

Fault Detection in the Blade and Pitch System of a Wind Turbine with \mathcal{H}_2 PI Observers

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Abstract. In this work, we present a fault detection strategy applicable to the blade and pitch system in offshore wind turbines. First, we model the system and possible faults and propose a PI observer to identify the faults. Then, the observer is designed accounting the sensors measurement noise, and addressing a trade off between the needs of false alarm rate, minimum detectable fault and detection time. By means of a well known benchmark, several simulations show the goodness of the approach and its flexibility to explicitly fix the fault detector performance.

1. Introduction

As a difference with respect to other energy production technologies, wind turbines are exposed to extremal and varying environmental conditions mainly due to wind nature. As a consequence, they are under permanently changing loads, what results in operation conditions with high variability hindering its correct operation. Recent studies, [1,2], show an increasing tendency on installing high power turbines, what results in a larger development of offshore wind farms. Its placement and size present a higher technological challenge than conventional wind turbines.

The periods of inactivity due to malfunctioning in wind turbines is in the order of a 3% of its life cycle, and the maintenance costs take between 10% and 30% of the total costs of a wind farm project [3]. A wind turbine consists of three main subsystems: the blade and pitch system, responsible of regulating the amount of wind energy that is transformed into mechanical one; the rotor, gathered by a drive train to the electrical generator, where the mechanical energy becomes electric; and the controller, who manages the operation of the previous systems. The authors in [4] explain that the pitch system is the main responsible of the downtimes of the machine.

In order to detect irregular operation of actuators and sensors, the control system can use fault detection techniques (FD) [5]. This methods will help the wind farm operator to detect operating problems and will help to avoid triggering critical faults. Thus, with FD not only downtimes can be reduced, but also the maintenance costs. The FD problem consists in the generation of residual signal that must be insensitive to noise measurements and disturbances, and a strategy to decide whether a fault has occurred by analysing the residual signal.

In this work, we assume that we have a sufficiently accurate model of the blade and pitch system and use a model-based FD technique [6–11]. We model the faults as additive signals, and the residual signal consists of an estimation of the fault signals. The design strategy of the fault detector consists in attenuating the \mathcal{H}_2 norm between both measurement noise and fault signal,



and the fault estimation error. This norm is formalized via linear matrix inequalities. First, we define the required performance metrics as well as the detection algorithm. Then, using these requirements, we present a design strategy based on an optimization problem.

The structure of this work is as follows. First, we state the problem in Section 2, where we include a mathematical model of the blade and pitch system and the considered fault signals. In Section 2.2 we propose a strategy to detect these faults, and specify the requirements to account in the detector design in Section 3. In Section 4 we include some simulation results and, finally, in Section 5 we summarize the main conclusions.

2. Problem Description

The problem of fault detection of this work is focused on a real problem which deals with the blade and pitch system of an offshore wind turbine. The wind turbine's model is based on the one proposed by Odgaard et al. at [12], which describes a three-bladed variable-speed offshore wind turbine. The complexity of this model is sufficient enough to achieve the precision required to meet the FD requirements. The relevant faults that affect the blade and pitch system are also specified in [12], which is referred for further modeling details. Hereafter, we define the necessary elements for the design of the detector.

2.1. System

Due to its hydraulic character, the close-loop operation of each of the blade and pitch systems can be modeled as a second order system. For ease of notation and due to the similarity between all three systems, we will only work with one of them. The transfer function between the real pitch angle, β , and the corresponding reference angle, β_{ref} , given by the controller, results in

$$\frac{\beta(s)}{\beta_{ref}(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}. \quad (1)$$

This dynamics may vary due to either a drop in pressure in the hydraulic supply system or high air content in the oil. In both cases, these faults will result in a variation of parameters ω_n and ξ . Thus, we rewrite this parameters as $\omega_n = \omega_{n0} + \Delta\omega_n$ and $\xi = \xi_0 + \Delta\xi$, where ω_{n0} and ξ_0 refer to the non-faulty operation and $\Delta\omega_n$ and $\Delta\xi$ describe the respective variations. Thus, the relation between the pitch angle and its reference taking into account the possible dynamics variation (faulty situation) can be realized in state-space form as follows:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\omega_{n0}^2 & -2\xi_0\omega_{n0} \end{bmatrix} x + \begin{bmatrix} 0 \\ \omega_{n0}^2 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f_a, \quad (2)$$

where $x = [\beta \ \dot{\beta}]^T$ represents the state vector and $u = \beta_{ref}$ is the input to the system. f_a is an additive signal that represents the faulty operation of the actuator (see [13]) and, in case of actuator faults, its value will be

$$f_{\dot{\beta}} = -2(\xi_0\Delta\omega_n + \omega_{n0}\Delta\xi + \Delta\xi\Delta\omega_n)\dot{\beta} + (\Delta\omega_n^2 + 2\omega_{n0}\Delta\omega_n)(\beta_{ref} - \beta). \quad (3)$$

In order to improve the system's reliability, each of the pitch angles is measured with two different sensors β_{m1} and β_{m2} . Each sensor is modeled by the sum of the actual variable measure, β_i , and a zero-mean gaussian noise of variance $\sigma_{\beta_m}^2$. These sensors can experiment either mechanical or electrical faults, what results, respectively, in a gain factor or an offset. An additive fault signal will take into account both situations and the measurements equation of the system results in

$$y = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} f_s + v, \quad (4)$$

with $y = [\beta_{m_1} \ \beta_{m_2}]^T$ the measurement vector and $v = [v_1 \ v_2]^T$ the white noises associated to each of the measurements. The additive signal f_s represents the malfunctioning of either of the sensors; so that, $f_s = f_{\beta_{m_1}}$ when the fault occurs in sensor β_{m_1} and $f_s = -f_{\beta_{m_2}}$ when the wrong measurements become from sensor β_{m_2} . If we assume that no simultaneous faults occur, this consideration will not influence the results.

Discretizing (2) and (4), the discrete state-space model results in

$$x_{k+1} = A_0 x_k + B_0 u_k + E_0 f_{a,k}, \quad (5)$$

$$y_k = C_0 x_k + H_0 f_{s,k} + v_k. \quad (6)$$

In the current work, we model the faults as slow-varying signals, i.e.,

$$f_{k+1} = f_k + \Delta f_k, \quad (7)$$

where f_k represents the vector $[f_{a,k} \ f_{s,k}]^T$ and Δf_k the corresponding variation of the fault from instant k to $k + 1$. Equation (7) allows modelling, for instance, step signals (Δf_k only takes a non-zero value at the fault appearance) or ramp signals (Δf_k takes a constant value). This kind of fault model have been widely used in the literature to analyze the behaviour of fault detection algorithms [13, 14]). Finally, we introduce an extended order model to include the fault dynamics as

$$z_{k+1} = A z_k + B u_k + B_f \Delta f_k, \quad (8)$$

$$y_k = C z_k + v_k, \quad (9)$$

where $z_k = [x_k^T \ f_k^T]^T$ is the extended state vector and

$$A = \begin{bmatrix} A_0 & [E_0 \ 0] \\ 0 & I \end{bmatrix}, \quad B = \begin{bmatrix} B_0 \\ 0 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad C = [C_0 \ [0 \ H_0]].$$

2.2. Fault Detector

To reach fault detection objectives, we propose a scheme that relies on model-based observers. Specifically, we propose the following algorithm that estimates the extended state of the system,

$$\hat{z}_k = \hat{z}_{k-1} + L(y_k - C \hat{z}_{k-1}), \quad (10)$$

where L is the gain matrix that defines the relationship between the new measurements and the prediction obtained from the model. Defining the estimation error as $\tilde{z}_k = z_k - \hat{z}_k$, its dynamics is given by

$$\tilde{z}_{k+1} = (I - LC) A \tilde{z}_k - L v_k + (I - LC) B_f \Delta f_k \quad (11)$$

The estimation error of the fault signals is then $\tilde{f}_k = C_f \tilde{z}_k$. Following the approach developed at [15], we propose to use the estimated faults ($\hat{f}_k = C_f \hat{z}_k$ with $C_f = [0 \ I]$) to compute a quadratic residual at each instant k as

$$r_k = \hat{f}_k^T F^{-1} \hat{f}_k. \quad (12)$$

In all, the fault detection decision is given by

$$\begin{cases} \text{if } r_k \leq \delta & \text{No fault} \\ \text{if } r_k > \delta & \text{Fault} \end{cases} \quad (13)$$

where δ is a threshold to be defined. This means that fault detection is achieved by means of estimating the fault signals that affect the states and the measurements.

To evaluate the performance of a fault detector we should take into account different parameters. Firstly, according to [16, 17] the minimum detectable fault is defined as the fault that drives the residual to its threshold when no other fault occurs. We denote the minimum detectable fault of channel l as $f_{\min,l}$.

Note 1. According to the previous definition, given (12) and (13) and assuming no estimation error ($\hat{f}_k = f_k$), the minimum detectable fault satisfies $f_{\min,l}^2 F^{-1}(l, l) = \delta$, for $l = 1, \dots, n_f$, where n_f is the number of faults that affect the system and $F^{-1}(l, l)$ is the l -th diagonal element of matrix F^{-1} .

Secondly, the False Alarm Rate (FAR) is defined in [16] as the average probability of rising false alarms over an infinite-time window, i.e.,

$$\phi = \lim_{K \rightarrow \infty} \frac{1}{K} \sum_{k=0}^{K-1} \Pr\{r_k > \delta \mid f_k = 0\}. \quad (14)$$

Note 2. Markov's inequality states that given x any positive random variable and $a > 0$, the inequality $\Pr\{x > a\} \leq \frac{\mathbf{E}\{x\}}{a}$ holds. Using this inequality and the definition of the FAR (14), we have that the FAR (14) of (12)-(13) is bounded by

$$\phi \leq \frac{1}{\delta} \mathbf{E}\{r_k \mid f_k = 0\}. \quad (15)$$

Thirdly, we define Integral of the Squared Error (ISE) of a fault estimation as

$$\varphi_l = \sum_{k=1}^{\infty} \tilde{f}_k^2(l). \quad (16)$$

The ISE characterizes the temporal behaviour of the detector against faults in accordance to the evaluation of disturbance rejection in control systems.

The design of a detector implies a trade-off between the FAR, the minimum detectable fault and the ISE. The objective of this work is to provide a method to compute the gain matrix L that satisfies this trade-off in a desirable and optimal way.

3. Design of the detector

To design matrix L , which regulates the estimator, and matrix F , which defines the residual, we will follow an approach based on the \mathcal{H}_2 norm of (11).

Theorem 1. Consider the estimation algorithm (10) applied to system (8)-(9). If there exist any symmetric matrices P , F , Γ_f , Γ_v and a full matrix X fulfilling

$$\begin{bmatrix} P & \mathcal{A} & 0 \\ \mathcal{A}^T & P & C_f^T \\ 0 & C_f & F \end{bmatrix} \succeq 0, \quad \begin{bmatrix} P & \mathcal{B}_\mu \\ \mathcal{B}_\mu^T & \Gamma_\mu \end{bmatrix} \succeq 0, \quad \mu = \{f, v\}, \quad (17)$$

with $\mathcal{A} = (P - X C) A$, $\mathcal{B}_f = (P - X C) B_f$ and $\mathcal{B}_v = -X$; then, defining the observer gain matrix as $L = P^{-1} X$, the following statements hold:

- In the absence of faults and measurement noises the extended state estimation error (11) converges to zero.

- The residual signal (12) is bounded by

$$\mathbf{E}\{\|r\|_{RMS}\} \leq \text{tr}(\Gamma_v V) + \text{tr}(\Gamma_f \Delta_f), \quad (18)$$

where $\Delta_f = \mathbf{E}\{\Delta f_k \Delta f_k^T\}$; then, in the absence of faults, the residual signal satisfies $\mathbf{E}\{\|r\|_{RMS}\} \leq \text{tr}(\Gamma_v V)$.

Proof. See Appendix A.1. □

The next theorem extends the results of the previous one to bound the FAR of the residual.

Theorem 2. For a given threshold $\delta > 0$ and a positive number $\bar{\phi} \in (0, 1)$, under the premisses of Theorem 1 if

$$\text{tr}(\Gamma_v V) \leq \bar{\phi} \delta, \quad (19)$$

is fulfilled then, the following additional statement holds:

- In the absence of faults and under null initial conditions, the fault detection algorithm assures a FAR bounded by $\bar{\phi}$.

Proof. See Appendix A.2. □

Finally, the following theorem extends the previous results by bounding the ISE of the estimation of the faults.

Theorem 3. Given a certain value $\bar{\varphi}_l > 0$, if the premisses of Theorem 1 are satisfied and the constraint

$$F(l, l) \Gamma_f(l, l) \leq \bar{\varphi}_l, \quad (20)$$

is fulfilled, where $F(l, l)$ and $\Gamma_f(l, l)$ denote the l -th diagonal element of matrices F and Γ_f , then, the following additional statement holds:

- In the absence of faults and under null initial conditions, the ISE of each element of the fault estimation vector, $\hat{f}_k(l)$, is bounded by $\bar{\varphi}_l$.

Proof. See Appendix A.3. □

Note 3. From the previous theorems we deduce that, given a certain threshold δ , the parameters $\bar{\phi}$ and F bound, respectively, the FAR and the minimum detectable faults. This determines the variability of the estimation signal caused by the noise in the absence of faults. Besides, the parameters $\bar{\varphi}_l$ ($l = 1, \dots, n_f$) bound the ISE after step faults in the absence of noise, i.e., they are proportional to the time of detection. For a certain value of F , the elements of matrix Γ_f allow to bound the ISE, i.e., to modify the time of detection.

Depending on the desired performance of the detector, its design should be done following different strategies. Thereby, for a certain FAR, we may look either for the minimal estimation error ensuring certain minimum detectable faults or for the minimal value of the minimum detectable faults provided a certain estimation error. Since we do not require any determined estimation error in this case, we will fix the minimum detectable faults ($\bar{f}_{\min, l}$) and minimize the ISE, what we can write as:

$$\begin{aligned} & \text{minimize } \gamma \\ & \text{subject to } \mathcal{X}_1 = \begin{cases} (17), (19), \text{tr}(\Gamma_f) \leq \gamma, \\ F \preceq \mathcal{F} \end{cases} \end{aligned} \quad (21)$$

along the variables $\gamma, \Gamma_f, F, \Gamma_v, P$ y X , where \mathcal{F} is the matrix whose diagonal elements contain the minimum detectable faults, i.e., $\mathcal{F}(l, l) = \delta^{-1} \bar{f}_{\min, l}^2$.

It should be noticed that Note 2 uses Markov's inequality to bound the FAR. As it is well-known, this bound may be too conservative (see [18]) because it does not take into account the kind of probability distribution of the bounded signal. Under the assumption of gaussian noises, the estimation error of a fault signal will also follow a gaussian distribution. Thus, if we know this distribution we can recalculate a new threshold that will give us a tighten bound of the FAR.

Theorem 4. *Given a detector designed as shown in (21), if the threshold in (13) is redefined as*

$$\underline{\delta} = \delta \delta_{\Sigma} \underline{\lambda}(F) / \bar{\lambda}(\Sigma_{\hat{f}}), \quad (22)$$

then $\bar{\phi}$ is a tighten bound of the FAR. Here, δ is the parameter used in (19) and

$$\delta_{\Sigma} = CDF_{\chi_{n_f}^2}^{-1}(1 - \bar{\phi}), \quad (23)$$

with $CDF_{\chi_{n_f}^2}^{-1}$ the inverse cumulative distribution function of a chi-squared random variable with n_f degrees of freedom. $\underline{\lambda}(F)$ and $\bar{\lambda}(\Sigma_{\hat{f}})$ represent, respectively, the minimum and maximum eigenvalues of F and matrix $\Sigma_{\hat{f}}$ defined as

$$\text{vec}(\Sigma_{\hat{f}}) = (I - (I - LC) A \otimes (I - LC) A)^{-1} (L \otimes L) \text{vec}(V). \quad (24)$$

Proof. See Appendix Appendix A.4. □

Note 4. *According to Theorem 1, if the detector (12) uses the threshold $\underline{\delta}$ instead of δ , the minimum detectable faults turn out to satisfy $f_{\min,l} f_{\min,l}^2 F^{-1}(l, l) = \underline{\delta}$. These values will be smaller to those $\bar{f}_{\min,l}^2$ initially included in problem (21). To achieve a minimum detectable fault close to $\bar{f}_{\min,l}^2$ when we use $\underline{\delta}$ as a threshold we can perform an iterative procedure by incrementing \mathcal{F} in (21).*

4. Simulations

In this section, we apply the FD strategy presented herein before to the blade and pitch system of a wind turbine. For this purpose, we use the parameters recommended in [6,12], i.e., $\xi_0 = 0.6$, $w_{n,0} = 11.11$ rad/s and $\sigma_{\beta_m} = 0.2^\circ$. The sampling time used in the simulations is $T_s = 0.01$ s. We will simulate an operation of 35 s with a fault affecting the actuator and each of the sensors. Thus, we can write the simulated fault signals as:

$$f_k = \begin{cases} \begin{bmatrix} \epsilon & 0 \end{bmatrix}^T & \text{if } 2 \leq t < 4 \\ \begin{bmatrix} 0 & \epsilon \end{bmatrix}^T & \text{if } 14 \leq t < 16 \\ \begin{bmatrix} 0 & -\epsilon \end{bmatrix}^T & \text{if } 25 \leq t < 27 \\ \begin{bmatrix} 0 & 0 \end{bmatrix}^T & \text{otherwise} \end{cases}, \quad (25)$$

where $\epsilon = 0.25$.

Figure 1 shows the simulation results of detectors with different guaranteed performances. Detector 1 has been designed with a FAR restricted to $\bar{\phi}_1 = 2.5 \cdot 10^{-6}$ and minimum detectable faults smaller than $\mathcal{F}_1(l, l) = 0.05$. For the same minimum detectable faults, $\mathcal{F}_2(l, l) = 0.05$, and a FAR limited to $\bar{\phi}_2 = 1 \cdot 10^{-6}$ we have obtained Detector 2. Finally, Detector 3 satisfies the same FAR as Detector 1, $\bar{\phi}_3 = 2.5 \cdot 10^{-6}$, and detects faults bigger than $\mathcal{F}_2(l, l) = 0.03$.

If we compare the performance of detectors 1 and 2 ($\mathcal{F}_1(l, l) = \mathcal{F}_2(l, l)$ and $\bar{\phi}_1 > \bar{\phi}_2$) we verify that the ISE increases when we try to estimate the same faults with a lower ISE ($\varphi_2(l) =$

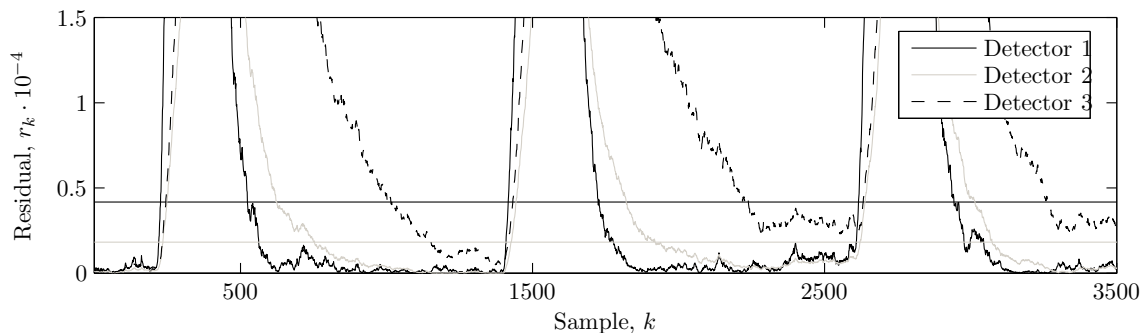


Figure 1. Residual signal and threshold of designs with different performances.

$2.31 \varphi_1(l)$). This increment of the ISE with the same minimum detectable faults means a higher time of detection. Similarly, if we compare detectors 1 and 3 ($\mathcal{F}_1(l, l) > \mathcal{F}_3(l, l)$ and $\bar{\phi}_1 = \bar{\phi}_3$) we observe that the ISE increases when we try to detect smaller faults ($\varphi_3(l) = 4.55 \varphi_1(l)$). This increment of ISE in the case of equal minimum detectable faults implies a larger time of detection.

5. Conclusion

In this work, we have developed a strategy to design a fault detection algorithm for the blade and pitch system of a wind turbine. The main goal of the approach is to provide a model-based observer that ensures an optimal detection while it guarantees certain performance requirements. Thus, we have proposed a design algorithm based on optimization techniques over the \mathcal{H}_2 norm that permits to fix, depending on the needs, the false alarm rate and the speed of detection, the accurateness of the estimations or the minimum detectable faults that will set off an alarm. The proposed technique has been validated on an international benchmark. Further research may extend this approach to other systems of the wind turbine such as the one that integrates the rotor, the drive train and the generator, including also uncertainties on the model.

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Appendix A. Proofs of Theorems

Appendix A.1. Proof of Theorem 1

Let us define a Lyapunov function at each instant k as $V_k = \tilde{z}_k^T P \tilde{z}_k$

- In the absence of noises or faults, after taking Schurs complements on the first linear matrix inequality presented in (17), premultiplying the result by \tilde{z}_k^T and postmultiplying by its transpose, we obtain that $V_{k+1} - V_k \leq 0$ what assures that the extended state estimation error (11) converges to zero.
- Performing similar steps on the other linear matrix inequalities in (17) (Schurs complements and operations with v_k^T y Δf_k^T), taking expected value on the results and adding the obtained constraints with the previous one, we get

$$\mathbf{E}\{V_{k+1}\} - \mathbf{E}\{V_k\} + \mathbf{E}\{\tilde{f}_k^T F^{-1} \tilde{f}_k\} - \mathbf{E}\{v_{k+1}^T \Gamma_v v_{k+1}\} - \Delta f_k^T \Gamma_f \Delta f_k \leq 0, \quad (\text{A.1})$$

where we have considered the non correlation between \tilde{x}_k , v_{k+1} and Δf_k . Let w be a stochastic vector with mean μ and a covariance matrix W ; if P is a symmetric matrix, then $\mathbf{E}\{w^T P w\} = \mu^T P \mu + \text{tr}(P W)$. Considering null initial conditions, applying the previous result to (A.1) and adding the result from $k = 0$ to $K - 1$, then

$$\sum_{k=0}^{K-1} \mathbf{E}\{\tilde{f}_k^T F^{-1} \tilde{f}_k\} \leq \sum_{k=0}^{K-1} \Delta x_k^T \Gamma_f \Delta x_k + K \text{tr}(\Gamma_v V)$$

where we have taken into account that $P \succeq 0$, $\mathbf{E}\{v_k\} = 0$ and $\sum_{k=0}^{K-1} \text{tr}(\Gamma_v V) = K \text{tr}(\Gamma_v V)$. Dividing the above expressions by K and taking the limit when $K \rightarrow \infty$, it leads to (18), which concludes this proof.

Appendix A.2. Proof of Theorem 2

Following Theorem 1, if the system is fault-free, we have $\mathbf{E}\{\|r\|_{RMS}\} \leq \text{tr}(\Gamma_v V)$. Applying the results of Lemma 2, we have $\phi \leq \delta^{-1} \text{tr}(\Gamma_v V)$. Then, no matter the probabilistic distribution of r_k , the FAR (14) is bounded by $\bar{\phi}$ when constraint (19) is satisfied.

Appendix A.3. Proof of Theorem 3

As stated in Theorem 1, under null initial conditions we have

$$\sum_{k=0}^{\infty} \tilde{f}_k^T F^{-1} \tilde{f}_k \leq \sum_{k=0}^{\infty} \Delta f_k^T \Gamma_f \Delta f_k$$

when no noise exist. If a fault $f(l)$ appears and we can model it as a unitary step signal ($\Delta f_k(l) = 1$ at instant $k = 0$) then, $\sum_{k=0}^{\infty} \tilde{f}_k^2(l) \leq \gamma_{f_l} f_l^2 \Delta f_0(l)$ and the ISE is bounded by

$$\sum_{k=0}^{\infty} \tilde{f}_k^2(l) \leq \gamma_{f_l} f_l^2. \quad (\text{A.2})$$

Then, bounding the product $\Gamma_f F$ we restrict the ISE of the fault estimations $f(l)$, what proves (20).

Appendix A.4. Proof of Theorem 4

We define $\Sigma_{\hat{f}}$ as the covariance matrix of the fault estimation signal, i.e., $\Sigma_{\hat{f}_k} = \mathbf{E}\{\hat{f}_k^T \hat{f}_k\}$. In the absence of faults, $\Sigma_{\hat{f}_k} = \mathbf{E}\{\tilde{f}_k^T \tilde{f}_k\}$, so that by the equation (11) we arrive to the expression included in (24).

In the absence of faults, a signal defined as $s_k = \hat{f}_k^T \Sigma_{\hat{f}}^{-1} \hat{f}_k$ follows a chi-squared distribution of n_f degrees of freedom, $\chi_{n_f}^2$; so, by definition,

$$\mathbf{Pr}(\hat{f}_k^T \Sigma_{\hat{f}}^{-1} \hat{f}_k > \delta_{\Sigma}) = \bar{\phi}.$$

Given a matrix M , we have $\underline{\lambda}(M) I \prec M \prec \bar{\lambda}(M) I$; thus we can bound s_k by

$$s_k = \hat{f}_k^T \Sigma_{\hat{f}}^{-1} \hat{f}_k > \hat{f}_k^T \hat{f}_k \underline{\lambda}(\Sigma_{\hat{f}}^{-1}),$$

and the residual r_k (defined in (12)) with

$$r_k = \hat{f}_k^T F^{-1} \hat{f}_k < \hat{f}_k^T \hat{f}_k \bar{\lambda}(F^{-1}).$$

From these two bounds we establish the relationship

$$s_k > \frac{\lambda(\Sigma_f^{-1})}{\lambda(F^{-1})} r_k.$$

For a signal $s'_k < s_k$ we have that $\Pr(s'_k > \delta_\Sigma) < \bar{\phi}$, what demonstrates that the threshold (22) bounds the FAR by $\bar{\phi}$.

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