$

Let us consider that heat only enters in the first control volume (i.e. cylinder), respectively doesn't flow from it to another element as in the previous case (*i*=2,..,*(n-1)*). Flow through the first control volume is given by the flow through cylinder surface. Let us defined the first element volume:

$$V[1] = \pi\left(\left(\frac{R[2]+R[1]}{2}\right)^2 - R[1]^2\right) \tag{14}$$

and surface through which heat enters/leaves the element:

$$S[1] = \underbrace{2\pi\frac{R[2]+R[1]}{2}}_{2\pi r}\underbrace{1}_{v}. \tag{15}$$

Then is possible to derive the equation for calculation of temperature in the first node (in the geometric rotor center):

$$T_1^k = T_1^{k-1} + \frac{a.\Delta t.\pi}{V[1]}\left[\frac{R[2]+R[1]}{R[2]-R[1]}\left(T_2^{k-1}-T_1^{k-1}\right)\right] \tag{16}$$

2.2.2. *Temperature calculation on the rotor surface*

Finally, it remains to determine the relationship for calculation of temperature in node *(n+1)*. Temperature of this node corresponds to the temperature of rotor surface. It is worthy to mention that the temperature of node *(n+1)* doesn‘t belong to the solution of partial differential equation, because the boundary condition is the temperature in node *n* (value measured by thermocouple). The first manner of temperature calculation of the rotor surface is based on a formally rewritten equation (12) for the layer *n*, i.e. $i = n$:

$$T_n^k = T_n^{k-1} + \frac{a.\Delta t.\pi}{V[n]}\left[\frac{R[n+1]+R[n]}{R[n+1]-R[n]}\left(T_{n+1}^{k-1} - T_n^{k-1}\right) - \frac{R[n]+R[n-1]}{R[n]-R[n-1]}\left(T_n^{k-1} - T_{n-1}^{k-1}\right)\right], \tag{17}$$

where the temperature T_n^k is measured, term T_{n+1}^{k-1} is to be determined and $V[n]$ is given by expression (9). By modifying of previous equation we get a required relationship to the calculation of temperature in the layer *(n+1)*:

$$T_{n+1}^{k-1} = T_n^{k-1} + \left(\frac{R[n+1]-R[n]}{R[n+1]+R[n]}\right)\left[\frac{V[n]}{a.\Delta t.\pi}(T_n^k - T_n^{k-1}) + \frac{R[n]+R[n-1]}{R[n]-R[n-1]}(T_n^{k-1} - T_{n-1}^{k-1})\right] \tag{18}$$

Calculation of temperature in the layer *(n+1)* according to (18) may come across the following problem. Thermal processes are generally slow processes. In the case of rough quantisation level of measured temperature, the values of T_n in two consecutive time steps may equal. If such situation occurs, the resulting evolution of calculated variable T_{n+1} over time shows significant fluctuations (high variance of signal), see the gray curve on fig. 2. This behaviour does not correspond to slow thermal processes.

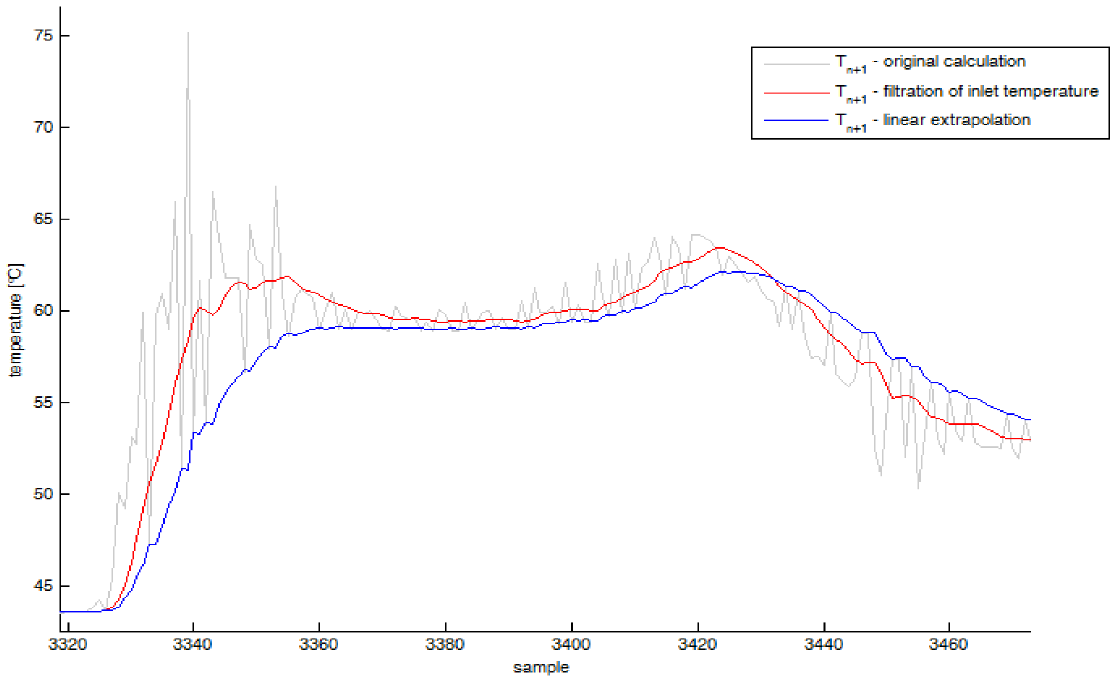

fig. 2: Temperature progress in the *(n+1).* layer

One option for elimination of this problem consists of filtration of measured temperature T_n by appropriate filter. This filtering smooths T_n and avoid the temperature equality in two consecutive time steps. In our work we used second order filter:

$$F(p) = \frac{1}{(\tau p+1)^2}, \tag{19}$$

where τ = 1s. After discretization by Tustin approximation the resulting equation for filtered temperature calculation in time step *k* is as follows:

$$T\_filt_n^k = \frac{-2(\Delta t-2\tau)T\_filt_n^{k-1}}{(2\tau+\Delta t)} - \frac{(\Delta t-2\tau)^2 T\_filt_n^{k-2}}{(2\tau+\Delta t)^2} + \frac{\Delta t^2 T_n^k}{(2\tau+\Delta t)^2} + \frac{2\Delta t^2 T_n^{k-1}}{(2\tau+\Delta t)^2} + \frac{\Delta t^2 T_n^{k-2}}{(2\tau+\Delta t)^2} \tag{20}$$

where T_n^k is the measured temperature, $T\_filt_n^k$ is the filtered inlet temperature and Δt is the sampling period. From fig. 2 it is evident that the resulting temperature T_{n+1} has a significantly smoother character than temperature waveform T_{n+1} in case without filtered T_n, see red curve.

As mentioned, the node *(n + 1)* is located at the border of control volume (not in the geometric center). In the case of finite volume method values are on the border of control volumes obtained by interpolation. Because the node *(n + 1)* is boundary node of the edge control volume, it is necessary to use the extrapolation. Extrapolation was chosen as linear – line passing in the points $R[n]$ and $R[n-1]$ through values T_n and T_{n-1}. The equation for calculating of temperature in the layer *(n+1)* by linear extrapolation is as follows [8]:

$$\underbrace{T_{n+1}^{k-1}}_{y} = \underbrace{\frac{(T_n^{k-1}-T_{n-1}^{k-1})}{(R[n]-R[n-1])}}_{a}\underbrace{R[n+1]}_{x} + \underbrace{T_n^k - \frac{(T_n^{k-1}-T_{n-1}^{k-1})}{(R[n]-R[n-1])}R[n]}_{b} \tag{21}$$

The resulting temperature waveform is shown in fig. 2 (blue curve). From the figure it is evident that the linear extrapolation eliminates the significant fluctuations problem. Trends of both curves (according to (18) and linear extrapolation) are similar. Therefore the linear extrapolation can be also used for calculating of temperature in the layer *(n+1).*

2.3. Explicit and implicit version of finite volume method

In this section the major difference between implicit and explicit version of the above method will be marginally mentioned. Note, that the above method is an explicit version. During deriving of formula for temperature calculation in the layer *i* in the time point *k*, the partial derivative $\frac{\partial T_{S[i-1]}}{\partial r}$ resp. $\frac{\partial T_{S[i]}}{\partial r}$ was approximated by difference, which containes the temperatures T_{i-1}^{k-1}, T_i^{k-1}a T_{i+1}^{k-1} in time *(k-1),* see (12). If the temperatures T_{i-1}^k, T_i^k and T_{i+1}^k (in the time point *k*) were inserted into relevant expression then the calculation of temperature distribution in the rotor cross section would lead to the system of equations (vs. a simpler aforementioned recursive relationship). Both versions of the method have advantages and disadvantages. Of course, solving of the system of equations is more memory and computationally demanding, but on the other hand the stability of the explicit version isn't always guaranteed.

2.4. Thermal stress and mean integral temperature

As mentioned, the thermal stress occurs due to nonuniform temperature field. Thus, let's define thermal stress as a deviation of highest temperature in the rotor cross section from the average of the temperature field [1], [3], [7]. Because the thermal stress is particularly significant during the turbine startup (external rotor heating), it is often assumed that the highest temperature occurs just in the layer *(n+1)*. Thermal stress can be written as a following relationship:

$$TS^k = T_{n+1}^k - TMI^k \tag{22}$$

where TMI^k is the mean integral temperature representing the average temperature across all layers (control volumes). The every control element has different volume – different weight, therefore the calculation of TMI^k is carried out as a weighted average. Mean integral temperature is given by:

$$TMI^k = \frac{\prod_{i=1}^{n+1} T_i^k V[i]}{\sum_{i=1}^{n+1} V[i]} = \frac{T_1^k V[1]+T_2^k V[2]+..+T_n^k V[n]+T_{n+1}^k V[n+1]}{V[1]+V[2]+..+V[n]+V[n+1]} \quad (23)$$

where the volume $V[n+1]$ is:

$$V[n+1] = \pi\left(R[n+1]^2 - \left(\frac{R[n+1]+R[n]}{2}\right)^2\right) \quad (24)$$

2.5. Application and evaluation on the 60 MW steam turbine
Firstly, the application and evaluation is based on the above knowledge of simulated heat propagation in the rotor cross section. This simulation was compared with performed simulation in software Ansys. Note that the results of both simulations were perfectly coincided – the deviations of calculated temperatures (in space and in time) were in the order of hundredths of a °C or less. Then the above knowledges were applied for analysis of signals measured on the 60 MW steam turbine. The application results are shown in data comprising startup of the turbine from the cold state. Note that the rotor cross section was discretized into 26 layers (i.e. *n = 25*) and the thermocouple was located 8 mm under the inner turbine body surface – stator. Calculation of temperature field together with thermal stress was cyclically carried with a period of 0.5 sec (i.e. Δt = 0.5). In fig. 3 the time progresses of temperature in the various layers using color scale (in °C) are shown, where the progress of layer 25 corresponds to the measured values of the thermocouple. Red curve shows the waveform of mean integral temperature *TMI* and the blue one shows the thermal stress *TS*. Due to external rotor heating the *TMI* is gradually increasing and then stabilizes at around 428 °C. Thermal stress increases till 8000 sec., then returns to zero by heating the rotor gradually.

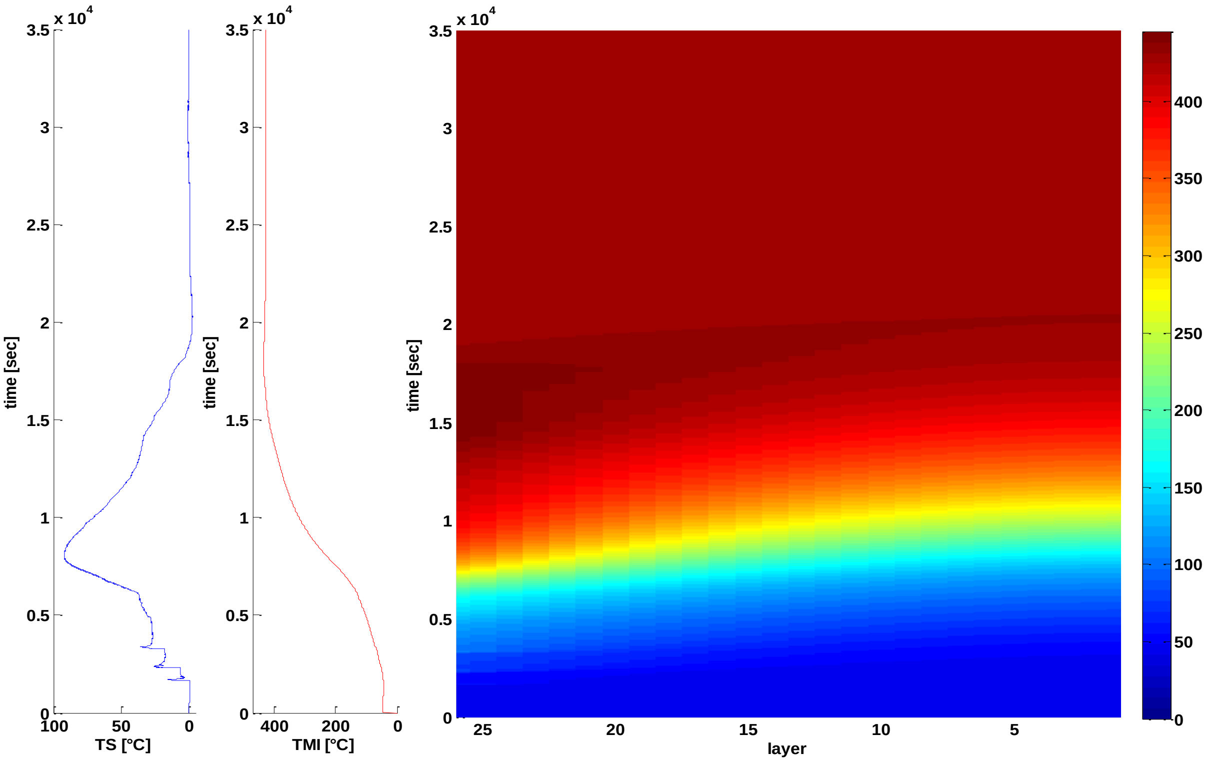

fig. 3: Progress of the rotor temperarture field and *TMI* and *TS* waveforms

3. Generating an optimal trend of steam turbine loading

One of the basic requirements for the turbine startup is achieving of the operating speed and corresponding nominal power in the shortest possible time without exceeding the specified thermal stress limits. Thermal stress is closely related to the amount of obtained/lost heat per time (i.e. change of temperature field). This change describes a heat flux variable [4], [10]:

$$q(t) = \frac{dQ}{dt} \tag{25}$$

Previous formula can be modified and applied to our case (dQ - unknown). From equation for calculating of the specific heat capacity, [11]:

$$c_r = \frac{1}{m}\frac{dQ}{dT} \tag{26}$$

the dQ can be expressed and put into the (25) where c_r is the specific heat capacity and dQ is the amount of heat that the body (in our case rotor) obtain/lost with mass *m* to its temperature changed by dT. Then we can get:

$$q(t) = \frac{c_r.m.dT}{dt} \tag{27}$$

Previous equation can be discreetly rewritten as follows:

$$q^k = \frac{c_r.m.\Delta T}{\Delta t} = \frac{c_r.\rho.V.(T^k - T^{k-1})}{\Delta t} \tag{28}$$

where Δt corresponds to the sampling period ($\Delta t = 0.5$), ρ is the density of material and *V* represents the volume which is given as a sum of all the elementary volumes *V[i]*. Because the temperature field of the rotor is generally inhomogeneous in the time point *k* resp. *k-1*, we can replace the temperature of the body (rotor) T^k resp. T^{k-1} with the average temperature – mean integral temperature TMI^k resp. TMI^{k-1}. Equation (28) can be rewritten into the following form:

$$q^k = \frac{c_r.\rho.\sum_{i=1}^{n+1} V[i].\left(\frac{\prod_{i=1}^{n+1} T_i^k V[i]}{\sum_{i=1}^{n+1} V[i]} - \frac{\prod_{i=1}^{n+1} T_i^{k-1} V[i]}{\sum_{i=1}^{n+1} V[i]}\right)}{\Delta t} = \frac{c_r.\rho.\left(\prod_{i=1}^{n+1} T_i^k V[i] - \prod_{i=1}^{n+1} T_i^{k-1} V[i]\right)}{\Delta t} \tag{29}$$

According to equation (29) the heat flux into the rotor based on the known temperature field in the time *k* can be calculated, thus the real increase (or decrease) of heat in the rotor per time Δt. Note that the volume *V* (resp. partial volumes *V[i]*) is defined as a cylinder with unit height in our case. Then the variable q^k (see (29)) can also be called as the linear heat flux density.

As mentioned, the thermal stress is related to the time change of heat in the rotor. The formula (29) will be utilized to generate such a trend of heat flux (resp. linear heat flux density), that would be optimal from the turbine startup point of view – i.e. the actual rotor thermal stress reaches the upper boundary of the established limit (without exceeding this limit). Because there must be satisfied a number of operational rules during the turbine startup, then the optimal value of heat flux can be generated just one time step Δt forward. In the each time point k it is therefore necessary to calculate the value of the heat flux q^{k+1}, which will lead in the following time point *(k+1)* to equality of actual thermal stress TS^{k+1} with the desired value TS_{max}^{k+1} (e.g. the upper limit of thermal stress):

$$q^{k+1} = \frac{c_r.\rho.\left(\prod_{i=1}^{n+1} T_i^{k+1} V[i] - \prod_{i=1}^{n+1} T_i^k V[i]\right)}{\Delta t} \tag{30}$$

The temperatures T_i^k *(i=1,..,n+1)* in the above relationship are known. However, the problem occurs with temperatures T_i^{k+1} *(i=1,..,n+1)*, because the future progress of temperature field leading to equality $TS^{k+1} = TS_{max}^{k+1}$ is unknown. Let's start from the equation for calculation of thermal stress in the time *(k+1)*.

$$TS^{k+1} = T_{n+1}^{k+1} - TMI^{k+1} \tag{31}$$

Using TMI^{k+1} according to (23) and $TS^{k+1} = TS_{max}^{k+1}$ in the previous equation we will get:

$$TS_{max}^{k+1} = T_{n+1}^{k+1} - \frac{\prod_{i=1}^{n+1} T_i^{k+1} V[i]}{\sum_{i=1}^{n+1} V[i]} \tag{32}$$

The unknown variables in equation (32) are again the temperatures T_i^{k+1} *(i=1,..,n+1)*. Notice that the future progress of temperatures $T_1^{k+1}, \ldots, T_{n-1}^{k+1}$ can be forward counted in the time point *k* in the case of explicit version, see equation (12). The same can be performed in the case of implicit version, but the calculation again will lead to solve the equation system. In formula (32) only variables T_n^{k+1} and T_{n+1}^{k+1} are remaining as unknown. If T_{n+1} is also function of T_n:

$$T_{n+1}^{k+1} = f(T_n^{k+1}), \tag{33}$$

where *f(.)* is a general function, e.g linear extrapolation (other expression than (21)):

$$T_{n+1}^{k+1} = T_n^{k+1} \frac{R[n+1]-R[n-1]}{R[n]-R[n-1]} - T_{n-1}^{k+1} \frac{R[n+1]-R[n]}{R[n]-R[n-1]} \tag{34}$$

then it is possible to substitute the T_{n+1}^{k+1} in equation (32) and express remaining unknown T_n^{k+1}. Specifically, in the case of linear extrapolation we will get:

$$T_n^{k+1} = \frac{TS_{max}^{k+1}(R[n]-R[n-1])(\sum_{i=1}^{n+1} V[i]) - T_{n-1}^{k+1}(R[n]-R[n+1])(\sum_{i=1}^{n+1} V[i])}{(R[n+1]-R[n-1])(\sum_{i=1}^{n+1} V[i]) - V[n](R[n]-R[n-1]) - V[n+1](R[n+1]-R[n-1])} + \frac{(\prod_{i=1}^{n-1} T_i^{k+1} V[i])(R[n]-R[n-1]) - V[n+1].T[n-1](R[n]-R[n+1])}{(R[n+1]-R[n-1])(\sum_{i=1}^{n+1} V[i]) - V[n](R[n]-R[n-1]) - V[n+1](R[n+1]-R[n-1])} \tag{35}$$

Finally, it remains to calculate the optimal value of the heat flux q^{k+1} in the time point *k*, see (30), because all temperatures T_i^{k+1} *(i=1,..,n+1)* are already known (T_{n+1}^{k+1} was calculate according to (33) resp.(34)). If the actual heat flux into the rotor is successively equal to the generated optimum value during the turbine startup (by regulation of amount and temperature of steam), then the actual rotor thermal stress moves around the desired value. This situation is demonstrated in fig. 4, where the blue line shows the actual waveform of the rotor thermal stress and the red one shows the desired waveform of thermal stress (gray line represents the waveform of temperature T_n). For the desired waveform of thermal stress the upper limit of the maximum of permissible thermal stress was elected. Note that the maximum of steam temperature is limited, therefore the temperature T_n also stabilizes at the appropriate value, see fig. 4. Of course, the thermal stress returns to zero after the entire rotor is heated. The heat flux into the rotor is influenced by the temperature and amount of steam – regulation of temperature and pressure.

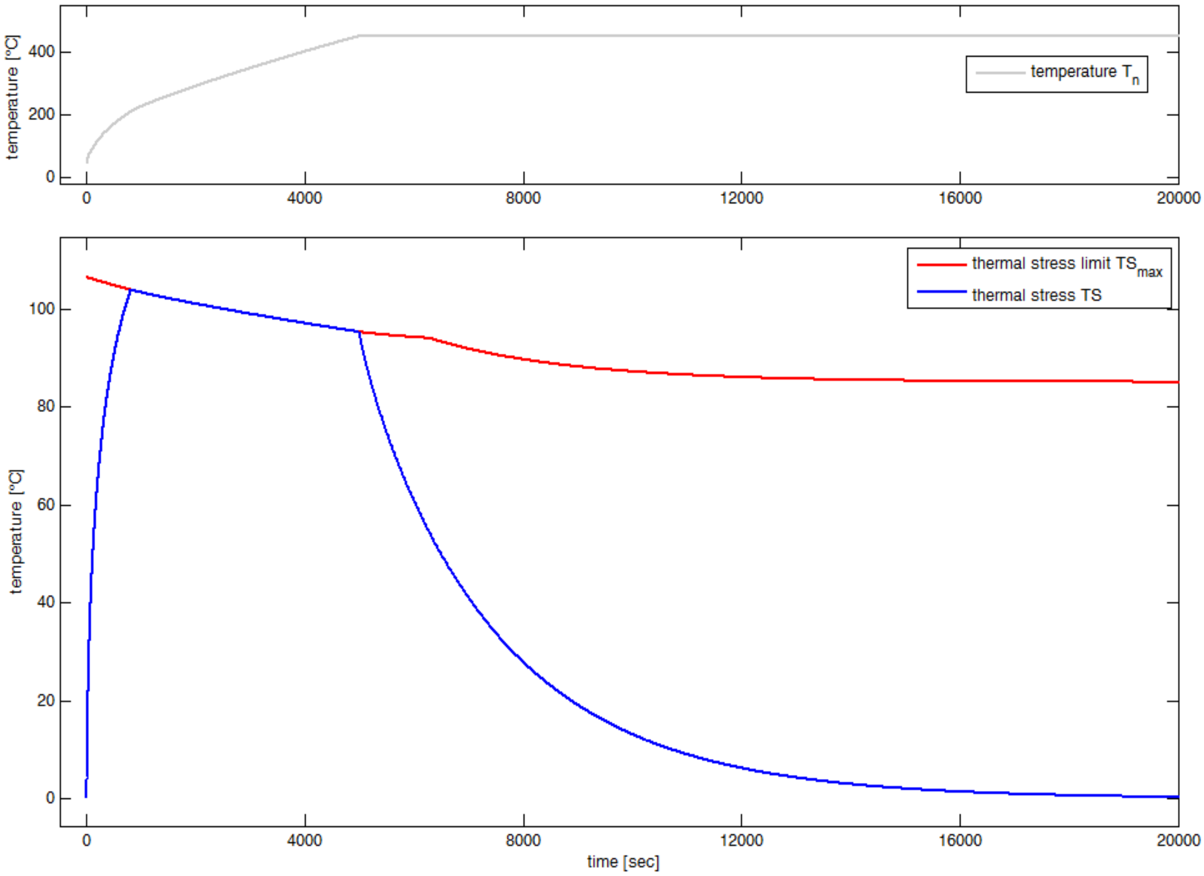

fig. 4: Optimal startup of turbine

4. Conclusion

This paper deals with the calculation of thermal stress of the rotor and generation of optimal steam turbine loading trend leading to equality of the actual thermal stress with the desired value. The entire problem is based on the calculation of the temperature field in the rotor cross section based on the temperature measurement in the inner turbine body – i.e. in the stator. First, using the finite volume method the partial differential equations of heat conduction is solved. The formula for calculation of temperatures in the inner layers of the rotor was obtained in its geometric center and on the rotor surface. Then there was also described the formula for calculation of thermal stress. The obtained calculation methods were afterthat applied to analyse signals from the 60 MW steam turbine. Finally, a method for generation of the optimum heat flux trend or linear heat flux density into the rotor, that leads to the desired behavior of actual thermal stress during the turbine startup, was proposed. Heat flux into the rotor is influenced by temperature and pressure of steam in the turbine.

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